Accelerated Algorithm for Compressed Sensing Using Nonlinear Sparsifying Transform in CT Image Reconstruction

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In our previous paper (J Dong, H Kudo: Proposal of compressed sensing using nonlinear sparsifying transform for CT image reconstruction. Medical Imaging Technology. Vol. 34 pp235–244, 2016), we showed that nonlinear sparsifying transform provides a new framework of compressed sensing (CS) for sparse-view CT image reconstruction. Furthermore, it was experimentally demonstrated to have superiority in improving image quality compared to total variation (TV) minimization, which is the most standard approach in CS. The image quality improvement appears in removing patchy artifacts, preserving accurate object boundaries, and preserving image textures. The TV uses the gradient transform which considers only correlations between adjacent pixels, while the nonlinear sparsifying transform can consider evaluating intensity variations among a specified relatively large search window. This property can be considered to be the key reason for achieving image quality improvement by the nonlinear sparsifying transform. However, the iterative algorithm developed in our previous paper, which can be viewed as a special case of standard iterative-thresholding (IT) algorithm, suffers from a drawback that it converges very slowly leading to a long computation time. The main reason of slow convergence is that the IT algorithm belongs to a class of simultaneous iterative algorithms, in which all projection data are used simultaneously (in parallel) for each image update. However, as is well-known in past research activities of CT image reconstruction, it is expected that the convergence can be significantly accelerated by introducing a class of row-action or block-iterative algorithm. Based on this observation, in this paper, we propose an accelerated algorithm of the CS using nonlinear sparsifying transform. By using proximal splitting framework, we succeeded in performing image update with a row-action-type program. The row-action-type update showed an encouraging acceleration such that both the iteration number and the computation time were reduced significantly compared to our previous simultaneous iterative algorithm. We investigated the efficiency of proposed accelerated algorithm using a numerical phantom and a practical CT image.

Key words: Computed tomography (CT), Image reconstruction, Compressed sensing (CS), Nonlinear sparsifying transform, Row-action-type acceleration

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1. Introduction

Potential radiation risk in computed tomography (CT) examinations has raised growing concerns from patients, radiologists, and medical physics community [1]. It is highly desirable to decrease radiation dose of CT scans while maintaining satisfactory image quality for a specific clinic task. An effective means for reducing radiation dose is to reduce a number of projection views per rotation of the scan. Therefore, sparse-view CT image reconstruction has been widely studied as a potential strategy [2], and the compressed sensing (CS) in signal restoration

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area is attracting much attention thanks to its power in solving underdetermined ill-posed inverse problems [3, 4].

Generally, in the framework of CS, a cost function consisting of data fidelity term and penalty term called regularization is designed. Then, an optimal solution can be obtained by minimizing the cost function based on the convex optimization theory. It is known that the regularization plays an important role so that image quality differs largely by different designs of regularizations. The crucial process of regularization is a concept called "sparsifying transform" which enforces majority of image components to be approximately zeros. In the early stage, regularizations using the Gibbs distribution based on Markov random fields were studied [5, 6], but they were defective in preserving object boundaries. Subsequently, regularizations by total variation (TV) minimization were proposed and applied to sparse-view CT image reconstruction [7–9], which created the trend in image reconstruction field. However, the TV minimization can observe noticeable gains only on piecewise constant images. In practical CT instances, images reconstructed by the TV usually suffer from distortions such as patchy artifacts, improper serrate edges, and loss of image textures.

We recognized that all the aforementioned regularizations were using linear transforms to promote sparsity. The main reason causing the distortions mainly originates from using linear transforms, which consider correlations between adjacent pixels but fail to take discontinuities such as edges and image textures into account. In such a situation, we have already proposed a novel CS framework using nonlinear sparsifying transforms, which is called nonlinear filter based compressed sensing (NLF-CS) method, for sparse-view CT image reconstruction [10]. In this work, we replaced the linear transform by a nonlinear transform, and investigated its power on denoising, i.e. the benchmark problem of CS, and CT reconstruction problem. Exciting experiment results were observed, and the distortions occurred in the TV results were significantly reduced. It was concluded that the CS using a nonlinear sparsifying transform is effective in removing patchy artifacts, preserving object boundaries accurately, and preserving image textures.

In our previous study [10], to solve the corresponding sparse recovery problem, an iterative-thresholding (IT) iterative algorithm was used, which was derived based on the majorization-minimization (MM) principle. In this method, each image update was performed by a steepest descent method using the data fidelity term followed by a thresholding operation. The IT iterative algorithm including the proposed NLF-CS algorithm in [10] belong to a class of simultaneous iterative algorithms [11, 12], in which all projection data are used simultaneously (in parallel) for each image update. In tomographic reconstruction fields, it is known that this class of iterative algorithms converge very slowly leading to a long computation time. Actually in [10], several thousand iterations were usually needed until the convergence. On the other hand, an alternative class of iterative tomographic reconstruction algorithms having block-iterative or row-action structures is known to converge very rapidly. Its basic idea is to split the projection data into many small subsets and image update is performed by using only a subset according to some pre-determined access order (in sequential). Based on this observation, in this paper, we accelerated the iterative algorithm by splitting the cost function into a number of sub-cost functions, from which a row-action iterative algorithm was developed based on proximal splitting framework [13, 14]. Thanks to the row-action structure of new algorithm, the convergence rate was improved significantly. The objective of this paper was to propose a row-action-type accelerated algorithm for our published NLF-CS algorithm. Both a numerical phantom and a practical CT image were utilized to confirm the acceleration efficiency.

2. Methodology

1) Problem definition

The aim of CT image reconstruction is to recover an object from measured projection data. When iterative methods are used for image reconstruction, the problem can be formulated as solving a linear equation \( \mathbf{A}\hat{x} = \hat{b} \), where \( \hat{b} = (b_1, b_2, ..., b_I)^T \) denotes the measured projection data, \( \hat{x} = (x_1, x_2, ..., x_J)^T \) represents the attenuation...
coefficients of object to be reconstructed, and $A = \{a_{ij}\}$ is the $I \times J$ system matrix. When the dimension of $\hat{b}$ is smaller than the dimension of $\hat{x}$ (i.e. $I < J$), the problem becomes underdetermined so that reconstructing an accurate image becomes challenging, which is just the mathematical setup of sparse-view CT reconstruction. In this situation, CS is usually utilized for obtaining a feasible image, in which a cost function consisting of a data fidelity term $L(\hat{x})$ and a regularization term $U(\hat{x})$ as in Eq. (1) is minimized.

$$f_\beta(\hat{x}) = L(\hat{x}) + \beta U(\hat{x}),$$  \hspace{1cm} (1)

where $L(\hat{x})$ is defined by the least-squares error $\|A\hat{x} - \hat{b}\|^2$, $U(\hat{x})$ is defined by Eq. (2), and $\beta$ is the hyperparameter to control the strength of regularization.

$$U(\hat{x}) = \|W\hat{x}\|_1 = \sum_{t=1}^{T} \| (W\hat{x})_t \|, \hspace{1cm} (2)$$

where $W$ represents the sparsifying transform which converts $\hat{x}$ into a sparse vector, and $T$ denotes the dimension of sparsifying transformed coefficients vector $W\hat{x}$. In Eq. (2), the expression like $\|z\|_1 = \sum_{t=1}^{T} |z_t|$ is called $L1$ norm which is known to have superior ability in picking up sparse solutions [15]. The sparsifying transform $W$ is usually designed as a linear high-pass filter which extracts high frequency components of object (i.e. object boundaries and textures). In many researches on CS image reconstruction, the TV defined by Eq. (3) was used.

$$U(\hat{x}) = \sum_{i,j} \sqrt{(x_{i,j} - x_{i,j+1})^2 + (x_{i,j} - x_{i,j+1})^2}, \hspace{1cm} (3)$$

where $x_{i,j}$ denotes a pixel value of image $\hat{x}$ at pixel $(i,j)$. The meaning of Eq. (3) can be interpreted as $\|\nabla \hat{x}\|_1$, which is the $L1$ norm of the magnitude of intensity gradient.

2) Brief review of our proposed NLF-CS algorithm

Since the sparsifying transform $W$ is usually designed as linear high-pass filter, we can redefine it as $W = 1 - L$, where $I$ is an identity operator and $L$ is a corresponding linear low-pass filter. Therefore, the regularization term can be expressed as Eq. (4).

$$U(\hat{x}) = \|(I - L)\hat{x}\|_1 = \|\hat{x} - L\hat{x}\|_1. \hspace{1cm} (4)$$

Inspired by the success of nonlinear filters in image processing such as denoising, we intended to replace the linear filter $L$ by a nonlinear filter $M$ to achieve improved image quality. Consequently, the cost function of Eq. (1) could be expressed in detail by

$$f_\beta(\hat{x}) = \|A\hat{x} - \hat{b}\|^2 + \beta \|\hat{x} - M\hat{x}\|^2 = \|A\hat{x} - \hat{b}\|^2 + \beta \sum_{j=1}^{J} |x_j - (M\hat{x})_j|. \hspace{1cm} (5)$$

In our previous paper [10], IT iterative algorithm was used to optimize Eq. (5), and it was derived based on the MM technique. In this process, a surrogate function $Q(\hat{x}; \hat{x}^{(k)})$ satisfying Eq. (6) is constructed where $k$ denotes the iteration number.

$$Q(\hat{x}^{(k)}; \hat{x}) = f_\beta(\hat{x}^{(k)}), \hspace{0.5cm} Q(\hat{x}; \hat{x}^{(k)}) \geq f_\beta(\hat{x}) \hspace{0.5cm} \forall \hat{x}. \hspace{1cm} (6)$$

After a series of equality and inequality transformation, the surrogate function can be yielded as Eq. (7) (see [10]), where $C$ denotes a term independent of $\hat{x}$.

$$Q(\hat{x}; \hat{x}^{(k)}) = C + \beta \sum_{j=1}^{J} |x_j - (M\hat{x})_j| + \frac{1}{2} \alpha \|\hat{x} - \bar{d}(\hat{x}^{(k)})\|^2, \hspace{0.5cm} \bar{d}(\hat{x}^{(k)}) = \hat{x}^{(k)} - \frac{2}{\alpha} \hat{A}^T(A\hat{x}^{(k)} - \hat{b}) \hspace{1cm} (7)$$

By minimizing $Q(\hat{x}; \hat{x}^{(k)})$ at each iteration number $k$, the iterative algorithm shown in Algorithm 1 is obtained, which monotonically decreases $f_\beta(\hat{x})$. In this algorithm derivation, to simplify the computation, $M\hat{x}$ was approximated by $M\hat{x}(\hat{x}^{(k)})$ (i.e. independent of $\hat{x}$). This approximation is reasonable because, when $\alpha$ is large enough, the unknown $\hat{x}$ is approximated by $\bar{d}(\hat{x}^{(k)})$, which approaches in changing the regularization term to a separable form.
3) Row-action-type accelerated NLF-CS algorithm

NLF-CS algorithm possesses a structure that all samples of projection data \( b_1, b_2, \ldots, b_\ell \) are used simultaneously in each image update. In CT reconstruction fields, such iterative algorithms are called simultaneous iterative reconstruction method, which is known to converge very slowly \([11, 12]\). The popular approach to accelerate the convergence dramatically is to introduce a class of iterative algorithms having row-action or block-iterative structure \([16]\). In this paper, we propose a row-action acceleration technique which can be applied to the case where the regularization term \( U(\hat{x}) \) is a non-differentiable \( L^1 \) norm function derived from the CS using the nonlinear sparsifying transform.

Before explaining algorithm acceleration in this paper, let us introduce two important mathematical tools.

**Proximity operator:** For a proper lower-semi-continuous (l.s.c) convex function \( f(\hat{x}) : \mathbb{R}^N \to \mathbb{R} \cup \{+\infty\} \), no matter whether it is differentiable or not, the proximity operator is defined by Eq. (8).

\[
\text{prox}_{\gamma f}(\hat{x}) = \arg \min_{\hat{x} \in \mathbb{R}^N} (f(\hat{x}) + \frac{1}{2\gamma} \| \hat{x} - \tilde{x} \|^2),
\]

(8)

where \( \tilde{x}, \hat{x} \in \mathbb{R}^N \) and \( \gamma > 0 \) is the step-size parameter. It is known that \( \text{prox}_{\gamma f}(\hat{x}) \) is uniquely determined, i.e. single-valued, for any \( \hat{x} \) when \( f(\hat{x}) \) is proper l.s.c convex, even for non-differentiable \( f(\hat{x}) \) such as the \( L^1 \) norm and TV functions. Furthermore, it is a non-expansive operator such that its fixed point satisfying \( \hat{x} = \text{prox}_{\gamma f}(\hat{x}) \) coincides with a minimizer of \( f(\hat{x}) \). These facts mean that the iteration formula in the form of \( \hat{x}(k+1) = \text{prox}_{\gamma f}(\hat{x}(k)) \) necessarily converges to a minimizer of \( f(\hat{x}) \) if and only if \( f(\hat{x}) \) is a possibly non-differentiable convex function. Such an iterative algorithm is called the proximal minimization algorithm \([13]\).

**Proximal splitting:** Let us consider the convex minimization problem expressed in Eq. (9).

\[
\min_{\tilde{x} \in \mathbb{R}^N} f_1(\tilde{x}) + \ldots + f_n(\tilde{x}),
\]

(9)

where \( f_1(\tilde{x}), \ldots, f_n(\tilde{x}) \) are possibly non-differentiable convex functions from \( \mathbb{R}^N \) to \( [-\infty, +\infty] \). We consider the situation where the proximity (prox) operator \( \text{prox}_{\gamma f_i}(\hat{x}) \) corresponding to \( f_i(\hat{x}) \) is intense to compute, but \( \text{prox}_{\gamma f_i}(\hat{x}) \) \((i = 1, \ldots, n)\) corresponding to sub-cost functions \( f_i(\hat{x}) \) \((i = 1, \ldots, n)\) can be easily computed. The proximal splitting is a framework to construct an iterative algorithm to minimize \( f(\hat{x}) \) under such situations. Normally, it is used for the case where the number of splitting \( n \) is two \([13]\), but we use a multi-splitting version described in the old paper by Passty \([14]\) because it naturally leads to a row-action-type iterative algorithm as clarified below. Basically, the algorithm is constructed by computing the prox operator \( \text{prox}_{\gamma f_i}(\hat{x}) \) corresponding to each \( f_i(\hat{x}) \)
sequentially according to the ascending order $i = 1, ..., n$. The processing procedure can be summarized as

$$
\begin{align*}
\hat{x}^{(k,i+1)} &= \text{prox}_{\gamma f_j} (\zeta^{(k,i)}), \quad (i = 1, ..., n) \\
\hat{x}^{(k+1,1)} &= \hat{x}^{(k,n+1)},
\end{align*}
$$

where $k$ denotes the main iteration number and $\gamma^{(k)} > 0$ is the step-size parameter. It is known that Eq. (10) converges to the minimizer of $f(\hat{x})$ under the diminishing step-size control $\gamma^{(k)} \to 0 \ (k \to \infty)$ satisfying $\sum_{k=1}^{\infty} \gamma^{(k)} = \infty$, $\sum_{k=1}^{\infty} (\gamma^{(k)})^2 < \infty$ and some additional conditions on $f_1(\hat{x}), ..., f_n(\hat{x})$ [14].

Below, we explain how to use the proximal splitting framework to construct an iterative algorithm to accelerate the original NLF-CS algorithm, i.e. how to minimize $f_p(\hat{x})$ of Eq. (5) by using proximal splitting framework. First, we split the data fidelity term into $I$ sub-cost functions as

$$
f_p(\hat{x}) = \| A\hat{x} - \hat{b} \|^2 + \beta \sum_j |x_j - (M\hat{x})_j| = \sum_{i=1}^{I} f_i(\hat{x}) + g(\hat{x})
$$

where $I$ is the dimension of projection data, $\hat{a}_i$ denotes the vector corresponding to the $i$-th row of system matrix $A$ (i.e., the $j$-th component of vector $\hat{a}_i$, $a_{ij}$ represents the probability that the $j$-th pixel of $x_j$ contributes to the $i$-th projection data $b_i$), $\hat{a}_i^T$ denotes the transpose of vector $\hat{a}_i$, and we used $g(\hat{x})$ to represent the regularization term. According to the above splitting, each iteration of the algorithm proceeds by applying the prox operators corresponding to $f_i(\hat{x})$, ..., $f_I(\hat{x})$ successively, i.e. processing of data fidelity term, followed by computing the prox operator corresponding to $g(\hat{x})$, i.e. processing of regularization term. A more general algorithm using a span parameter $S$, intended to perform the smoothing more frequently, i.e. $I/S$ times, during each main iteration $k$, was introduced and its processing procedure can be summarized in Eq. (13). We remark that each image update indexed by $(k, i)$ uses only one projection data $b_i$, so this algorithm possesses the row-action-type structure such as ART (algebraic reconstruction technique) method, which is known to converge rapidly by using the special data-access order [16].

$$
\text{loop } i = 1, 2, ..., I
\begin{cases}
\hat{x}^{(k,i+1)} = \text{prox}_{\gamma f_j} (\zeta^{(k,i)}) \\
\text{if}(i \ mod \ S) = 0 \quad \hat{x}^{(k,i+1)} \leftarrow \text{prox}_{\gamma g/(2S)} (\zeta^{(k,i+1)})
\end{cases}
\hat{x}^{(k+1,1)} = \hat{x}^{(k,n+1)}
$$

The span parameter $S$ was determined as follows in our implementation. The meaning of $S$ can be interpreted as the interval to perform smoothing during each main iteration $k$. Therefore, when $S$ is small, the total number of smoothing during iteration becomes large so that the smoothing (regularization) takes effect after only a small number of iterations leading to a fast convergence. In this sense, a small value of $S$ is preferred. However, the price of using small value of $S$ is an increase of computation time necessary to perform the smoothing. In this sense, a large value of $S$ is preferred. We experimentally determined the optimal value of $S$ in the following way. With several possible candidate values of $S$, image reconstructions were performed. Among them, we chose the best value at which the total computation time to reach to a sufficient reconstructed image is shortest. The best values of $S$ in our simulation studies determined in this way will be shown in Section 3. Next, we derive an explicit expression of each prox operator involved in the algorithm. First, the prox operator corresponding to $f_j(\hat{x})$ is defined by

$$
\hat{x}^{(k,i+1)} = \text{prox}_{\gamma f_j} (\zeta^{(k,i)}) = \arg \min_x (|\hat{a}_i^T x - b_i|^2 + \frac{1}{2\gamma^{(k)}} \|x - \zeta^{(k,i)}\|^2).
$$

Here, we introduce a slack variable $z = \hat{a}_i^T \hat{x}$, then Eq. (14) changes to
\[ x(k,i) = 0, \ k = 0 \]

while \( f_y(x(k))-f_y(x(k+1)) > \text{tol} \) do

\[ y(k) = \frac{y(k)}{1+\varepsilon}, \quad x(k,i) = x(k) \]

for \( (i=0; i<N\times M; i++) \) do

\[ x(k,i) = x(k,i) + \frac{y(k) - M_j(x(k,i))}{\mu} \]

end for

end while

return \( x(k+1) \)

The row-action-type accelerated NLF-CS algorithm is summarized in Algorithm 2.

Algorithm 2  Row-action-type accelerated NLF-CS algorithm (N: number of radial bins, M: number of views).

**Input:** A measured sparse-view projection data \( \hat{b} \) as an \( N\times M \)-dimensional vector, step-size control parameters \( (\gamma (0), \varepsilon) \), a hyperparameter \( \beta > 0 \), an integer span parameter \( S > 0 \), and an algorithm tolerance \( \varepsilon > 0 \).

**Output:** The reconstructed image \( \hat{x} \) as an \( N\times N \)-dimensional vector.

Since Eq. (15) is a typical constrained minimization problem, it can be solved in closed form by using the Lagrange function

\[ L(\hat{x}, z, \lambda) = \frac{1}{2\gamma(k)} \| \hat{x} - x(k,i) \|^2 + (z - b_i)^2 + \lambda (z - \hat{a}_i^T \hat{x}), \]

where \( \lambda \) is the Lagrange multiplier. Finally, by optimizing the Lagrange function, the image update equation can be obtained as

\[ x(k,i+1) = x(k,i) + \frac{y(k) - (S\beta \sum_j |x_j - (M_j^T x)| + \frac{1}{2\gamma(k)} \| \hat{x} - x(k+1) \|^2)}{S\gamma(k) \beta} \text{ (otherwise)} \]

The row-action-type accelerated NLF-CS algorithm is summarized in Algorithm 2.
3. Experimental results

1) Numerical phantom

In this simulation, we implemented the original NLF-CS, row-action accelerated NLF-CS algorithms and the conventional simultaneous iterative reconstruction technique (SIRT) [11, 12] for comparison. The numerical spot phantom containing circles and rectangles having various contrasts and sizes was designed for this comparison. The image size was $256 \times 256$ pixels and the projection data was computed by parallel-beam geometry with $256$ radial bins and $32$ views over $180^\circ$ angular range. No statistical noise was added to the projection data in this simulation. For the NLF-CS and row-action accelerated NLF-CS algorithms, the median filter was used mainly, because the simple median filter with a proper window size was enough to obtain successful images for this simple phantom, where the experimentally determined window size was $7 \times 7$. In this simulation, we set the parameter values as $\alpha = 500$, $\beta = 0.08$ for NLF-CS and $\gamma(k) = 10/(1 + 1000 \times k)$, $\beta = 2.0$, $S = 5 \times 256$ for the row-action accelerated NLF-CS. The root mean square error (RMSE) was used for evaluating errors between phantom and reconstructed images. The reason to use different values of $\beta$, i.e. $\beta = 0.08$ in NLF-CS and $\beta = 2.0$ in row-action accelerated NLF-CS, is explained as follows. Of course, theoretically speaking, both the algorithms converge to the same minimizer of cost function $f_\beta(\tilde{x})$ with same value of $\beta$. However, the actual algorithm behavior when operated with a finite number of iterations is rather different. The original NLF-CS uses non-diminishing (constant) step-size $1/\alpha$, whereas row-action accelerated NLF-CS uses the diminishing step-size rule $\gamma(k) \to 0(k \to \infty)$. Therefore, due to the effect of diminishing step-size, we observed a tendency that the degree of smoothing achieved...
by row-action accelerated NLF-CS is a bit weaker compared with that in NLF-CS even if the same value of $\beta$ is used. To compensate this effect, we used a bit larger value of $\beta$ in row-action accelerated NLF-CS compared with NLF-CS, where these comparable values leading to the nearly same degree of smoothing was experimentally determined.

The reconstruction results are shown in Fig. 1. It shows that the row-action-type NLF-CS algorithm converged rapidly so that only 10 iterations were enough to obtain satisfactory reconstruction. In Fig. 2, the convergence property is shown where RMSE value was plotted as a function of iteration number and also as a function of actual computation time. We observe that the row-action accelerated NLF-CS algorithm can lead to convergence with approximately 20 iterations, 65 s, while the original NLF-CS algorithm requires 1,200 iterations, 800 s. The computation time was reduced to approximately 1/12. We also performed an additional simulation study using the projection data with statistical Poisson noise corresponding to $1.2 \times 10^4$ blank-scan photon counts (the results are not shown here). Theoretically speaking, it is guaranteed that row-action-type accelerated NLF-CS algorithm converges to the minimizer of cost function $f_\beta(\tilde{x})$ even if the noise is contained in the projection data, as can be proved by the mathematical technique of [14]. As expected from this theoretical result, row-action-type accelerated NLF-CS algorithm decreased the value of cost function $f_\beta(\tilde{x})$ almost monotonically and converged to a reconstructed image very closer to that obtained by NLF-CS algorithm based on the IT structure. In particular, by using the diminishing step-size rule $\gamma(k) \to 0$ ($k \to \infty$) described in Section 2, the convergence to the limit-cycle (oscillating solutions), which is a major drawback of block-iterative and row-action iterative algorithms [16], could be avoided in a successful way.

2) Dental CT image

In this study, a slice of dental CT image was used to evaluate convergence speed of the accelerated NLF-CS method. The image size was $512 \times 512$ and projection data was computed with 48 views over $180^\circ$ angular range. Similarly to the case of numerical phantom, the SIRT, original NLF-CS, and row-action accelerated NLF-CS methods were compared. In this simulation, nonlocal means filter, which was best in terms of image quality in our previous paper [10], was used, because the NLF-CS with a simple median filter as well as the standard TV regularization fail in producing good enough images for this image. With respect to parameters of non-local mean filter, both the sizes of search window and patch window were set to $7 \times 7$, and the Gauss parameter $\delta_1$ and the intensity similarity parameter $\delta_2$ were set to 10.0. The other parameters were set as $\alpha = 500$, $\beta = 70$ for NLF-CS and $\gamma(k) = 10/(1 + 1000 \times k)$, $\beta = 800.0$, $S = 5 \times 512$ for row-action accelerated NLF-CS. Figure 3 shows reconstruction results, where small box areas are zoomed-in in the bottom-right part. Similarly to the case of numerical phantom, we observe that the row-action accelerated NLF-CS can achieve a much higher convergence even in the practical CT image instance.
4. Conclusions

In this paper, we proposed an accelerated algorithm for our previously proposed nonlinear filter based compressed sensing (NLF-CS) algorithm. Thanks to the application of proximal multi-splitting [14], we succeeded in developing an iterative algorithm which allows row-action-type update. In the simulation study using the numerical phantom and the practical CT image, this algorithm achieved an encouraging acceleration so that both the iteration number and the execution time were reduced significantly compared to our previous simultaneous iterative algorithm.

We applied the row-action accelerated algorithm to sparse-view CT image reconstruction in this paper. The algorithm can also be used in the low-dose and limited-angle CT reconstructions which occurs in, for example, C-arm angiography and electron tomography. We leave it as a future work. Furthermore, we are aiming at developing a more powerful nonlinear filter which can selectively remove streak artifacts. This will contribute to further reducing the number of projection views.

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References

CT画像再構成における非線形スパーシファイ変換を用いた圧縮センシングの高速化アルゴリズム

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要 旨：スパースビューCTにおける新しい画像再構成の枠組みである非線形スパーシファイ変換を用いた圧縮センシング（CS）は，トークルバリエーション（TV）など既存のCSと比較して画質性能に優れる。具体的には，筆者らの先行研究において，パッチ状アーチファクトの削減，エッジの正確な再現，テクスチャーセーブなどの画質向上が実現できることができた。これらの画質向上は，おもにTVにおけるスパーシファイ変換が接続画素間の相関を考慮しないのに対して，非線形スパーシファイ変換では大きなサイズの窓の中から類似性が高い画素のみを適応的に抜き出して信号スパース化に用いる性質により，実現される。一方，問題点としては，先行研究で使用された反復閾値処理法に基づく反復アルゴリズムは同化反復型の構造をもつため，収束が非常に遅く計算量が膨大になる点が挙げられる。この問題点は，CT画像再構成分野で高速に収束することが知られるブロック反復型やローラクション型の反復法を導入することで，解決できると考えられる。そこで，本論文では，非線形スパーシファイ変換を用いたCSに用いる高速に収束する反復アルゴリズムを提案する。具体的には，近接スプリッティングと呼ばれるローラクション型の反復アルゴリズムを導入可能な数値的枠組みを利用して，ローラクション型の高速に収束する画像更新に可能な反復アルゴリズムを構築する。提案手法により，先行研究で提案した同化反復型の構造をもつ反復アルゴリズムと比較して，反復回数と収束に要する計算時間を大幅に削減することができた。提案手法の有効性を，数値ファントムと歯科用CT画像を用いた画像再構成実験により検証した。

キーワード：CT，画像再構成，圧縮センシング，非線形スパーシファイ変換，ローラクション型高速化

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