R.I.イメージのデジタル処理について

放射線医学総合研究所 飯 沼 武

1）目的

・R.I.イメージの定量化・相互比較
・R.I.イメージの図形処理による認識の客観化
・R.I.イメージの自动生成による診断補助
・動態イメージの処理

2）デジタル・データ収集

・デジタル・イメージ
コリメータの半値巾（1cm直径）の\(\frac{1}{4}\)をいれ\(\frac{1}{5}\)の大きさの絵素（2mm〜3mm）からなる
計数値の行列。スキャナーとカメラで異なる。

・データ収集法

スキャナー・オフ・ライン
カメラ・オン・ライン

放射線学計画中のシステムについて

3）イメージ処理

・smothing
・restoration
・differentiation

4）イメージ表示

オン・ライン・・・C.R.T. display
オフ・ライン・・・Line Printer X-y plotter

・maximumを100%に規格化。0〜100%を適当な段階に区分。等間隔、対数区分、二乗区分

・輝度変調、等高線、濃度、色

5）自動認識の考え方

・正常なイメージの棄却
・イメージの大きさ、形状、normalization
・hot, cold spotの大きさ、場所

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Image Restoration in Radioisotope Imaging Systems

T. A. INUMA, PH.D. and T. NAGAI, M.D.
National Institute of Radiological Sciences, Chiba, Japan

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ABSTRACT. In the radioisotope image visualization, the final image must be interpreted by the human visual system. However, an inherent blurring of the image limits the amount of useful information which can be extracted from it. Attempts have been made to restore the image by compensating for the degradation.

By expressing the radioisotope image as a convolution integral of a true radioisotope distribution, with the spatial resolution of the imaging system, and by using an iterative approximation method for the evaluation of the integral, a 'restored image' is obtained. An experimental result on a profile scan demonstrates the applicability of the method to radioisotope imaging systems.

1. Introduction

The fundamental problem in radioisotope imaging systems is the contradiction between resolving power and detection efficiency of the system. Indeed, the best spatial resolution would only be obtained with a detector of zero sensitivity (infinite collimation). Therefore, the observed image in a practical system is necessarily degraded to some extent, and becomes less meaningful. If the image can be restored by compensating for the degradation a better compromise between the two opposing factors can be reached.

Recently we reported a preliminary result on a profile scan and an experimental result on an area scan (Nagai, Inuma and Kida 1967) which were 'computer-focused' in terms of the point-source response of the scanning system, but the mathematical background of the 'computer-focusing' was not fully described.

The purpose of the present paper is to clarify the mathematical relationship between an observed (blurred) image and a true radioisotope distribution (ideal image), and to describe the mathematical formulation of image restoration. A detailed experimental result of a profile scan is also presented.

2. Formation of an observed image

Radioisotope imaging systems are essentially analogous to optical systems, but there are several important differences. One of them is that an object in three-dimensional space is inevitably viewed by a practical image system. Taking this into account, an observed radioisotope image (blurred image) is expressed in analogy to the optical system as follows:

\[ G(x', y', z') = \int \int \int_{-\infty}^{\infty} F(x-x, y-y, z-z) R(x, y, z) \, dx \, dy \, dz \]  \hspace{1cm} (1)

where \( G(x, y, z) \) is the observed image as a function of three object-volume co-ordinates, \( F(x, y, z) \) is the ideal image i.e. true radio-isotope distribution as a
function of three co-ordinates and \( R(x, y, z) \) is the point spread function as a function of three co-ordinates.

Here we assume that the \( R \) is invariant over the region of object volume (organ volume) to be observed. This assumption is roughly satisfied with the section scanning method developed by Kuhl (1964). In this case \( R \) can be measured by scanning a point source of a particular radioisotope under consideration in three-dimensions.

In the case of an area image obtained with conventional moving and stationary devices, we have the following equation, assuming that the point spread function does not change over the image plane, i.e. surface of an organ:

\[
G(x', y') = \int F(x' - x, y' - y) \cdot R(x, y) \, dx \, dy \quad (2)
\]

where \( G, F \) and \( R \) are the identical quantities as those in eqn. (1), but in two-dimensional co-ordinates.

Strictly speaking, the above assumption is not valid in a practical system but it is approximately valid in some circumstances, for example in positron scanning and when scanning with two opposing focused detectors. When the organ depth is small, the assumption is roughly satisfied with any type of imaging systems.

Let us consider the case of a profile scanner, i.e. one dimensional imaging. The following equation can be derived with the same assumption as above:

\[
G(x') = \int F(x' - x) \cdot R(x) \, dx \quad (3)
\]

The integral eqns. mentioned above mean that an observed image, \( G \), can be expressed as a convolution integral of the ideal image, \( F \), and the point spread function, \( R \), i.e. the spatial resolution of the system, provided \( R \) is invariant over the space to be examined.

In order to restore the image it is necessary to solve eqns. (1), (2) and (3) for \( F \) under conditions where \( G \) and \( R \) are known.

3. The mathematical formulation of image restoration

The solution of the convolution integral equations is often simplified by the use of Fourier transformations as has been shown in some optical problems (Elias, Grey and Robinson 1952).

For simplicity, we shall discuss the case of profile scanning, but the same method can be applied to the other cases. The application of Fourier transformation to eqn. (3) yields the following equation:

\[
G(\nu) = F(\nu) \cdot R(\nu) \quad (4a)
\]

where

\[
G(\nu) = \int_{-\infty}^{\infty} G(x) \exp (-i2\pi\nu x) \, dx \quad (4b)
\]

\( F(\nu) \) and \( R(\nu) \) are expressed by the same equation as above, and \( \nu \) is the spatial-frequency.
$R(v)$ is the Fourier transform of the point spread function which is identical with the 'modulation transfer function' (M.T.F). From eqn. (4) it is seen that the M.T.F. is one of the important factors characterizing the resolving power of an imaging system (Beck 1964; Craddock, Fedoruk and Reid 1966).

From eqn. (4) we can also recognize that image restoration is theoretically possible by taking the inverse Fourier transform of the following equation:

$$ F(v) = G(v)/R(v) \quad (5a) $$

$$ F(x) = F^{-1}[G(v)/R(v)] \quad (5b) $$

where $F^{-1}$ indicates the inverse Fourier transformation.

However, there are several problems arising in attempts to make practical use of eqn. (5) as has been pointed out by several authors (Quittner 1966; Harris 1966). The greatest difficulty is that in the presence of noise (such as statistical fluctuations) in $G(x)$, the oscillations associated with $F(x)$, the restored image, may be so high as to make the image physically meaningless.

As an alternative tool, the iterative approximation method (Skarsgard, Johns and Green 1961) has been employed in an attempt to solve the integral equations of (1), (2) and (3).

Procedures for the iterative calculation are as follows:

The 1st iteration,

$$ G^0(x') = G^0(x') + \left\{ G(x') - \int_{-\infty}^{\infty} G^0(x'-x)R(x) \, dx \right\} \quad (6) $$

where $G^0(x')$ is an arbitrary function, but it is practical to use the original image $G(x')$, as $G^0(x')$. $G^1(x')$ is the 1st approximated image.

The $n$th iteration is expressed as follows:

$$ G^n(x') = G^{n-1}(x') - \left\{ G(x') - \int_{-\infty}^{\infty} G^{n-1}(x'-x)R(x) \, dx \right\} \quad (7a) $$

and,

$$ H^n(x') = \int_{-\infty}^{\infty} G^n(x'-x)R(x) \, dx. \quad (7b) $$

If $G^n(x')$ converges to $F(x)$, the value of $\{G(x') - H^n(x')\}$ should gradually approach zero. However, one cannot continue the iterative approximation until $G(x) = H^n(x)$, because the same difficulties as occurred in the Fourier transform method are encountered. Due to the presence of noise in $G$ and $R$, the iteration must be stopped at a certain point that is determined by the magnitude of the noise. The superiority of the present method over the Fourier transform method is due to the fact that the iteration can be terminated at an arbitrary level of approximation.

The following expression is used to define the limit of approximation:

$$ G(x') - H^n(x') \leq A. \quad (8) $$

This equation indicates the condition when iteration should be terminated. $A$ is a function of noise level of $G$ and $R$.

In the cases of area and three dimensional images, the equations corresponding to eqn. (7) are as follows:

For an area image,

$$ G^n(x', y') = G^{n-1}(x', y') - \left\{ G(x', y') - \int_{-\infty}^{\infty} G^{n-1}(x'-x, y'-y)R(x, y) \, dx \, dy \right\}. \quad (9) $$
For a three dimensional image,
\[ G^n(x', y', z') = G^{n-1}(x', y', z') + \int \int \int_{-\infty}^{\infty} G^{n-1}(x'-x, y'-y, z'-z) R(x, y, z) \, dx \, dy \, dz. \] (10)

In these cases the iteration should be terminated at a certain level of approximation that is defined by the similar expression to eqn. (8).

4. Experimental

4.1. Method

We have applied this method to profile and area scanning. In the present report a detailed discussion of the profile scan is presented. The experimental apparatus used for profile scanning was a whole body scanner equipped with two slit-collimated NaI(T1) crystals of 8 in. diameter and 4 in. thick, which scanned along the length of the body (Eto, Watanabe, Tanaka and Hiramoto 1962). The slit-collimator was made of 5 cm thick lead, and the slit-width length and depth were 10 cm, 25 cm and 15 cm respectively. With this scanner the assumption required in deriving eqn. (3) was approximately valid.

The whole-body distribution of $^{133}$Cs following an oral administration was measured in one subject. The 669 keV y-line of $^{133}$Cs was selected by a single channel analyser, and a profile scintigram was digitally recorded on a multiscaler of 128 channels synchronized with the movement of the detectors, so that the counts accumulated in each channel corresponded to those from a profile distance of 2.5 cm.

4.2. Results

Since an incremental length of 2.5 cm was chosen, the collimator resolution function ($R(x)$ in eqn. (3)) was determined by scanning a line source of $^{133}$Cs having a length of 2.5 cm. The source was located at the centre of both the NaI crystals and placed in a water phantom 20 cm thick. The result is plotted in fig. 1 (curve A), for which the actual values of $R(x)$ are also shown. The resolution function consists of 17 incremental figures, the sum of which is normalized to be unity.

In curve B the resolution of a 19 hole honeycomb collimator used in area scanning (Nagai, et al. 1967) is shown for comparison. It was obtained by scanning a $^{131}$I point source in air placed 6 cm from the collimator face (the focal distance of this collimator). Each incremental length was selected to be 1 mm in this case.

A profile scintigram for a subject obtained using a 128 channel multiscaler, consisted of about 70 incremental counts. each increment corresponding to an anatomical position from the top of the head down to the feet. Scanning was usually performed once a day after the administration of $^{133}$Cs, but a profile scintigram on the 20th day is shown as an example (fig. 2).

The histogram formed by the solid line in fig. 2 indicates the type of result obtained (blurred image $\hat{G}(x')$). Each channel (increment) corresponds to 2.5 cm. The iterative approximation starts as follows:
Fig. 1. Spatial response of the whole body scanner measured with a $^{132}$Cs source 2.5 cm long embedded in a 20 cm thick water bath (curve A), and that of honeycomb collimator of 19 holes measured with a $^{131}$I source 2 mm square (B). The incremental numbers of the $R(x)$ for the whole body scanner are also shown.

Fig. 2. Comparison between an original profile scintigram (solid line), and the corrected one (broken line). Each channel corresponds to a length of 2.5 cm, and important anatomical landmarks are also shown in the figure. This result shows the distribution at the 20th day after a single oral administration of $5\mu$Ci $^{132}$CsCl. The amount of whole body retention was less than 0.1 $\mu$Ci at the time of the measurement.
The 1st iteration:

\[ G^1(x') = G(x') + \left\{ G(x') - \sum_{x=-8}^{8} G(x'-x)R(x) \right\} \]  

where \( R \) consists of 17 incremental figures, \( R(-8), \ldots, R(8) \), (see fig. 1), and the convolution integral in eqn. (6) must be replaced by a form of summation. \( G(x') \) corresponds to an incremental count in the original histogram and so the above calculation must be performed for all increments (channels).

The nth iteration is:

\[ G^n(x') = G^{n-1}(x') + \left\{ G(x') - \sum_{x=-8}^{8} G^{n-1}(x'-x)R(x) \right\} \]  

and

\[ H^n(x') = \sum_{x=-8}^{8} G^n(x'-x)R(x). \]

The extent of convergence of \( G^n(x') \) to \( F(x) \) can be estimated by the following equation which is similar to eqn. (8):

\[ \sum_{x=-8}^{8} \{ G(x') - H^i(x') \}^2 = A^i \]  

where \( \sum \) refers to the summation of the error in each increment over all increments in the histogram, and \( A^i \) indicates the value of eqn. (13) at the \( i \)th iteration.

The iterative approximation is usually terminated when \( A^i \) satisfies the following condition:

\[ A^i \leq \sum_{x=-8}^{8} G(x'). \]  

This means \( H^i(x') \) on the average agree with \( G(x') \) within one statistical standard deviation of \( G(x') \), that is \( \{ G(x') \}^{1/2} \).

The calculation was made on the histogram shown as a solid line in fig. 2 and the value of eqn. (13) was calculated after each iteration. In this case \( \sum G(x') \) is about \( 10^6 \) counts. Taking \( A^1 \) as 100, arbitrarily, values of \( A^2 = 19.2 \), \( A^3 = 11.8 \), \( A^4 = 9.3 \) and \( A^6 = 8.1 \) were obtained. The convergence of the \( A^i \) was rapid at first, but became very slow at the 4th and 5th iterations. The result obtained after the 5th iteration is shown as the broken line of fig. 2, although the criterion of eqn. (14) was satisfied at the 3rd iteration.

As can be seen, the restored image (corrected histogram) shows more detailed structures of \( ^{132} \)Cs distribution than the original. This result was compared with that obtained by Palmer (1964) who carried out a profile scan with a sharper slit-collimator (5 cm wide) on a subject incorporating more than 1 \( \mu \)Ci of \( ^{137} \)Cs. Both results are in good agreement confirming the correctness of our method.

5. Discussion

As radioisotope image is ultimately examined by eye, and is used to obtain diagnostic information, all information contained in the image must be extracted
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efficiently by the human visual system. As pointed out by Harris (1966), however, the eyes can be extremely inefficient in extracting information under certain circumstances. The purpose of image restoration is to make a blurred image more intelligible by solving the convolution integral equations mentioned above. It should be noted here that the information content of an image is not increased by the calculation, but that the method of displaying the information only is changed so that the human visual system can extract this information more readily.

The iterative approximation procedure is a powerful method of solving the convolution integral equations and thus of restoring the degraded image. The Fourier transformation method failed to give a satisfactory solution. The iterative approximation calculation is quite simple and needs little programming and computational expense.

Harris (1966) has employed a modified Fourier transform method to solve the convolution integral of eqn. (a). By introducing an artificial modulation transfer function $R_m(x)$ into eqn. (3) he avoided the difficulties due to the fact that $R(x)$ may take a value of 0, but the restored image was deliberately blurred by the introduction of the $R_m(x)$.

Although the iterative method allows us to approach the ideal image, $F(x)$, it is practically impossible to obtain a noise-free estimate of $F(x)$ as has been described in section 3. Therefore, the iteration should be terminated at a limiting level that is controlled by the noise level in $G(x)$ and $R(x)$. Consequently, $G^n(x)$ is not a completely 'restored image', but still a 'partly blurred image'.

In radioisotope imaging systems, $G(x)$ is inherently noisy (due to statistical fluctuations) because only a restricted amount of radioisotope may be administered, and only a limited time for the measurement is available. On the other hand, $R(x)$ can be determined quite accurately by using a strong point source. So, the error is mainly determined by the statistical fluctuation in $G(x)$.

As a criterion at which the iteration should be terminated, it is convenient to adopt one statistical standard deviation of $G(x)$. This criterion results in the following expression:

$$
\sum_{x=1}^{N} \frac{(G(x) - F_n(x))^2}{G(x)} \leq N
$$

where $N$ is the total number of increments in the original image.

For the practical application of this method, it should be noted that the imaging system must satisfy the assumption mentioned in section 2. When a single detector system is used to obtain an image of a deep lying organ, the value of $R(x)$ may change so much that the convolution integral equation does not hold.

In addition, it is often better to smooth out the original image, $G(x)$, by an interpolation method (Kreisel 1949) or by other means (Brown 1963) in order to obtain a non-fluctuating solution. This is especially important when the original count-rate is low.

One of the disadvantages of this method is that the amount of calculation is so vast that the use of a digital computer is essential, and that time for the iterative computation might be quite long even with a fast computer. Therefore,
in some clinical applications, it might be advisable to stop the calculation at the 1st iteration. The result thus obtained is still nearer to the ideal image than the original one is.

The application of this method is not limited to the field of radioisotope imaging systems, but it may be applied to many other problems that can be expressed by convolution integral equations. They include photographic images of all kinds, dynamic (time-varying) phenomena observed by band-limited detectors, many kinds of spectra measured by detectors having limited resolving power and so on. In these cases, the term 'Image' should be interpreted in its broadest sense.

6. Conclusion

In attempts to remove the inherent 'blur' in the image observed by a radioisotope imaging system (moving or stationary), the following conclusions have been reached:

(1) under appropriate conditions, the observed (blurred) image can be expressed as a convolution integral between the ideal (restored) image and a point spread function of the system.

(2) Restoration of the blurred image is achieved to a satisfactory extent by the iterative approximation method, but the ultimate accuracy of the approximated result is determined by the statistical fluctuations incorporated in the original image.

(3) Application of this method to the profile scanning revealed fine details of the $^{132}$Cs distribution in the human body that were not visible in the original scintigram.

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Ein "wiederhergestelltes Bild" wird erhalten, indem man das Radioisotopenbild als Konvolutionsintegral der wirklichen Verteilung des Radioisotops, unter der spatialen Auflösung des Abbildesystems, sowie indem man für die Abschätzung des Integrals ein iteratives Annäherungsverfahren anwendet. Ein Experimentalergebnis der Profitastung bestätigt die Verwendbarkeit des Verfahrens für die Radioisotopen-Abbildesysteme.

Резюме

Восстановление изображения в системах, дающих изображение радиоактивного изотопа

При получении зрительного образа восстановления радиоактивного изотопа исходное изображение интерпретируется при помощи зрительной системы человека. Однако проработанная распыленность изображения ограничивает количество полезной информации, которую можно извлечь из него. Делались попытки восстановления изображения путем компенсирования деградации.

Возможность изображения радиоактивного изотопа в виде сверточного интеграла действительного распределения радиоактивного изотопа, с пространственным разрешением изобразительной системы, и применения итерационный метод приближения, получается "восстановленное изображение". Экспериментальный результат на профильной размерке иллюстрирует применимость этого метода к системам дающим изображение радиоактивного изотопа.

REFERENCES