The semistar operations on certain Prüfer domain, II

Ryûki MATSUDA*

Abstract

Let D be a 1-dimensional Prüfer domain with exactly two maximal ideals. We completely determine the star operations and the semistar operations on D .

Let G be a torsion-free abelian additive group. If G is not discrete, G is called indiscrete. If every non-empty subset S of G which is bounded below has its infimum $\inf(S)$ in G , then G is called complete. If G is not complete, G is called incomplete. Let D be a 1-dimensional Prüfer domain with exactly two maximal ideals M and N , let $V = D_M$ (resp., $W = D_N$), let v (resp., w) be a valuation belonging to V (resp., W), let Γ (resp., Γ') be the value group of v (resp., w), and let K be the quotient field of D . Let $\Sigma(D)$ (resp., $\Sigma'(D)$) be the set of star operations (resp., semistar operations) on D . In [1, Proposition 6 (2)], we proved that, if both Γ and Γ' are discrete, then $|\Sigma(D)| = 1$ and $|\Sigma'(D)| = 7$. In this paper, we prove the following,

Theorem Let D be a 1-dimensional Prüfer domain with exactly two maximal ideals M and N , let Γ (resp., Γ') be the value group belonging to the valuation ring D_M (resp., D_N).

- (1) If both Γ and Γ' are discrete, then $|\Sigma(D)| = 1$ and $|\Sigma'(D)| = 7$.
- (2) If Γ is discrete, and if Γ' is indiscrete, then $|\Sigma(D)| = 2$ and $|\Sigma'(D)| = 14$.
- (3) If both Γ and Γ' are indiscrete, then $|\Sigma(D)| = 4$ and $|\Sigma'(D)| = 30$.

In the above Theorem, (1) was proved in [1, Proposition 6 (2)] as was mentioned. (2) was proved in [2] in the case where Γ' is incomplete. (3) was studied in [2] in the case where both Γ and Γ' are incomplete. Unfortunately, [2, Theorem (4)] and its proof were incomplete. Section 1 of this paper contains preliminary results, and in Section 2, we prove Theorem (3). The proof of Theorem (2) is similar to that of [2, Theorem (2)].

Throughout the paper, we confer [2], p denotes an element of $M \setminus N$, q denotes an element of $N \setminus M$, and x denotes an element of $K \setminus \{0\}$.

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*Professor Emeritus, Ibaraki University, Mito, Ibaraki 310-8512, Japan (rmazda@adagio.ocn.ne.jp)

§1 Preliminary results

The set of non-zero fractional ideals of D is denoted by $F(D)$, and the set of non-zero D -submodules of K is denoted by $\bar{F}(D)$.

(1.1) Let $x \in K \setminus \{0\}$ and let $I \in \bar{F}(D)$. Then $x \in I$ if and only if $v(x) \in v(I)$ and $w(x) \in w(I)$.

(1.2) ([2, (1.1)]) (1) Each element $x \in D \setminus \{0\}$ can be expressed as 1 or p or q or pq uniquely up to associates, where $p \in M \setminus N$ and $q \in N \setminus M$.

(2) Each element $x \in K \setminus \{0\}$ can be expressed as 1 or p or q or pq or $\frac{1}{p}$ or $\frac{1}{q}$ or $\frac{q}{p}$ or $\frac{p}{q}$ or $\frac{1}{pq}$ uniquely up to associates.

An ideal A_0 of D is called with type α , if there exists a subset $\{p_\lambda \mid \lambda \in \Lambda\}$ of $M \setminus N$ so that A_0 is generated by the set $\{p_\lambda \mid \lambda \in \Lambda\}$, and there does not exist $\inf v(A_0)$. An ideal B_0 is called with type β , if there exists a subset $\{q_\sigma \mid \sigma \in \Sigma\}$ of $N \setminus M$ so that B_0 is generated by the set $\{q_\sigma \mid \sigma \in \Sigma\}$, and there does not exist $\inf w(B_0)$. If Γ (resp., Γ') is discrete, then Γ (resp., Γ') is complete. If Γ (resp., Γ') is complete, then there does not exist an ideal of D with type α (resp., β). Throughout the paper, $A_0, A_1, \dots$ (resp., $B_0, B_1, \dots$) denote ideals of D with type α (resp., β). The definition of type α (resp., β) is simpler than that in [2], and both are essentially the same.

Let $I, J \in \bar{F}(D)$. If $I = xJ$ for some $x \in K \setminus \{0\}$, we say that I and J are similar.

(1.3) (1) $F(D) = \{(x), xM, xN, xA_0, xB_0, xMN, xA_0N, xB_0M, xA_0B_0, K \mid x \in K \setminus \{0\}, A_0 \text{ (resp., } B_0) \text{ is an ideal with type } \alpha \text{ (resp., } \beta)\}$.

(2) $D, M, N, A_0, B_0, MN, A_1N, B_1M, A_2B_2$ are not similar with each other.

(3) We have $A_0 = A_0^\vee, B_0 = B_0^\vee$ and $A_0B_0 = (A_0B_0)^\vee$.

(1.4) Example. Let G_1 (resp., G_2) be non-zero torsion-free abelian additive groups. Let $G_1 \oplus G_2$ be their direct sum with the lexicographic order: Let $x := (a_1, a_2), y := (b_1, b_2)$ be elements of $G_1 \oplus G_2$ with $a_2 < b_2$, then let $x < y$. Let v_0 (resp., w_0) be the projection mapping of $G_1 \oplus G_2$ to G_1 (resp., G_2). Let k be a field, and let K be the quotient field of the semigroup ring $k[X; G_1 \oplus G_2]$. Let v (resp., w) be the canonical extension of v_0 (resp., w_0) to a valuation on K , and let V (resp., W) be the valuation ring on K belonging to v (resp., w), and let M' (resp., N') be the maximal ideal of V (resp., W). Let $D := V \cap W$, and let $M := M' \cap D$ and $N := N' \cap D$. Then we have $V = D_M, M' = MV, W = D_N$ and $N' = NW$. We have $\Gamma = G_1$, and $\Gamma' = G_2$.

(1.5) Let $\star$ be a star operation.

If $N^\star = D$, then $(A_0N)^\star = A_0$.

If $N^\star = N$, then $(A_0N)^\star = A_0N$.

If $M^\star = D$, then $(B_0M)^\star = B_0$.

If $M^\star = M$, then $(B_0M)^\star = B_0M$.

- (1.6) (1) $xMV \subseteq M$ is impossible.
 $xMV \subseteq N$ is impossible.
 $xNW \subseteq M$ is impossible.
 $xNW \subseteq N$ is impossible.
(2) $xMV \subseteq W$ is impossible.
 $xNW \subseteq V$ is impossible.

Let $I^{d'} = I$ (resp., $I^{v'} = (I^{-1})^{-1}, I^e = K$) for each $I \in \bar{F}(D)$. Then d' (resp., v' , e) is a semistar operation on D , and is called the d' -operation (resp., v' -operation, e -operation) on D .

- (1.7) $V^{v'} = K, W^{v'} = K$.

Let $I \subseteq J$ be an inclusion relation of elements of $\bar{F}(D)$. A mapping $\star$ from $\bar{F}(D)$ to $\bar{F}(D)$ is called monotone for $I \subseteq J$, if $I^\star \subseteq J^\star$.

- (1.8) (cf., [2, (1.8)]) Let $\star$ be a semistar operation on D .

- (1) $V^\star$ is either V or K . $W^\star$ is either W or K .
(2) $(MV)^\star$ is either MV or V or K . $(NW)^\star$ is either NW or W or K .
(3) If $V^\star = V$, then $(A_0V)^\star = A_0V$ for every ideal A_0 of D with type α . If $W^\star = W$, then $(B_0W)^\star = B_0W$ for every ideal B_0 of D with type β .

§2 Γ and Γ' are indiscrete

Throughout the section, we assume that Γ and Γ' are indiscrete.

- (2.1) (1) $M^v = D, N^v = D, (MN)^v = D$.
(2) $(A_0N)^v = A_0, (B_0M)^v = B_0$.
(3) $\{I \in F(D) \mid I \not\subseteq I^v\} = \{xM, xN, xMN, xA_0N, xB_0M \mid x \in K \setminus \{0\}, A_0 \text{ (resp., } B_0) \text{ is an ideal with type } \alpha \text{ (resp., } \beta)\}$.

- (2.2) Let $x \in K \setminus \{0\}$ and let A_0, A_1 (resp., B_0, B_1) be an ideal of D with type α (resp., β).

- (1) If $xM \subseteq D$, then $(x) \subseteq D$.
If $xM \subseteq M$, then $(x) \subseteq D$.
If $xM \subseteq N$, then $(x) \subseteq N$.
If $xM \subseteq A_0$, then $(x) \subseteq A_0$.
If $xM \subseteq B_0$, then $(x) \subseteq B_0$.
If $xM \subseteq MN$, then $(x) \subseteq N$.
If $xM \subseteq A_0N$, then $(x) \subseteq A_0N$.
If $xM \subseteq B_0M$, then $(x) \subseteq B_0$.
If $xM \subseteq A_0B_0$, then $(x) \subseteq A_0B_0$.
(2) If $xN \subseteq D$, then $(x) \subseteq D$.
If $xN \subseteq M$, then $(x) \subseteq M$.
If $xN \subseteq N$, then $(x) \subseteq D$.

If $xN \subseteq A_0$, then $(x) \subseteq A_0$.
 If $xN \subseteq B_0$, then $(x) \subseteq B_0$.
 If $xN \subseteq MN$, then $(x) \subseteq M$.
 If $xN \subseteq A_0N$, then $(x) \subseteq A_0$.
 If $xN \subseteq B_0M$, then $(x) \subseteq B_0M$.
 If $xN \subseteq A_0B_0$, then $(x) \subseteq A_0B_0$.
 (3) If $xMN \subseteq D$, then $(x) \subseteq D$.
 If $xMN \subseteq M$, then $(x) \subseteq D$.
 If $xMN \subseteq N$, then $(x) \subseteq D$.
 If $xMN \subseteq A_0$, then $(x) \subseteq A_0$.
 If $xMN \subseteq B_0$, then $(x) \subseteq B_0$.
 If $xMN \subseteq MN$, then $(x) \subseteq D$.
 If $xMN \subseteq A_0N$, then $(x) \subseteq A_0$.
 If $xMN \subseteq B_0M$, then $(x) \subseteq B_0$.
 If $xMN \subseteq A_0B_0$, then $(x) \subseteq A_0B_0$.
 (4) If $xA_0N \subseteq D$, then $xA_0 \subseteq D$.
 If $xA_0N \subseteq M$, then $xA_0 \subseteq M$.
 If $xA_0N \subseteq N$, then $xA_0 \subseteq D$.
 If $xA_0N \subseteq A_1$, then $xA_0 \subseteq A_1$.
 If $xA_0N \subseteq B_0$, then $xA_0 \subseteq B_0$.
 If $xA_0N \subseteq MN$, then $xA_0 \subseteq M$.
 If $xA_0N \subseteq A_1N$, then $xA_0 \subseteq A_1$.
 If $xA_0N \subseteq B_0M$, then $xA_0 \subseteq B_0M$.
 If $xA_0N \subseteq A_1B_0$, then $xA_0 \subseteq A_1B_0$.
 (5) If $xB_0M \subseteq D$, then $xB_0 \subseteq D$.
 If $xB_0M \subseteq M$, then $xB_0 \subseteq D$.
 If $xB_0M \subseteq N$, then $xB_0 \subseteq N$.
 If $xB_0M \subseteq A_0$, then $xB_0 \subseteq A_0$.
 If $xB_0M \subseteq B_1$, then $xB_0 \subseteq B_1$.
 If $xB_0M \subseteq MN$, then $xB_0 \subseteq N$.
 If $xB_0M \subseteq A_0N$, then $xB_0 \subseteq A_0N$.
 If $xB_0M \subseteq B_1M$, then $xB_0 \subseteq B_1$.
 If $xB_0M \subseteq A_0B_1$, then $xB_0 \subseteq A_0B_1$.

(2.3) (1) Set $M^{\star_1} := M, N^{\star_1} := N$. Then there is a unique star operation $\star_1$, and $\star_1 = d$.

(2) Set $M^{\star_2} := M, N^{\star_2} := D$. Then there is a unique star operation $\star_2$.

(3) Set $M^{\star_3} := D, N^{\star_3} := N$. Then there is a unique star operation $\star_3$.

(4) Set $M^{\star_4} := D, N^{\star_4} := D$. Then there is a unique star operation $\star_4$, and $\star_4 = v$.

(2.4) We have $|\Sigma(D)| = 4$ and $\Sigma(D) = \{\star_1, \star_2, \star_3, \star_4\}$.

(2.5) (1) $\bar{F}(D) \setminus F(D) = \{xV, xW, xMV, xNW, xA_0V, xB_0W, K \mid x \in K \setminus \{0\}, A_0 \text{ (resp., } B_0) \text{ is an ideal with type } \alpha \text{ (resp., } \beta)\}$.

(2) $V, W, MV, NW, A_0V, B_0W, K$ are not similar with each other.

(2.6) Let $\star^1, \star^2$ be semistar operations on D satisfying the followings:

(1) The restriction $\star'_1 := \star^1|_{F(D)}$ (resp., $\star'_2 := \star^2|_{F(D)}$) of $\star^1$ (resp., $\star^2$) to $F(D)$ is a star operation on D , and $\star'_1 = \star'_2$.

(2) We have that $V^{\star^1} = V^{\star^2}, W^{\star^1} = W^{\star^2}, (MV)^{\star^1} = (MV)^{\star^2}$ and $(NW)^{\star^1} = (NW)^{\star^2}$.

Then we have $\star^1 = \star^2$.

In the following (2.7), let $\star := \star_i^j$. Since $\star_i$ is a star operation on D , we have $(xI)^\star = xI^\star$ for every $I \in F(D)$. We define $(xV)^\star := xV^\star, (xW)^\star := xW^\star, (xMV)^\star := x(MV)^\star, (xNW)^\star := x(NW)^\star$ and $K^\star := K$. We define $(A_0V)^\star := A_0V$ (resp., $(A_0V)^\star := K$) if $V^\star = V$ (resp., $V^\star = K$), $(B_0W)^\star := B_0W$ (resp., $(B_0W)^\star := K$) if $W^\star = W$ (resp., $W^\star = K$), $(xA_0V)^\star := x(A_0V)^\star$, and $(xB_0W)^\star := x(B_0W)^\star$.

(2.7) Let $1 \leq i \leq 4$.

(1) Set $I_0^{\star_i^1} := I_0^{\star_i}$ for each $I_0 \in F(D)$, and set $V^{\star_i^1} := V, W^{\star_i^1} := W, (MV)^{\star_i^1} := MV$ and $(NW)^{\star_i^1} := NW$. Then there is determined a unique mapping $\star_i^1$ from $\bar{F}(D)$ to $\bar{F}(D)$.

(2) Set $I_0^{\star_i^2} := I_0^{\star_i}$ for each $I_0 \in F(D)$, and set $V^{\star_i^2} := V, W^{\star_i^2} := W, (MV)^{\star_i^2} := MV$ and $(NW)^{\star_i^2} := W$. Then there is determined a unique mapping $\star_i^2$ from $\bar{F}(D)$ to $\bar{F}(D)$.

(3) Set $I_0^{\star_i^3} := I_0^{\star_i}$ for each $I_0 \in F(D)$, and set $V^{\star_i^3} := V, W^{\star_i^3} := W, (MV)^{\star_i^3} := V$ and $(NW)^{\star_i^3} := NW$. Then there is determined a unique mapping $\star_i^3$ from $\bar{F}(D)$ to $\bar{F}(D)$.

(4) Set $I_0^{\star_i^4} := I_0^{\star_i}$ for each $I_0 \in F(D)$, and set $V^{\star_i^4} := V, W^{\star_i^4} := W, (MV)^{\star_i^4} := V$ and $(NW)^{\star_i^4} := W$. Then there is determined a unique mapping $\star_i^4$ from $\bar{F}(D)$ to $\bar{F}(D)$.

(5) Set $I_0^{\star_i^5} := I_0^{\star_i}$ for each $I_0 \in F(D)$, and set $V^{\star_i^5} := V, W^{\star_i^5} := K$ and $(MV)^{\star_i^5} := MV$. Then there is determined a unique mapping $\star_i^5$ from $\bar{F}(D)$ to $\bar{F}(D)$.

(6) Set $I_0^{\star_i^6} := I_0^{\star_i}$ for each $I_0 \in F(D)$, and set $V^{\star_i^6} := V, W^{\star_i^6} := K$ and $(MV)^{\star_i^6} := V$. Then there is determined a unique mapping $\star_i^6$ from $\bar{F}(D)$ to $\bar{F}(D)$.

(7) Set $I_0^{\star_i^7} := I_0^{\star_i}$ for each $I_0 \in F(D)$, and set $V^{\star_i^7} := K, W^{\star_i^7} := W$ and $(NW)^{\star_i^7} := NW$. Then there is determined a unique mapping $\star_i^7$ from $\bar{F}(D)$ to $\bar{F}(D)$.

(8) Set $I_0^{\star_i^8} := I_0^{\star_i}$ for each $I_0 \in F(D)$, and set $V^{\star_i^8} := K, W^{\star_i^8} := W$ and $(NW)^{\star_i^8} := W$. Then there is determined a unique mapping $\star_i^8$ from $\bar{F}(D)$ to $\bar{F}(D)$.

(9) Set $I_0^{\star_i^9} := I_0^{\star_i}$ for each $I_0 \in F(D)$, and set $V^{\star_i^9} := K$ and $W^{\star_i^9} := K$. Then there is determined a unique mapping $\star_i^9$ from $\bar{F}(D)$ to $\bar{F}(D)$.

(2.8) Each $\star_i^j$ satisfies the following conditions:

- (1) $(xI)^\star = xI^\star$ for every $x \in K \setminus \{0\}$ and $I \in \bar{F}(D)$;
- (2) $I \subseteq I^\star$ for every $I \in \bar{F}(D)$;
- (3) $(I^\star)^\star = I^\star$ for every $I \in \bar{F}(D)$.

(2.9) Let $x \in K \setminus \{0\}$, A_0, A_1 be ideals of D with type α , and B_0, B_1 be ideals with type β .

(1) If $xM \subseteq V$, then $(x) \subseteq V$.

If $xM \subseteq W$, then $(x) \subseteq W$.

If $xM \subseteq MV$, then $(x) \subseteq V$.

If $xM \subseteq NW$, then $(x) \subseteq NW$.

If $xM \subseteq A_0V$, then $(x) \subseteq A_0V$.

If $xM \subseteq B_0W$, then $(x) \subseteq B_0W$.

(2) If $xN \subseteq V$, then $(x) \subseteq V$.

If $xN \subseteq W$, then $(x) \subseteq W$.

If $xN \subseteq MV$, then $(x) \subseteq MV$.

If $xN \subseteq NW$, then $(x) \subseteq W$.

If $xN \subseteq A_0V$, then $(x) \subseteq A_0V$.

If $xN \subseteq B_0W$, then $(x) \subseteq B_0W$.

(3) If $xMN \subseteq V$, then $(x) \subseteq V$.

If $xMN \subseteq W$, then $(x) \subseteq W$.

If $xMN \subseteq MV$, then $(x) \subseteq V$.

If $xMN \subseteq NW$, then $(x) \subseteq W$.

If $xMN \subseteq A_0V$, then $(x) \subseteq A_0V$.

If $xMN \subseteq B_0W$, then $(x) \subseteq B_0W$.

(4) If $xA_0N \subseteq V$, then $xA_0 \subseteq V$.

If $xA_0N \subseteq W$, then $(x) \subseteq W$.

If $xA_0N \subseteq MV$, then $xA_0 \subseteq MV$.

If $xA_0N \subseteq NW$, then $(x) \subseteq W$.

If $xA_0N \subseteq A_1V$, then $xA_0 \subseteq A_1V$.

If $xA_0N \subseteq B_0W$, then $xA_0 \subseteq B_0W$.

(5) If $xB_0M \subseteq V$, then $(x) \subseteq V$.

If $xB_0M \subseteq W$, then $xB_0 \subseteq W$.

If $xB_0M \subseteq MV$, then $(x) \subseteq V$.

If $xB_0M \subseteq NW$, then $xB_0 \subseteq NW$.

If $xB_0M \subseteq A_0V$, then $(x) \subseteq A_0V$.

If $xB_0M \subseteq B_1W$, then $xB_0 \subseteq B_1W$.

(6) If $xV \subseteq V$, then $(x) \subseteq V$.

If $xV \subseteq MV$, then $(x) \subseteq MV$.

If $xV \subseteq A_0V$, then $(x) \subseteq A_0V$.

(7) If $xW \subseteq W$, then $(x) \subseteq W$.

If $xW \subseteq NW$, then $(x) \subseteq NW$.

If $xW \subseteq B_0W$, then $(x) \subseteq B_0W$.

(8) If $xMV \subseteq V$, then $(x) \subseteq V$.

If $xMV \subseteq MV$, then $(x) \subseteq V$.

If $xMV \subseteq A_0V$, then $(x) \subseteq A_0V$.

(9) If $xNW \subseteq W$, then $(x) \subseteq W$.

If $xNW \subseteq NW$, then $(x) \subseteq W$.

If $xNW \subseteq B_0W$, then $(x) \subseteq B_0W$.

- (2.10) (1) $\star_3^1, \star_4^1, \star_3^2, \star_4^2, \star_3^5, \star_4^5$ is not monotone for $M \subseteq MV$.
 (2) $\star_2^1, \star_4^1, \star_2^3, \star_4^3, \star_2^7, \star_4^7$ is not monotone for $N \subseteq NW$.

(2.11) Set $\mathcal{I} := \{M, N, MN, A_0N, B_0M, V, W, MV, NW \mid A_0 \text{ (resp., } B_0) \text{ is an ideal with type } \alpha \text{ (resp., } \beta)\}$ and $\mathcal{J} := \{V, W, MV, NW, A_1V, B_1W \mid A_1 \text{ (resp., } B_1) \text{ is an ideal with type } \alpha \text{ (resp., } \beta)\}$. The followings are equivalent.

- (1) $\star_i^j$ is a semistar operation.
 (2) $\star_i^j$ is monotone for every $xI \subseteq J$, where $I \in \mathcal{I}$ and $J \in \mathcal{J}$.

Proof. Assume (2). We confer (1.6) and (1.8). Let $\star := \star_i^j$, and let $xB_0W \subseteq J$ for some $J \in \bar{\mathcal{F}}(D)$.

The case $W^\star = W$. Then $(B_0W)^\star = B_0W$. Hence $\star$ is monotone for $xB_0W \subseteq J$.

The case $W^\star = K$. We may assume $J \in \{W, NW, B_1W, K\}$, where B_1 is with type β . Hence $J^\star = K$. Hence $\star$ is monotone for $xB_0W \subseteq J$.

(2.12)

- $\star_1^1$ is a semistar operation on D , and $\star_1^1 = d'$.
 $\star_1^2$ is a semistar operation on D .
 $\star_2^2$ is a semistar operation on D .
 $\star_1^3$ is a semistar operation on D .
 $\star_3^3$ is a semistar operation on D .
 $\star_1^4$ is a semistar operation on D .
 $\star_2^4$ is a semistar operation on D .
 $\star_3^4$ is a semistar operation on D .
 $\star_4^4$ is a semistar operation on D .
 $\star_1^5$ is a semistar operation on D .
 $\star_2^5$ is a semistar operation on D .
 $\star_1^6$ is a semistar operation on D .
 $\star_2^6$ is a semistar operation on D .
 $\star_3^6$ is a semistar operation on D .
 $\star_4^6$ is a semistar operation on D .
 $\star_1^7$ is a semistar operation on D .
 $\star_3^7$ is a semistar operation on D .
 $\star_1^8$ is a semistar operation on D .
 $\star_2^8$ is a semistar operation on D .
 $\star_3^8$ is a semistar operation on D .
 $\star_4^8$ is a semistar operation on D .
 $\star_1^9$ is a semistar operation on D .
 $\star_2^9$ is a semistar operation on D .
 $\star_3^9$ is a semistar operation on D .
 $\star_4^9$ is a semistar operation on D , and $\star_4^9 = v'$.

(2.10) and (2.12) imply the following,

(2.13) Let $\mathcal{S} := \{\star \in \Sigma'(D) \mid D^\star = D\}$. Then $|\mathcal{S}| = 25$.

$$(2.14) \quad |\Sigma'(D)| = 30.$$

Proof. We have $|\Sigma(V)| = 2$ and $|\Sigma(W)| = 2$. Let $\Sigma(V) = \{\sigma_1, \sigma_2\}$, and let $\Sigma(W) = \{\sigma_3, \sigma_4\}$. Let $[\sigma_1]$ (resp., $[\sigma_2]$) be the canonical extension of σ_1 (resp., σ_2) to a semistar operation on V . $[\sigma_1]$ (resp., $[\sigma_2]$) canonically introduces a semistar operation $\star^1$ (resp., $\star^2$) on D . We define $[\sigma_3], [\sigma_4], \star^3, \star^4$ similarly. Let $\star$ be a semistar operation on D such that $e \neq \star \notin \mathcal{S}$. Then $D^\star = V$ or $D^\star = W$, say $D^\star = V$. $\star$ canonically introduces a semistar operation $\alpha(\star)$ on V . Then $\alpha(\star) = [\sigma_1]$ or $\alpha(\star) = [\sigma_2]$, say $\alpha(\star) = [\sigma_1]$. For every $I \in \bar{F}(D)$, we have $I^{\star^1} = (IV)^{[\sigma_1]} = (IV)^{\alpha(\star)} = (IV)^\star = (ID^\star)^\star = I^\star$. Hence $\star^1 = [\star]$, and hence we have $\Sigma'(D) = \mathcal{S} \cup \{e, \star^1, \star^2, \star^3, \star^4\}$.

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