The $\cos\alpha$ method for X-ray residual stress measurement using two-dimensional detector

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Abstract

X-ray stress measurement is widely used as one of the most powerful nondestructive tools to measure residual stresses in polycrystalline solids. In most cases, the $\sin^2\psi$ method has been used to determine the stress. In recent years, however, the $\cos\alpha$ method has attracted engineers as a new method to measure the stress using two-dimensional detectors, such as imaging plates. The present article is the review of the state of the art of the $\cos\alpha$ method. For biaxial stress cases, the $\cos\alpha$ method utilizes the whole Debye-Scherrer ring recorded on a two-dimensional detector taken by single exposure of X-rays, and normal and shear stresses are determined simultaneously. The accuracy of the stress measurement of the $\cos\alpha$ method has been confirmed to be equivalent to that of the $\sin^2\psi$ method for various metals. The simple optical system of the $\cos\alpha$ method makes stress analyzers smaller, lighter and more convenient to use for on-site or field measurements. A recent portable stress analyzer adopting the $\cos\alpha$ method shortens the measurement time to 60 s. The method has been further developed to analyze triaxial residual stresses. Various advantages of the $\cos\alpha$ method are highlighted in comparison with the other methods of X-ray stress determination. Applications of the $\cos\alpha$ method to machines and engineering structures are presented, together with future perspectives of the method.

Keywords : X-ray stress measurement, The $\cos\alpha$ method, Two-dimensional detector, Imaging plate, Residual stress, Debye-Scherrer ring, X-ray single exposure, Triaxial stress

1. Introduction

Residual stresses have been playing an important role in fatigue and fracture accidents of machines and engineering structures, and also in their dimensional stability. Almost all machines and structures have residual stresses and their unexpected failure or distortion have often been ascribed to residual stresses. The prediction of residual stresses is sometimes very difficult and the development of measurement techniques is highly demanded.

The X-ray diffraction method is known as one of the most powerful nondestructive tools to measure residual stresses in polycrystalline materials and have been widely used in service as well as in laboratory researches (Noyan and Cohen, 1987, Tanaka et al., 2006). The fundamental principle of X-ray stress measurement is based on the diffraction of X-rays by crystals. The stress is determined from the lattice strain measured by X-ray diffraction using the theory of elasticity. In the measurement of residual stresses in polycrystals, the $\sin^2\psi$ method has been commonly used (Macherauch and Müller, 1961). The stress is obtained from the slope of the relation of the diffraction angle or lattice strain plotted against $\sin^2\psi$, where the diffraction angle is measured at several tilt angles ψ of X-ray incidence. In order to meet the demand of on-site measurements, several commercial stress analyzers have been produced, making them smaller, lighter, handier and quicker to obtain the stress. X-ray detectors used in these analyzers based on the $\sin^2\psi$ method are zero-dimensional detectors such as scintillation counters, or one-dimensional detectors such as position sensitive proportional counters (PSPCs) (Tanaka et al., 2006).

In the past two decades, two-dimensional detectors such as imaging plates (IPs), semiconductor detectors or 2D-PSPCs have been gradually used in commercial X-ray stress analyzers. One of them uses IPs and adopts the $\cos\alpha$ method

for stress determination. The principle of the $\cos\alpha$ method was first proposed by Taira, Tanaka, and Yamasaki (1978) for in-plane biaxial stress analysis. The method utilizes the whole Debye-Scherrer ring (D-S ring) recorded on a two-dimensional detector taken by single exposure of X-rays, and normal and shear stresses are obtained simultaneously. Using X-ray films as detectors, they applied the method to evaluate the local stress near fatigue crack tips. In 1980s, IPs were produced by Fuji Photo Film Co., Ltd. as a new X-ray detector. In 1990s, Yoshioka *et al.* (1992) introduced IPs for stress measurement by the $\cos\alpha$ method. Sasaki and Hirose (1995b) extended the method to triaxial stress analysis. In 2012, a first commercial stress analyzer based on the $\cos\alpha$ method was produced by Pulstec Industrial Co., Ltd. The simple optical system of the $\cos\alpha$ method makes stress analyzers portable, smaller, lighter and more convenient to use for on-site and field measurements. With the use of a recent $\cos\alpha$ -based analyzer, the time required for one stress measurement is shortened to 60 s. The $\cos\alpha$ method is a genuinely Japanese original product in the sense that both software (fundamental theory) and hardware (IP and equipment) were born in Japan. In recent years, the method has attracted engineers as a new method of X-ray stress measurement abroad (Ling and Lee, 2015, Robinson and Redington, 2015, Ramitec-Rico *et al.*, 2016) as well as in Japan (Tanaka, 2017), and applications of the method have been spreading to various engineering fields.

In this article, recent developments of stress measurement by the $\cos\alpha$ method using IPs are reviewed. The fundamentals of X-ray stress measurement are first described, and then a recommended procedure of experimental measurements of the $\cos\alpha$ method is summarized. Various advantages of the $\cos\alpha$ method are highlighted in comparison with the other methods of X-ray stress determination. Finally, applications of the $\cos\alpha$ method to the parts of machines and engineering structures are presented, together with future perspectives of the $\cos\alpha$ method.

2. Fundamentals of X-ray stress measurement

2.1 Strain measured by X-rays

Strain measurement by X-rays is based on Bragg diffraction by crystals. When a monochromatic X-ray beam is incident on polycrystals as shown in Fig. 1, the diffraction by crystals takes place at the angles satisfying

$$\lambda = 2d \sin \theta \quad (1)$$

where λ is the wavelength, d is the spacing of diffraction planes, and θ is the diffraction angle. Under stress, the lattice spacing changes from the stress-free value, d_0 , to $d_0 + \Delta d$. Since the wavelength λ is constant, the strain in the direction of the normal of the diffraction plane is obtained from the change of the diffraction angle as

$$\varepsilon = \frac{\Delta d}{d_0} = -\cot \theta_0 (\theta - \theta_0) \quad (2)$$

where θ_0 is the diffraction angle of stress-free crystals and θ is that for strained crystals. When the diffraction comes from many grains with random crystal orientation, the lattice strain measured by X-ray diffraction corresponds to macroscopic elastic strain and is related to the stress by isotropic elasticity.

Fig. 1 Strain measurement by X-ray diffraction.

Fig. 2 Normal strain in the x - y - z coordinates of specimen measured by X-ray diffraction.

Fig. 3 Debye-Scherrer ring recorded on a two-dimensional detector by a single-exposure of X-rays.

Figure 2 shows the direction, OP, of the normal strain measured by X-ray diffraction in the x - y - z coordinates where the x - y coordinates are on the specimen surface and the z -axis is the surface normal. The diffraction vector, $\mathbf{n} = (n_1, n_2, n_3)$, of the plane normal for the case of the tilt angle ψ and the rotation angle ϕ is given by

$$n_1 = \sin \psi \cos \phi, \quad n_2 = \sin \psi \sin \phi, \quad n_3 = \cos \psi \quad (3)$$

The strain $\varepsilon_{\phi\psi}$ is related to the strain components in the x - y - z coordinates as follows:

$$\varepsilon_{\phi\psi} = \varepsilon_x n_1^2 + \varepsilon_y n_2^2 + \varepsilon_z n_3^2 + \gamma_{yz} n_2 n_3 + \gamma_{zx} n_3 n_1 + \gamma_{xy} n_1 n_2 \quad (4),$$

where $\varepsilon_x, \varepsilon_y, \varepsilon_z$ are normal strains and $\gamma_{yz}, \gamma_{zx}, \gamma_{xy}$ are shear strains. By using Hooke's law of isotropic elasticity, the equation is expressed in terms of normal stresses, $\sigma_x, \sigma_y, \sigma_z$, and shear stresses, $\tau_{xy}, \tau_{yz}, \tau_{zx}$, as

$$\varepsilon_{\phi\psi} = \frac{1+\nu}{E} (\sigma_x n_1^2 + \sigma_y n_2^2 + \sigma_z n_3^2 + 2\tau_{yz} n_2 n_3 + 2\tau_{zx} n_3 n_1 + 2\tau_{xy} n_1 n_2) - \frac{\nu}{E} (\sigma_x + \sigma_y + \sigma_z) \quad (5)$$

where E is Young's modulus and ν is Poisson's ratio. The above equation is the fundamental of X-ray stress measurement by the $\cos \alpha$ method as well as the $\sin^2 \psi$ method. Using Eq. (5), the stress is determined from measured strains $\varepsilon_{\phi\psi}$ in several different directions of ϕ and ψ . In the $\sin^2 \psi$ method, strains are determined from peak positions of diffraction profiles recorded at several tilt angles ψ with zero- and one-dimensional detectors.

In the $\cos \alpha$ method, two-dimensional detectors are used to detect X-ray diffraction. Figure 3 shows an experimental set-up for recording a D-S ring when the X-ray beam is incident on the specimen surface at the tilt angle ψ_0 and the rotation angle ϕ_0 . The diffraction vector of the plane normal for the azimuthal angle α taken clockwise from the $-\eta$ direction of the D-S ring on the IP is expressed as follows (Taira *et al.*, 1978, Sasaki and Hirose, 1995b):

$$\begin{aligned} n_1 &= \cos \eta \sin \psi_0 \cos \phi_0 - \sin \eta \cos \psi_0 \cos \phi_0 \cos \alpha - \sin \eta \sin \phi_0 \sin \alpha \\ n_2 &= \cos \eta \sin \psi_0 \sin \phi_0 - \sin \eta \cos \psi_0 \sin \phi_0 \cos \alpha + \sin \eta \cos \phi_0 \sin \alpha \\ n_3 &= \cos \eta \cos \psi_0 + \sin \eta \sin \psi_0 \cos \alpha \end{aligned} \quad (6)$$

where 2η is the complimentary angle of 2θ . In the $\cos \alpha$ method, the strain $\varepsilon_{\phi\psi}$ measured by X-ray diffraction is expressed by ε_α because α specifies the direction of the measured strain.

The strain is determined from the radius of D-S rings recorded on an IP. (Hereafter, IP is used as a representative of two-dimensional detectors). Figure 4 illustrates the D-S ring obtained by the X-ray beam impinging at the tilt angle of ψ_0 , where the IP is placed perpendicular to incident X-ray beam. The diffraction angle θ is obtained from the ring radius r_α as follows:

$$2\theta = \pi - \tan^{-1} (r_\alpha / L) \quad (7)$$

where L is the distance between the IP and the specimen. From Eqs. (2) and (7), the strain ε_α at the azimuthal angle α is

expressed as

$$\varepsilon_\alpha = \frac{\cos^2 2\theta_0}{2L \tan \theta_0} (r_\alpha - r_0) \quad (8)$$

where r_α is the radius of the D-S ring at angle α and r_0 is the radius for stress-free crystals. As described later, the fundamental equations of the $\cos\alpha$ method do not require the exact value of stress-free diffraction angle to determine the stress. They only require relative difference of the radius of D-S rings. Therefore, the distance L between the IP and the specimen in Eq. (8) can be approximated by the value L_m which is calculated from the mean radius, r_m , of D-S rings by

$$L_m = r_m / \tan 2\eta_0 \quad (9)$$

where $2\eta_0 = \pi - 2\theta_0$. The strain, ε_α , is now determined by

$$\varepsilon_\alpha = \frac{\cos^2 2\theta_0}{2L_m \tan \theta_0} (r_\alpha - r_0) \quad (10)$$

Fig. 4 Strain measurement of a Debye-Scherrer ring in the $\cos\alpha$ method.

2.2 Determination of in-plane biaxial stresses by the $\cos\alpha$ method

In this section, the principle of the $\cos\alpha$ method for in-plane biaxial stress analysis is described. Since the penetration depth of X-rays below the surface is shallow (see Fig. 8), the stress state in most cases can be assumed to be biaxial, *i.e.* $\sigma_z = \tau_{yz} = \tau_{zx} = 0$. Furthermore, material is assumed to be isotropic and to have a uniform stress within the penetration depth of X-rays below the surface.

In the $\cos\alpha$ method, a special technique of data acquisition is adopted to minimize possible experimental error (Taira *et al.*, 1978). As shown in Fig. 4, a set of four strains, ε_α , $\varepsilon_{\pi+\alpha}$, $\varepsilon_{-\alpha}$, $\varepsilon_{\pi-\alpha}$, are measured for each α from the radius of D-S rings using Eq. (10). Two subtractions of strains, $(\varepsilon_\alpha - \varepsilon_{\pi+\alpha})$ and $(\varepsilon_{-\alpha} - \varepsilon_{\pi-\alpha})$, are used to determine the following two strain parameters. The first parameter is the mean of $(\varepsilon_\alpha - \varepsilon_{\pi+\alpha})$ and $(\varepsilon_{-\alpha} - \varepsilon_{\pi-\alpha})$:

$$\varepsilon_{\alpha 1} \equiv [(\varepsilon_\alpha - \varepsilon_{\pi+\alpha}) + (\varepsilon_{-\alpha} - \varepsilon_{\pi-\alpha})] / 2 \quad (11)$$

The second one is half the difference:

$$\varepsilon_{\alpha 2} \equiv [(\varepsilon_\alpha - \varepsilon_{\pi+\alpha}) - (\varepsilon_{-\alpha} - \varepsilon_{\pi-\alpha})] / 2 \quad (12)$$

Note that both $\varepsilon_{\alpha 1}$ and $\varepsilon_{\alpha 2}$ are determined from the relative difference of diffraction angles or radii and not directly dependent on stress-free value of θ_0 or r_0 , as seen from the following expression for $\varepsilon_{\alpha 1}$.

$$\varepsilon_{\alpha 1} = -\frac{(\theta_\alpha - \theta_{\pi+\alpha}) + (\theta_{-\alpha} - \theta_{\pi-\alpha})}{2} \cot \theta_0 = \frac{\cos^2 2\theta_0}{4L_m \tan \theta_0} [(r_\alpha - r_{\pi+\alpha}) + (r_{-\alpha} - r_{\pi-\alpha})] \quad (13)$$

This is very important for the accuracy of stress measurements since it is in most cases difficult to determine exactly the diffraction angle of stress-free state.

For the case of rotation angle $\varphi_0 = 0$, the first strain parameter is related only to the normal stress σ_x as

$$\varepsilon_{\alpha 1} = -\frac{1+\nu}{E} \sigma_x \sin 2\eta \sin 2\psi_0 \cos \alpha \quad (14)$$

From this relation, the normal stress σ_x is determined from the slope, M_1 , of the linear relation between $\varepsilon_{\alpha 1}$ and $\cos \alpha$ as

$$\sigma_x = K_1 \frac{\partial \varepsilon_{\alpha 1}}{\partial \cos \alpha} = K_1 M_1 \quad (15)$$

$$K_1 = -\frac{E}{1+\nu} \frac{1}{\sin 2\eta \sin 2\psi_0} \quad (16)$$

The second strain parameter is related to the shear stress τ_{xy} as

$$\varepsilon_{\alpha 2} = \frac{2(1+\nu)}{E} \tau_{xy} \sin 2\eta \sin \psi_0 \sin \alpha \quad (17)$$

Similarly, the shear stress τ_{xy} is determined from the slope, M_2 , of the linear relation between $\varepsilon_{\alpha 2}$ and $\sin \alpha$ as follows:

$$\tau_{xy} = K_2 \frac{\partial \varepsilon_{\alpha 2}}{\partial \sin \alpha} = K_2 M_2 \quad (18)$$

$$K_2 = \frac{E}{2(1+\nu)} \frac{1}{\sin 2\eta \sin \psi_0} \quad (19)$$

In the above equations, K_1 and K_2 are called the stress constant; K_1 is negative and K_2 is positive.

Figure 5 illustrates $\cos \alpha$ and $\sin \alpha$ diagrams. From the slope of the regression line, the normal and shear stresses are determined. For a tensile stress, the slope M_1 of $\cos \alpha$ diagram is negative. The strain parameter $\varepsilon_{\alpha 1}$ is zero at $\cos \alpha = 0$ and minimum at $\cos \alpha = 1$. For a positive shear stress, the slope M_2 of $\sin \alpha$ diagram is positive. The strain parameter $\varepsilon_{\alpha 2}$ is zero at $\sin \alpha = 0$, and is maximum at $\sin \alpha = 1$. The deviation from the regression line can be used as an indicator of reliability of the measured stress.

Sasaki and Hirose (1995a) proposed a method to determine all three stress components of in-plane biaxial stress field from a D-S ring obtained by single exposure. Further, Miyazaki and Sasaki (2015) noticed that ε_{α} was given as a linear combination of $\cos \alpha$, $\sin \alpha$, $\cos 2\alpha$, $\sin 2\alpha$, and proposed a method to determine stresses, σ_x and τ_{xy} , on the bases of Fourier analysis of the relation of ε_{α} as a function of α . They claimed their Fourier method applicable to imperfect D-S rings (Fujimoto *et al.*, 2015).

The elastic constant, $(1+\nu)/E$, in stress constants K_1 and K_2 is the X-ray value, often called diffraction elastic constant, which is usually different from mechanical value and dependent on the diffraction plane as well as crystals. When reliable data of the X-ray elastic constant is not available, the value can be calculated by the Kröner's model (1958) from single crystal elastic constants. Experimentally, the X-ray elastic constant, $(1+\nu)/E$, is determined from the change of slope M_1 with the applied stress, σ_A , by using the following equation (Shimize *et al.*, 2014):

$$\frac{1+\nu}{E} = -\frac{1}{\sin 2\eta \sin 2\psi_0} \cdot \frac{\partial M_1}{\partial \sigma_A} \quad (20)$$

(a) The $\cos \alpha$ diagram for σ_x determination

(b) The $\sin \alpha$ diagram for τ_{xy} determination.

Fig. 5 The $\cos \alpha$ and $\sin \alpha$ diagrams.

2.3 Determination of triaxial stresses by the $\cos\alpha$ method

The stress measured by X-rays is the weighted average of the stress distributed within the penetration depth of X-rays below the surface (Tanaka *et al.*, 2006). In X-ray stress measurements, non-zero values of out-of-plane stresses, σ_z , τ_{yz} and τ_{zx} , are often measured for composite materials (Tanaka *et al.*, 1992), uni-directionally machined surfaces (Hanabusa and Fujiwara, 1982, Wakashima *et al.*, 1983), and bearing surfaces fatigued by rolling contact (Kamura *et al.*, 2015, 2018). For triaxial stress cases, the first and second strain parameters at the rotation angle $\varphi_0 = 0^\circ$ in Fig. 3 are related to the stress as follows (Sasaki and Hirose, 1995b, Sasaki *et al.*, 2009):

$$\varepsilon_{\alpha 1} = -\frac{1+\nu}{E} [(\sigma_x - \sigma_z) \sin 2\psi_0 + 2\tau_{zx} \cos 2\psi_0] \sin 2\eta \cos \alpha \quad (21)$$

$$\varepsilon_{\alpha 2} = 2 \frac{1+\nu}{E} [\tau_{xy} \sin \psi_0 + \tau_{yz} \cos \psi_0] \sin 2\eta \sin \alpha \quad (22)$$

For normal incidence, $\psi_0 = 0$, the first term in the square bracket of the above two equations disappears, and then following equations are derived (Sasaki *et al.*, 2009):

$$\tau_{zx} = -\frac{E}{2(1+\nu)} \frac{1}{\sin 2\eta} \left(\frac{\partial \varepsilon_{\alpha 1}}{\partial \cos \alpha} \right) = K_0 \left(\frac{\partial \varepsilon_{\alpha 1}}{\partial \cos \alpha} \right) \quad (23)$$

$$\tau_{yz} = \frac{E}{2(1+\nu)} \frac{1}{\sin 2\eta} \left(\frac{\partial \varepsilon_{\alpha 2}}{\partial \sin \alpha} \right) = -K_0 \left(\frac{\partial \varepsilon_{\alpha 2}}{\partial \sin \alpha} \right) \quad (24)$$

$$K_0 = -\frac{E}{2(1+\nu)} \frac{1}{\sin 2\eta} \quad (25)$$

where K_0 is the stress constant. Out-of-plane shear stresses, τ_{zx} and τ_{yz} , are determined from $\cos\alpha$ and $\sin\alpha$ diagrams.

In order to determine the other stress components, we have to obtain D-S rings for four arrangements, IP-2, IP-3, IP-4 and IP-5, as shown in Fig. 6 (Sasaki *et al.*, 2009). IP-1 is for normal incidence, and the other four, IP-2 to IP-5, are for oblique incidence. The rotation angle φ_0 is set to be four values: 0, 90, 180, and 270°. For the case of $\varphi_0 = 0^\circ$, the slopes of the $\cos\alpha$ and $\sin\alpha$ diagrams are related to stresses as follows:

$$\sigma(0) \equiv (\sigma_x - \sigma_z) + 2\tau_{zx} \cot 2\psi_0 = K_1 \frac{\partial \varepsilon_{\alpha 1}(0)}{\partial \cos \alpha} = K_1 M_1(0) \quad (26)$$

$$\tau(0) \equiv \tau_{xy} + \tau_{yz} \cot \psi_0 = K_2 \frac{\partial \varepsilon_{\alpha 2}(0)}{\partial \sin \alpha} = K_2 M_2(0) \quad (27)$$

where $\sigma(0)$ and $\tau(0)$ are used for stresses determined from $\cos\alpha$ and $\sin\alpha$ diagrams in order to distinguish them from those of the biaxial case, and (0) attached is to indicate the values for $\varphi_0 = 0^\circ$. The stress constants, K_1 and K_2 , are defined by Eqs. (16) and (19). Similar equations are derived for the case of $\varphi_0 = 180^\circ$. From these relations for the 0-180° pair, three components of stresses, σ_x - σ_y , τ_{xy} , τ_{yz} , are determined by

Fig. 6 Arrangement of five IPs for triaxial stress measurement.

$$\begin{aligned}
\sigma_x - \sigma_z &= [\sigma(0) + \sigma(180)]/2 \\
\tau_{xy} &= [\tau(0) + \tau(180)]/2 \\
\tau_{yz} &= [\tau(0) - \tau(180)] \tan \psi_0 / 2
\end{aligned} \tag{28}$$

Similar relations are derived for the case of the 90-270° pair,

$$\begin{aligned}
\sigma_y - \sigma_z &= [\sigma(90) + \sigma(270)]/2 \\
\tau_{xy} &= -[\tau(90) + \tau(270)]/2 \\
\tau_{zx} &= -[\tau(90) - \tau(270)] \tan \psi_0 / 2
\end{aligned} \tag{29}$$

On the basis of the reliability of measured stresses, τ_{xy} and τ_{yz} are better determined from $\sin \alpha$ diagrams of the 0-180° pair, and τ_{xy} and τ_{zx} are from those of the 90-270° pair (Tanaka, 2018). For τ_{xy} , the mean of two calculated values is adopted. The out-of-plane shear stresses, τ_{yz} and τ_{zx} , are also determined from D-S rings obtained for normal incidence.

The out-of-plane normal stress, σ_z , has to be measured to determine in-plane normal stresses, σ_x and σ_y , because only deviatoric components of the normal stress are obtained from $\cos \alpha$ and $\sin \alpha$ diagrams. Tanaka (2018) proposed a method to determine σ_z using double exposure under normal incidence of X-rays as described below.

The mean strain along the whole circumference of D-S ring is determined by

$$\bar{\varepsilon}_\alpha = \frac{1}{2\pi} \int_0^{2\pi} \varepsilon_\alpha d\alpha \tag{30}$$

The relation between the stress σ_z and the mean strain $\bar{\varepsilon}_\alpha$ for normal incidence is obtained as

$$\sigma_z = \frac{E}{1-2\nu} \bar{\varepsilon}_\alpha - \frac{(1+\nu) \cos^2 \theta_0 - 2\nu}{2(1-2\nu)} \cdot [(\sigma_x - \sigma_z) + (\sigma_y - \sigma_z)] \tag{31}$$

The terms of normal stress difference, $(\sigma_x - \sigma_z)$ and $(\sigma_y - \sigma_z)$, are already obtained by the $\cos \alpha$ method described above, so the mean strain has to be measured to determine σ_z . From Eqs. (10) and (30), the mean strain is determined from the difference in radius between the mean value, r_m , of the D-S rings and the stress-free value, r_0 , as

$$\bar{\varepsilon}_\alpha = \frac{\cos^2 2\theta_0}{2L_m \tan \theta_0} (r_m - r_0) = -\frac{\sin 4\theta_0}{4L_m \tan \theta_0} (L_m - L_0) = \frac{\sin 4\theta_0}{4 \tan \theta_0} \left(\frac{L_0}{L_m} - 1 \right) \tag{32}$$

where L_0 is the true value of IP-specimen distance, $L_0 = r_0 / \tan 2\eta_0$, and L_m is the distance calculated from the mean radius r_m by Eq. (9). Figure 7 illustrates IP recording D-S rings at normal incidence in a double-exposure experiment.

Fig. 7 Double exposure for out-of-plane normal stress determination before and after specimen movement. L_0 is the true value of IP-specimen distance and L_m is the converted distance from the mean radius r_m . 2η is the complimentary angle of the diffraction angle 2θ , and suffix 0 indicates those of stress-free values.

After the first D-S ring has been recorded, the specimen is moved by distance Δz and the second D-S ring is recorded. By using the difference of the radii of two D-S rings, Δr_m , and $\Delta L_m = \Delta r_m / \tan 2\eta$, L_0 is expressed as

$$L_0 = r_m / \tan 2\eta = (\Delta z / \Delta r_m) \cdot r_m = (\Delta z / \Delta L_m) \cdot L_m \quad (33)$$

The substitution of Eq. (33) into Eq. (32) gives

$$\bar{\varepsilon}_\alpha = \frac{\sin 4\theta_0}{4 \tan \theta_0} \left(\frac{\Delta z}{\Delta L_m} - 1 \right) \quad (34)$$

3. Measurement procedure

3.1 Experimental set-up and the determination of distance between IP and specimen

The strain is calculated from the radius of D-S rings recorded on an IP which is placed perpendicular to the incident X-ray beam (see Fig. 4). In strain measurement, the center of D-S rings should be located precisely. The distance between the IP and the specimen is required for strain calculation. In published reports, standard powders were sometimes used to determine the center and the IP-specimen distance. For example, in strain measurements using Fe 211 diffraction, Cr powder (Taira *et al.*, 2018) or Ag powders (Akiniwa *et al.*, 2004) were recorded on the same IP, because the diffractions of Cr 211 or Ag 311 were close to Fe 211 diffraction. In commercial $\cos\alpha$ -based analyzers, the distance is calculated from the mean radius of D-S rings using Eq. (9).

3.2 Penetration depth and Lorenz polarization factor

Before determining the diffraction angle or strain, the measured profile of X-ray diffraction intensity is corrected by Lorenz polarization (LP) and absorption factors, and also by geometrical factor (Tanaka, 2017). The penetration depth of X-rays for the $\cos\alpha$ method is obtained as

$$T(\psi_0, 2\theta, \alpha) = \frac{1}{\mu} \left[\frac{1}{\cos \psi_0} + \frac{1}{-\cos 2\theta \cos \psi_0 + \sin 2\theta \sin \psi_0 \cos \alpha} \right]^{-1} \quad (35)$$

where μ is the linear absorption coefficient. Figure 8 show the change of the penetration depth with azimuthal angle α for Fe 211 diffraction by Cr-K α radiation calculated using $\mu = 890 \text{ cm}^{-1}$. The depth decreases with increasing incident angle ψ_0 , and for a given ψ_0 it is the maximum at $\alpha = 0^\circ$ and 360° ($-\eta$ side) and the minimum at $\alpha = 180^\circ$ ($+\eta$ side).

Measured diffraction profiles are the convolution of diffractions by K α 1 and K α 2. In stress measurements, smoothing of profiles is normally carried out. Then, angular resolution is decreased and doublet peak of diffractions is not separated. The peak position of profiles is determined as a single peak. The method of the determination of peak position in Japanese standard JSMS-SD-10-05 (2005) is the center of the full-breadth at half-maximum.

Fig. 8 Variation of penetration depth with tilt angle ψ_0 of X-ray incidence (Tanaka, 2018).

4. Applications of the $\cos\alpha$ method

4.1 Advantages of the $\cos\alpha$ method

In comparison with the other methods of X-ray stress measurement, the advantages of the $\cos\alpha$ method using two-dimensional detectors are listed as follows:

- (1) The exact value of the stress-free diffraction angle is not required for stress determination.
- (2) By single exposure of X-rays, in-plane normal and shear stresses can be obtained from the slopes of $\cos\alpha$ and $\sin\alpha$ diagrams, respectively. The linearity of these diagrams indicates the reliability of measured stress values.
- (3) Because of single exposure, the optical system is simple and does not require scanning mechanism of goniometer. Stress analyzers become smaller, lighter, and more convenient to use for on-site or field measurements.
- (4) The time required for one measurement is shorter, because of single exposure.
- (5) It is suitable for stress measurement at local small area and narrow portion of structures, because the irradiated area does not change owing to single exposure.
- (6) Measurement of triaxial residual stresses is much simpler and quicker, compared with the Dölle-Hauk method based on the $\sin^2\psi$ method.
- (7) Features of D-S rings provide microstructural information such as grain sizes and texture.

Items (1) and (2) are common advantages to the $\sin^2\psi$ method, and furthermore, in the $\cos\alpha$ method, in-plane shear stress is obtained simultaneously. Items (3) and (4) are superior points of the $\cos\alpha$ method, enabling quick, multiple or 100% on-site inspection. Because of Item (5), it is unnecessary to re-adjust the irradiated area in measurement, which is suitable for stress measurement at local area such as notches or cracks (Akiniwa *et al.*, 2004). Item (7) is characteristic of two-dimensional detectors. Features of D-S rings will give valuable information of microstructure. It is also possible to record simultaneously multiple diffractions from constituent phases (Sasaki *et al.*, 1996) or to determine the amount of retained austenite in steels by single exposure.

Since a pinhole collimator is adopted in the optical system in $\cos\alpha$ -based stress analyzers, the irradiated area of X-rays is around a few millimeters in diameter. Continuous D-S rings are sometimes difficult to obtain for the cases of coarse grained materials. The limited irradiated area of the $\cos\alpha$ method sometimes hampers accurate stress determination of coarse grained materials. Various oscillation techniques are used to overcome this difficulty as described later.

Figure 9 shows examples of D-S rings of Fe 211 taken from a carbon steel (JIS-S50C) using Cr-K α radiation under normal incidence, where the full-width at half-maximum (FWHM) averaged around the whole circumference of rings is indicated in the caption of each figure (Tanaka, 2017, 2018). In (a) for annealed surface steel, the variation of intensity around the circumference is large because of coarse grains. The intensity variation is reduced very much in (b) fine-particle bombarded (FPB) surface, suggesting no texture development by FPB. In contrast, some texture development is detected by non-uniform intensity distribution of D-S rings as seen in (c) milled surface and (d) planed surfaces. The FWHM is smallest for annealed surface, and increases with increasing degree of plastic deformation.

Fig. 9 Debye-Scherrer rings of various machined surfaces of carbon steel (S50C). FMHM is the full-width at half maximum averaged around the whole circumference of rings (Tanaka, 2017, 2018).

4.2 Reliability of the measured stress value

One way to judge the reliability of the stress measured by the $\cos\alpha$ method is to compare the measured stress value with that determined by the $\sin^2\psi$ method. Another way is to measure the stress on the surface of four-point bent specimens, and to compare the stress data with those calculated elastically from the applied load. In the early stage of the development of the $\cos\alpha$ method, the equipment was built up in each laboratory, while, in recent years, commercial $\cos\alpha$ -based analyzers are used. The accuracy of stress measurement by the $\cos\alpha$ method is proved to be equivalent to that by the $\sin^2\psi$ method for various metallic materials, such as steels (Shimizu *et al.*, 2014, Kohri *et al.*, 2014., Maruyama *et al.*, 2015, Ling and Lee, 2015, Ramirec-Rice *et al.*, 2016), stainless steels (Kohri *et al.*, 2014), nickel alloys (Kohri *et al.*, 2014), and aluminum alloys (Kohri *et al.*, 2014, Nozue *et al.*, 2015).

The stress on the polished surface of fine-particle bombarded (FPB) S50C steel plate was measured by the $\cos\alpha$ method under the application of four-point bending load (Tanaka, 2017). Diffraction of Fe 211 by Cr-K α was used for stress measurement. Figure 10 shows $\cos\alpha$ diagrams taken at several applied stresses, σ_A . Good linearity between $\varepsilon_{\alpha 1}$ and $\cos\alpha$ is seen at each stress, even though there is a small deviation. The linear regression line is inclined upward to the right, indicating the compressive stress. The slope is decreased with increasing applied tensile stress. The stress value, σ_x (MPa), calculated from the slope multiplied with the stress constant $K_1 = -465$ GPa is plotted against the applied stress, σ_A (MPa), in Fig. 11. The linear regression gives

$$\sigma_x = 0.98\sigma_A - 362 \quad (36)$$

where the constant term, -362 MPa, is the compressive residual stress due to FPB. The proportional constant of 0.98 confirms the accuracy of the stress measurement by the $\cos\alpha$ method. Similar good correlation between the shear stress determined from $\sin\alpha$ diagrams and the applied shear stress has been reported (Tanaka, 2017). In these experiments, it should be noted that the proportional constant in the relation between the X-ray measured stress and the applied stress is very sensitive to the roughness of the specimen surface.

Fig. 10 The $\cos\alpha$ diagram of FPB peened steel subjected to tension (Tanaka, 2017).

Fig. 11 Change of X-ray stress, σ_x , with applied stress for FPB peened steel (Tanaka, 2017).

4.3 Stress measurement of coarse-grained materials

The $\cos\alpha$ method assumes D-S rings continuous. However, D-S rings are sometimes spotty when the irradiated area does not contain enough number of grains because of coarse grain sizes or small irradiated area. To obtain continuous D-S rings, the irradiated area has to contain enough number of diffracting grains. For the case of the $\sin^2\psi$ method using zero-dimensional detectors, the minimum number of grains within X-ray irradiated area required for reliable stress measurements is known to be around 1000 to 5000 (Tominaga and Akiniwa, 2014). This number may depend on grain misorientation and measurement conditions. Since it is rather difficult to expand the irradiated area in the $\cos\alpha$ method, several oscillation techniques have been used to overcome the difficulty of stress measurement due to spotty rings.

Four techniques have been proposed to increase the number of diffracting grains.

- (1) The oscillation of azimuthal angle α on D-S rings recorded by two-dimensional detectors (software oscillation) (Sasaki *et al.*, 1997, Tanaka, 2017).
- (2) Back-and-forth planer motion of the specimen during exposure of X-rays at the fixed angle (Sasaki *et al.*, 1997), or in-plane averaging of X-ray intensity data (Hayashi *et al.*, 2014) or of strain data (Miyazaki *et al.*, 2016).
- (3) The oscillation of the specimen within a small angle range around the specimen axis during X-ray exposure at the fixed X-ray incidence (Takizawa and Soyama, 2013, Tominaga and Akiniwa, 2014).
- (4) The oscillation of the incident X-ray beam within a small angle range during X-ray exposure on the fixed specimen (Miyazaki *et al.*, 2015, Tanaka, 2017).

Using recorded D-S rings, the strain data can be averaged within a small range of azimuthal angle α during data processing. This α -oscillation does indeed reduce the deviation of the stress data. However, averaging over a large angle range can decrease the measured value. The range of α -oscillation should be below 10° to 15° (Tanaka, 2017). Back-and-forth planer motion of the specimen during exposure can increase the number of diffracting grains. Sasaki *et al.* (1997) used planer motion during exposure of X-rays at the fixed angle to measure the stress of coarse-grained steels. Planer oscillation is illustrated in Fig. 12(a). As an alternative, D-S rings are recorded at several positions, and

then X-ray intensity distribution data at each azimuthal angle are averaged to determine the strain (Hayashi *et al.*, 2014) or the strain data are averaged to determine the stress (Miyazaki *et al.* 2016). Basically, these in-plane averaging techniques are equivalent to the averaging technique by back-and-forth planer motion. They are only applicable to the region with uniform residual stresses, and not applicable for the region with stress gradient like near cracks or notches.

The oscillation of the specimen under fixed X-ray incidence is possible around three axes of goniometers. Figure 12(b) shows three oscillation axes: φ -oscillation is around the specimen normal, ω -oscillation is around the axis perpendicular to the diffraction plane made by the incident X-rays and the surface normal, and χ -oscillation is around the line intersected by the diffraction plane and specimen surface. A proper range of oscillation is around twice FWHM. In the stress measurement based on the $\sin^2\psi$ method, Tominaga and Akiniwa (2014), and Takizawa and Soyama (2013) reported that ω -oscillation is more effective than φ - or χ -oscillation.

In stress measurements of large structures, the oscillation of structures is difficult, so α -oscillation of D-S data is first recommended. The oscillation of the incident X-ray beam is possible around three axes of φ , χ , and ω . Figure 12 (c) illustrates ψ_0 -oscillation of X-ray incidence, which is equivalent to ω -oscillation in (b). In the $\cos\alpha$ method, ψ_0 -oscillation was confirmed to be very powerful to improve the reliability of stress data (Miyazaki *et al.*, 2015, Tanaka, 2017). Proper oscillation techniques have to be selected depending on actual measurement conditions.

Fig. 12 Various oscillation techniques to increase the number of diffracting grains.

4.4 Influences of stress gradient and texture on stress measurement

The influence of steep stress gradient on $\sin^2\psi$ diagrams and $\cos\alpha$ diagrams are different. In $\sin^2\psi$ diagrams, steep stress gradient makes $\sin^2\psi$ diagrams nonlinear (Tanaka *et al.*, 2006), while nonlinearity rarely appears in $\cos\alpha$ diagrams. In the $\cos\alpha$ method, the penetration depth changes with the azimuthal angle as shown in Fig. 8. The strain parameter ε_{a1} and ε_{a2} are determined from the difference between strains at different penetration depths. Therefore, the steep stress gradient does not have a direct influence on the linearity of $\cos\alpha$ and $\sin\alpha$ diagrams. When there is a steep gradient of residual stress beneath surfaces, the stress value obtained at different tilt angles may change in the $\cos\alpha$ method, because the penetration depth decreases with increasing tilt angle (Tanaka, 2018).

With respect to the influence of texture on stress determination, not much work has been conducted with the $\cos\alpha$ method, compared with the $\sin^2\psi$ method. The degree of texture can be estimated from the intensity distribution around the circumference of D-S rings. Stress measurements may be possible from the strain data obtained from the profile with enough intensity. Future studies are required for the influence of texture on stress measurement.

4.5 Triaxial residual stress measurement

The out-of-plane shear stress was measured on uni-directionally machined surfaces (Hanabusa and Fujiwara, 1982, Wakashima *et al.*, 1983), and bearing surfaces fatigue by rolling contact (Kamura *et al.*, 2015, 2018). In the $\sin^2\psi$ method, out-of-plane shear stresses induce so-called ψ -splitting, which has been analyzed by Dölle-Hauk method (Dölle & Hauk, 1976). The triaxial stress analysis by the $\cos\alpha$ method is much simpler and quicker. Even when the out-of-plane shear stress exists, both $\cos\alpha$ and $\sin\alpha$ diagrams are linear, and stresses are determined by the slope of the linear relation of $\cos\alpha$ and $\sin\alpha$ diagrams as described in section 2.3.

Sasaki and Hirose (1995b) measured the triaxial residual stress on machined surfaces and found that the accuracy of the $\cos\alpha$ method was equivalent to that of the Dölle-Hauk method (Dölle & Hauk, 1976) of the $\sin^2\psi$ method. Kamura *et al.* (2015, 2018) measured triaxial residual stresses on the bearing race fatigued by rolling contact. Sasaki *et al.* (2016)

conducted the measurement of triaxial residual stresses in rails in service to detect the progress of damages.

Tanaka (2018) measured triaxial residual stresses on uni-directionally machined surfaces of carbon steel. Figure 13 shows examples of $\cos\alpha$ and $\sin\alpha$ diagrams of the milled surface of a carbon steel (JIS-S50C). Both diagrams show some irregularity because of coarse grains and texture. Linear regression lines are drawn in the figure. From the slope of linear regression lines, stresses were calculated. The measured stresses of the milled surface are summarized in Table 1, together with the standard deviation of the linear regression as an indicator of the reliability of measurement. On the basis of the standard deviation, recommendations regarding the experimental conditions are proposed. The out-of-plane shear stress is accurately determined by $\cos\alpha$ and $\sin\alpha$ diagrams measured under normal incidence of the X-ray beam. For stress determination by oblique incidence, the incident angle ψ_0 is recommended to be above 35° . The out-of-plane shear stress along the cutting direction, τ_{zx} is positive. A positive value of the out-of-plane shear stress is characteristic of uni-directionally machined surfaces and increases according to the degree of cutting severity in the order of grinding, milling and planning (Hanabusa and Fujiwara, 1982, Wakashima *et al.*, 1983). In-plane normal stresses were compressive, and the magnitude of compression was larger in the direction perpendicular to the cutting direction.

The out-of-plane normal stress σ_z should be zero on the free surface because of force balance. In X-ray measurement of out-of-plane residual stresses in uni-directionally machined surface of steels, Hanabusa and Fujiwara (1982) reported compressive residual stress in ferrite phase and tensile stress in cementite phase. Compressive residual stresses measured in ferrite may be balanced by tensile stresses in cementite to give rise to zero macro-stress perpendicular to the surface. Similar balance of out-of-plane shear stresses, τ_{yz} and τ_{zx} , may be satisfied between constituent phases.

Fig. 13 The $\cos\alpha$ and $\sin\alpha$ diagrams of milled surface of carbon steel (Tanaka, 2018).

Table 1 Residual stress and standard deviation of milled surface of carbon steel (n/a = not applicable, SD = standard deviation) (Tanaka, 2018).

Specimen	ψ_0 degree	$\sigma_x - \sigma_z$ MPa	SD	$\sigma_y - \sigma_z$ MPa	SD	τ_{xy} MPa	SD	τ_{yz} MPa	SD	τ_{zx} MPa	SD
Milled surface	0	n/a	n/a	n/a	n/a	n/a	n/a	6	6	50	4
	15	-210	24	-391	27	8	26	5	30	59	22
	25	-303	9	-375	9	11	14	7	15	48	13
	35	-298	7	-367	12	5	8	1	8	63	9
	45	-263	9	-410	9	3	8	-5	9	65	7
	55	-298	11	-517	13	2	7	-5	6	70	8

4.6 Applications to on-site and field measurements

Residual stress measurements of welded structures are one of mostly demanded fields. Since welded parts are sometimes accompanied with grain growth and texture, D-S rings are often spotty or have highly non-uniform intensity distribution. Various oscillation techniques described in section 4.3 are used to reduce intensity variation of D-S rings. The stress is sometimes determined from the data of limited azimuthal angle ranges of D-S rings.

Wang *et al.* (2014) developed their own system of the $\cos\alpha$ method to measure local area of welded parts inside large scale pressure vessels made of Ni-base super alloy. The diffraction of Ni 311 by Mn-K α radiation was used for stress analysis. Though there was intensity variation in D-S rings, they succeeded in obtaining the stress data which were equivalent to those by the $\sin^2\psi$ method using a part of circumference of D-S ring with high intensity. After calibration

of the accuracy of measured stress, they applied the system to large engineering structures.

The application of the $\cos\alpha$ method to circular cylinders was studied by Oguri *et al.* (2016). The influence of the curvature of the cylinder surface on the stress value is small when the irradiated area is below the one-sixth of the radius of curvature of the surface. This limitation is about the same as the $\sin^2\psi$ method.

For on-site or field measurements, analyzers need to be portable, small, light, handy, and quick in obtaining stress with high accuracy. A recent $\cos\alpha$ -based stress analyzer takes only 60 s for one stress measurement. Such portable analyzers do not require water supply because X-ray tube is air-cooled, and only need household power of 100V-240V and 130 W. The X-ray tube, IP and IP reader are mounted as a unit, and the stress is calculated from in-situ reading of the whole D-S rings in the unit (Maruyama *et al.*, 2015). In each measurement, the IP-specimen distance is determined from the mean radius of measured D-S rings, which minimizes the positioning error (Nozue *et al.*, 2015). Because of these advantages, portable analyzers are now widely used in various fields, including on-site mapping of residual stress distribution in engineering structures (Tsutsumi, 2016). Figure 14 shows an example of mapping of the longitudinal residual stress distribution near weld measured by the $\cos\alpha$ method (Gadallah *et al.*, 2015). The weld is V-shaped with the line width of 12 mm on the welding surface, and the longitudinal residual stress was measured across the weld line on both welding surface and back surfaces. X-ray data are used to examine the accuracy of the prediction of residual stress distribution by the thermal elastic-plastic finite element method (TEP-FEM) and the contour method (CM).

Two-dimensional mapping of residual stresses will be effective for the area with a large variation of microstructures and residual stresses. A global picture of residual stress distributions obtained by areal mapping will provide valuable information to improve the performance of engineering structures.

Fig. 14 Residual stress distribution perpendicular to weld bead. Longitudinal stresses, σ_x , in the direction of bead are measured across the weld line by the X-ray method, and are used to check the accuracy of the prediction by the thermal-elastic-plastic finite element method (TEP-FEM) and the contour method (CM) (Gadallah *et al.*, 2015).

5. Concluding remarks

The $\cos\alpha$ method utilizes the whole strain data around the circumference of D-S rings recorded on two-dimensional detectors such as imaging plates (IPs). In biaxial stress cases, normal and shear stresses can be determined from $\cos\alpha$ and $\sin\alpha$ diagrams by single exposure. The accuracy of the $\cos\alpha$ method is equivalent to that of the conventional $\sin^2\psi$ method. The simple optical system makes stress analyzers portable, smaller, lighter, handier to obtain the stress in short time. Because of these advantages, the method has wide application fields, such as in-situ measurement of changing stresses or on-site stress measurement at narrow and multiple parts. In combination with robotic technology, mapping of areal distribution of residual stress can be carried out automatically. For triaxial residual stress analysis, the $\cos\alpha$ method is far simpler than the $\sin^2\psi$ method and has equivalent accuracy of the stress data. The $\cos\alpha$ method is rather new, so its applications are yet rather limited. In the near future, a large expansion of application fields is expected.

The measurements by the $\cos\alpha$ method is quick as well as accurate for fine-grained materials such as those shot-peened or quenched and tempered, because their D-S rings have rather uniform intensity showing less preferred orientation. Stress measurements can be difficult for materials with coarse grains or high texture, because their D-S rings are spotty or have high non-uniformity of the intensity around the circumference of D-S rings. To overcome such difficulty, several oscillation techniques have been proposed.

Features of D-S rings recorded on IP itself will provide microstructural information, such as grain size and texture. This information will be valuable for a nondestructive evaluation of material microstructures. The FWHM is a useful

parameter to indicate the degree of plastic deformation of materials.

Parallel with the above-mentioned studies, an activity of the standardization of stress measurement by the $\cos\alpha$ method is in progress in the X-ray Committee of the Society of Materials Science, Japan, which will promote further developments and wider applications of the method to various engineering fields.

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