Computational Accuracy of Radiation Emerging from the Ocean Surface in the Model Atmosphere-Ocean System

by

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Abstract

Computational accuracy of the Cox-Munk ocean surface model is discussed using the energy conservation of the incident radiation on it. The computational scheme in the atmosphere-ocean system is based on the doubling-adding method. Firstly, with the aid of the simple modification of the refraction matrix, computational accuracy is significantly improved even in the small discrete system of (15, 24), where the first and second numerals in the parentheses denote the discrete coordinates in the atmosphere and the ocean, respectively. This modification is necessary especially for the computations of the radiation including polarization effect which require a great deal of computer time and memory space. Secondly, the effects of shadows, multiple reflections, and refractions by the ocean surface are discussed to improve the accuracy. Finally, a computational example of albedo, the upwelling radiance and the degree of linear polarization at the top of the atmosphere-ocean system are discussed at wavelength $0.5\mu\text{m}$.

1. Introduction

Computer simulation of radiative transfer in the atmosphere-ocean system is useful not only for the interpretation of the remote sensing data observed by satellites, airplanes and ships, but also for the development of remote-sensing techniques. Upon computing the radiation in the visible or its adjacent wavelength regions, the reflection and the refraction by the ocean surface should appropriately be treated, together with the scattering and the absorption in the atmosphere and the ocean. Here the computational accuracy of the radiation reflected or transmitted by the atmosphere can separately be investigated using the energy conservation law for a pure scattering medium. It was found that a computational accuracy would be satisfactory in a rather small system of 15 discrete quadrature points.

As for the ocean surface, the radiation refracted by the surface tends to become large

at the direction connected by Snell's law. Thus to simulate the refracted radiation accurately, large discrete coordinates would be required. Previously a system of (15, 24, 45) was adopted by Takashima and Masuda (1985) in their model computations, where the numerals in the parentheses represent the discrete directions in the atmosphere, in the ocean and from the atmosphere into the ocean, respectively in that order. In this system, to avoid the above overestimation, an empirical technique to smooth out the extreme peak was adopted. Thus the coordinates for the refracted radiation was additionally considered. This procedure is tiresome, and yet the size of the system could not be reduced very much.

In this paper, therefore, computational accuracy of radiation reflected and refracted by the Cox-Munk model surface (1955) is discussed using the energy conservation of the incident radiation onto the surface. This model surface is simulated by many facets whose

slopes are changed according to the Gaussian distribution with respect to wind speed. A numerical example is carried out at wavelength $0.5\mu\text{m}$ which is within the most transparent water region. To improve accuracy, the shadowing effect of the incident radiation onto the surfaces is also discussed. This effect becomes significant at a solar zenith angle of $\sim 90^\circ$. Furthermore, the effect of the multiple reflections and refractions by the model ocean surface are discussed. Finally, a computational example of albedo, the upwelling radiance and the degree of linear polarization at the top of the atmosphere are shown with respect to the effect of the ocean surface, the atmospheric and the oceanic constituents.

2. Model Atmosphere-Ocean System

Fig. 1 shows the diagram of the atmosphere-ocean system used in the present computations. The vertical concentration profile and size distribution of aerosols are given by Selby and McClatchey (1972). Visibil-

Fig. 1. Diagram showing the atmosphere-ocean system.

ity of the atmosphere is in the clear model. The aerosols are assumed to be spherical, and their size distribution function is the same at all altitudes, given by the form :

$$n(r) = \begin{cases} c \times 10^4 & \text{for } 0.02 \mu\text{m} \leq r \leq 0.1 \mu\text{m} \\ c \times r^{-4} & \text{for } 0.1 \mu\text{m} \leq r \leq 10.0 \mu\text{m} \\ 0 & \text{for } r \leq 0.02 \mu\text{m} \text{ or } r \geq 10.0 \mu\text{m} \end{cases} \quad (1)$$

where the constant c is normalized to 0.08824 for the case of 100 particles. The scattering phase matrix is computed at wavelength $0.5 \mu\text{m}$ based on the refractive index of water ($m = 1.335 - i0.0$) given by Hale and Querry (1973). The optical thicknesses of atmospheric constituents such as aerosols (τ_a), molecules (τ_r) and ozone (τ_o) are given by $\tau_a = 0.194$, $\tau_r = 0.146$ and $\tau_o = 0.00931$. The values of τ_r and τ_o are compiled for the summer mid-latitude region of the LOWTRAN 5 model atmosphere (Kneizys et al., 1980). Upon computing the emergent radiation, the inhomogeneous atmosphere is simulated by nine homogeneous sub-layers. The ocean surface is simulated by many facets whose slopes show an isotropic Gaussian distribution (Cox and Munk, 1955). The case of wind speed 2m/sec is considered. The ocean is assumed to be homogeneous and its bottom to absorb all radiation incident on it.

A pure water model is assumed for the molecular scattering and absorption. The scattering coefficient of pure water is given by Morel (1974), where the effect of the dissolved salts is ignored. This effect is usually neglected and can be of importance only for the clearest water. The absorption coefficients are obtained from the imaginary part of the refractive index of water given by Hale and Querry (1973). Little information is available for hydrosols, therefore in the present work, the data compiled by Tanaka and Nakajima (1977) is adopted, where the size distribution of hydrosols is given by the form :

$$n(r) = \begin{cases} c_s \times r^{-4} & \text{for } 0.1 \mu\text{m} \leq r \leq 22 \mu\text{m} \\ 0 & \text{for } r \leq 0.1 \mu\text{m} \text{ or } r \geq 22 \mu\text{m} \end{cases} \quad (2)$$

where $n(r)dr$ is the number density of hydrosols with radii between r and $r+dr$ and c_s is a constant representing the turbidity condition. In this paper $c_s = 30000$ is adopted, which

corresponds to the hydrosol's number density of 10^7 particles/cm³. For the complex index of refraction of hydrosols, $m = 1.16 - i0.0$ is adopted.

The incident radiation just beneath the ocean surface is attenuated 99.3% to become only 0.7% of the initial energy when it travels through ocean water of an optical thickness of 5. Therefore, the contribution of radiation below this depth is neglected. This assumption is equivalent to the condition that the ocean water is bounded by a black body at the depth of this optical thickness.

3. Method of Calculation

The doubling-adding method is used for calculations reported in this paper. Theoretical exposition and applications of the theory to an inhomogeneous atmosphere-ocean system have previously been presented by Takashima (1985). On computing the radiation, it is necessary to determine a set of discrete directions (quadrature points) appropriate to the atmosphere-ocean system. Fig. 2 shows a schematic diagram of the set of quadrature points. The region in the atmosphere is covered by n_1 quadrature points determined as zeros (μ_i^a) of the shifted n_1 -th Legendre polynomial of range (0,1), where μ_i^a are the cosines of the incident angles. The integral between the limits 0 and 1 is expressed by the Legendre-Gauss quadrature formula:

Fig. 2. Schematic diagram of the set of quadrature points.

$$\int_0^1 f(\mu^a) d\mu^a = \sum_{i=1}^{n_1} w_i^a f(\mu_i^a) \quad (3)$$

where w_i^a are the weights corresponding to μ_i^a . The quadrature points (μ_i^a) within the ocean are found by Snell's law, namely

$$\mu_i^o = (1 - (1 - \mu_i^{a2})/n^2)^{1/2} \quad (4)$$

where μ_i^o are the cosines of the refracted angles within the ocean and n is the index of refraction of water relative to air. The corresponding weights are

$$w_i^o = w_i^a \mu_i^a / (\mu_i^o n^2) \quad (5)$$

An additional set of $(n_2 - n_1)$ quadrature points is mapped into the region from the critical angle to the horizon. They are determined as zeros of the shifted $(n_2 - n_1)$ -th Legendre polynomial of range $(0, (1 - n^{-2})^{1/2})$.

4. Reflection and Refraction Matrices Expressed by the Cox-Munk Model.

Cox and Munk made measurements of the sun glitter from aerial photographs (1954a,b). Their measurements covered wind speeds ranging from 0 to 14 m sec⁻¹. They found that there are thousands of dancing highlights in the photographs. At each highlight there must be a small water facet which is so inclined as to reflect an incoming ray from the sun towards the observer. To study these surface characteristics, let us consider the average brightness of the sea surface over a sufficiently long time and a sufficiently wide surface area to smooth out fluctuations due to individual glitter sparkles of sunglint. The average is then essentially independent of time but varies smoothly with the azimuth and the elevation of the sea surface under consideration. With this consideration, the ocean surface can be treated as a homogeneous air-water interface. Hence the surface can be numerically simulated by many facets, whose slope components show the Gaussian distribution with respect to surface wind (Cox and Munk, 1955). It is isotropic in the case of distribution independent of wind direction, but anisotropic in the case of distribution depending upon wind direction. It should be noted that with the increase of wind speed, the probability of white cap occurrence, which is the formation of innumerable bubbles, in-

creases in the field of view. At present the optical properties of the white cap is not well known. Therefore in the present work, the effect of the white cap on the reflection and the refraction matrices of radiation is not yet undertaken. The density function of the wave slope $p(z_x, z_y)$ obeying an isotropic Gaussian distribution is expressed as

$$p(z_x, z_y) = (\pi\sigma^2)^{-1} \exp(-(z_x^2 + z_y^2)/\sigma^2) \quad (6)$$

where z_x and z_y represent the slope components in the X - and Y -directions in the sun vertical and the plane perpendicular to it, respectively. These are measured from the averaged smooth ocean surface. Cox and Munk (1955) found from airplane photographs that the mean square slope of $\sigma^2 = \langle z_x^2 + z_y^2 \rangle_{AV}$ increases, regardless of the direction, with the masthead wind speed of v (m/sec) according to

$$\sigma^2 = 0.003 + 0.00512 \times v \pm 0.004 \quad (7)$$

The reflection and the refraction matrices can be converted to parameters regarding to the incident and the reflected or refracted directions with the aid of spherical trigonometry. The reflection matrix is expressed by the form (Takashima, 1985) :

$$\begin{aligned} & \overline{R}_s(\mu_1, \mu_0, \phi_1 - \phi_0) \\ &= \frac{4a^2}{\sigma^2} \exp\left(\frac{1-2a}{\sigma^2}\right) \overline{R}_c(\pi - \beta') \\ & \times \overline{R}_{sp}(\chi_i, \chi_t, n) \overline{R}_c(-\alpha') \end{aligned} \quad (8)$$

where parameter a is given by

$$a = \frac{1 + \mu_0\mu_1 - \sqrt{1 - \mu_0^2} \sqrt{1 - \mu_1^2} \cos(\phi_1 - \phi_0)}{(\mu_0 + \mu_1)^2} \quad (9)$$

where ϕ_0 and ϕ_1 are the azimuth angles, μ_0 and μ_1 are cosines of incident and reflected directions in the meridian coordinates, respectively (Fig. 3). $\overline{R}_{sp}(\chi_i, \chi_t, n)$ is the reflection matrix of a specular surface expressed in Chandrasekhar's representation of the Stokes vector (Chandrasekhar, 1950) :

$$\overline{R}_{sp} = \begin{pmatrix} r_{//}^2 & & & \\ & r_{\perp}^2 & & \\ & & -r_{//}r_{\perp} & \\ & & & -r_{//}r_{\perp} \end{pmatrix} \quad (10)$$

where elements $r_{//}$ and $r_{\perp}$ are given by Born and Wolf (1964) in the forms :

$$r_{//} = -\frac{n \cos \chi_i - \cos \chi_t}{n \cos \chi_i + \cos \chi_t} \quad (11)$$

$$r_{\perp} = \frac{\cos \chi_i - n \cos \chi_t}{\cos \chi_i + n \cos \chi_t} \quad (12)$$

Fig. 3. Diagram showing the reflection by the ocean surface. I_0 and I_1 are the incident and the reflected radiation, respectively. n is the normal to the facet. ϕ_0 and ϕ_1 are the azimuth angles, μ_0 and μ_1 are cosines of the incident and the reflected direction in the meridian coordinates, respectively. χ_i is the incident or reflection angle to the facet. α and β are the azimuth angle and tilt of the facet, respectively. α' and β' are rotation angles.

where χ_i and χ_t are the incident and the refracted angle, respectively, and n is the index of refraction of medium (II) relative to that of medium (I). $\bar{R}_c$ is the rotation matrix which is given by Chandrasekhar (1950) in the form :

$$\bar{R}_c(\phi) = \begin{pmatrix} (1+\cos 2\phi)/2 & (1-\cos 2\phi)/2 & 0 & 0 \\ (1-\cos 2\phi)/2 & (1+\cos 2\phi)/2 & 0 & 0 \\ -\sin 2\phi & \sin 2\phi & \sin 2\phi/2 & 0 \\ 0 & 0 & -\sin 2\phi/2 & 0 \\ \cos 2\phi & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad (13)$$

In the scalar form, $\bar{R}_{sp}$ becomes $(r_{\parallel}^2 + r_{\perp}^2)/2$, and the rotation matrices are omitted. Similarly, the refraction matrix is expressed as

$$\begin{aligned} \bar{R}_T(\mu_2, \mu_0, \phi_2 - \phi_0) = & \frac{4}{\sigma^2} \left(\frac{d-1/n}{c} \right) \exp \left(\frac{1 - (1+1/n^2 - 2d/n)/c^2}{\sigma^2} \right) \\ & \times \left(\frac{b}{c} \right) \bar{R}_c(\pi - \beta') \bar{T}_{sp}(\chi_i, \chi_t, n) \bar{R}_c(-\alpha') \quad (14) \end{aligned}$$

where

$$b = (1 - \mu_0 \mu_2 / n - (1/n) \sqrt{1 - \mu_0^2} \times \sqrt{1 - \mu_2^2} \cos(\phi_2 - \phi_0)) / c^2 \quad (15)$$

$$c = \mu_2 - \mu_0 / n \quad (16)$$

$$d = \mu_0 \mu_2 + \sqrt{1 - \mu_0^2} \sqrt{1 - \mu_2^2} \cos(\phi_2 - \phi_0) \quad (17)$$

Where ϕ_0 and ϕ_2 are the azimuth angles, μ_0 and μ_2 are cosines of incident and refracted directions in the meridian coordinates, respectively (Fig.4). $\bar{T}_{sp}(\chi_i, \chi_t, n)$ is the refraction matrix of the specular surface in the form :

$$\bar{T}_{sp} = |n| \frac{\cos \chi_t}{\cos \chi_i} \begin{pmatrix} t_{\parallel}^2 & 0 & 0 \\ 0 & t_{\perp}^2 & 0 \\ 0 & 0 & -t_{\parallel} t_{\perp} \\ 0 & 0 & -t_{\parallel} t_{\perp} \end{pmatrix} \quad (18)$$

where elements $t_{\parallel}$ and $t_{\perp}$ denote

$$t_{\parallel} = \frac{2 \cos \chi_i}{n \cos \chi_i + \cos \chi_t} \quad (19)$$

and

$$t_{\perp} = \frac{2 \cos \chi_i}{\cos \chi_i + n \cos \chi_t} \quad (20)$$

In the scalar form, $\bar{T}_{sp}$ becomes $|n| (\cos \chi_t / \cos \chi_i) (t_{\parallel}^2 + t_{\perp}^2) / 2$, and the rotation matrices are omitted. The orientation of a facet is given as a function of $(\mu_0, \mu_1, \phi_1 - \phi_0)$ or $(\mu_0, \mu_2, \phi_2 - \phi_0, n)$. The effective area must be calculated when the relation between the direction of the incident radiation and the orientation of the facet is in the case of Fig. 5(b). The ratio of the effective area to the total area of the facet is given by (see Appendix):

Fig. 4. Diagram showing the refraction by the ocean surface. I_0 and I_2 are the incident and the refracted radiation, respectively. χ_i and χ_t are the incident and the refraction angle to the facet, respectively. n , ϕ_0 , μ_0 , ϕ_2 , μ_2 , α , β , α' and β' are similar to those of Fig. 3.

Fig. 5. Relation between the direction of the incident radiation and the orientation of the facet.

$$r = 2\mu_0 \cos\beta / \cos\chi_i \quad (21)$$

Therefore $\bar{R}_s$ and $\bar{R}_T$ should be corrected by a factor of r . It should be noted that the reflectance ρ_j and the transmittance t_j can be related to

$$\rho_j + t_j = 1 \quad (22)$$

in agreement with law of conservation of energy. ρ_j and t_j in eq (22) are expressed by

$$\rho_j = \frac{2\pi}{\mu_{0j}} \int_0^1 \frac{1}{4\pi\mu_1} \left\{ \frac{1}{2\pi} \int_0^{2\pi} \bar{R}_s(\mu_1, \mu_{0j}, \phi_1 - \phi_{0j}) d(\phi_1 - \phi_{0j}) \right\} \mu_1 d\mu_1 \quad (23)$$

$$t_j = \frac{2\pi}{\mu_{0j}} \int_0^1 \frac{1}{4\pi\mu_2} \left\{ \frac{1}{2\pi} \int_0^{2\pi} \bar{R}_T(\mu_2, \mu_{0j}, \phi_2 - \phi_{0j}) d(\phi_2 - \phi_{0j}) \right\} \mu_2 d\mu_2 \quad (24)$$

Equations (23) and (24) are expressed in the discrete coordinate system described in Chap. 3 as

$$\rho_j = \frac{1}{2\mu_{0j}} \sum_{i=1}^{n_1} w_i \bar{R}_s(\mu_{1i}, \mu_{0j}) \quad (25)$$

$$t_j = \frac{1}{2\mu_{0j}} \sum_{i=1}^{n_2} w_i \bar{R}_T(\mu_{2i}, \mu_{0j}) \quad (26)$$

where w_i are the weight functions of the Legendre-Gauss quadrature formula, and $\bar{R}_s$ and $\bar{R}_T$ are azimuth averaged radiance, which are expressed as

$$\begin{aligned} \bar{R}_s(\mu_{1i}, \mu_{0j}) &= \frac{1}{2\pi} \int_0^{2\pi} \bar{R}_s(\mu_{1i}, \mu_{0j}, \phi_{1i} - \phi_{0j}) d(\phi_{1i} - \phi_{0j}) \\ &= \frac{1}{2\pi} \sum_{k=1}^{n_3} w_k \bar{R}_s(\mu_{1i}, \mu_{0j}, (\phi_{1i} - \phi_{0j})_k) \end{aligned} \quad (27)$$

$$\begin{aligned} \bar{R}_T(\mu_{2i}, \mu_{0j}) &= \frac{1}{2\pi} \int_0^{2\pi} \bar{R}_T(\mu_{2i}, \mu_{0j}, \phi_{2i} - \phi_{0j}) d(\phi_{2i} - \phi_{0j}) \\ &= \frac{1}{2\pi} \sum_{k=1}^{n_3} w_k \bar{R}_T(\mu_{2i}, \mu_{0j}, (\phi_{2i} - \phi_{0j})_k) \end{aligned} \quad (28)$$

The Simpson rule was adopted for the integrations in eqs.(27) and (28). Eqs.(25) and (26) are for cases where the radiation is transmitted from the atmosphere to the ocean; when the transmission is reverse, n_1 and n_2 are exchanged.

5. Computational Results and Discussions.

Fig. 6 shows the reflectance ρ , the transmittance t , and their sum $\rho + t$ computed by eqs.(21), (25), (26), (27) and (28). n_1 and n_2 are numbers of quadrature points in the atmosphere and in the ocean, respectively. Fig. 7 shows the azimuth averaged radiances of the reflected radiation ($\bar{R}_s(\mu_1, \mu_{0j})/4\pi\mu_1$) and those of the refracted radiation ($\bar{R}_T(\mu_2, \mu_{0j})/4\pi\mu_2$), respectively. In the case of $(n_1, n_2) = (100, 140)$ in Fig. 6, the sum $\rho + t$ in eq.(22) becomes 1, which is mostly satisfied, whereas in the cases of $(n_1, n_2) = (15, 24)$ or $(30, 45)$, the sum becomes $\rho + t > 1$, when θ_0 is small. This results mainly from the overestimation of the transmittance t . These facts are explained by using Fig. 7 as follows. The azimuth averaged radiances of the reflected radiation are relatively smooth, whereas those of the refracted radiation have sharp peaks at the directions connected by Snell's law; therefore, the degree of the Legendre-Gauss quadrature formula of eq. (26) is too small to simulate the integration of eq.(24). In order to overcome this insufficiency, n_1 and n_2 should be fairly large. However, the computations with large values of n_1 and n_2 , such as (100, 140), require a great deal of computer time and memory space, especially when the polarization effects are consid-

Fig. 6. Reflectance ρ , transmittance t and their sum $\rho + t$. (a) and (c) indicate the case where the incident radiation is from above the ocean surface, whereas (b) and (d) are from below it. θ_0 in abscissae denotes the incident angle. $n = 1.335$ at $\lambda = 0.5\mu\text{m}$ and $v = 2\text{m/sec}$.

ered. Therefore, we modified the refraction matrix to be so efficient in computation that the condition $\rho_j + t_j = 1$ might be satisfied:

$$1 = \rho_j + t_j = \frac{1}{2\mu_{0j}} \sum_{i=1}^{n_1} w_i \tilde{R}_s(\mu_{1i}, \mu_{0j}) + \frac{1}{2\mu_{0j}} \sum_{i=1}^{n_2} w_i \tilde{R}_T(\mu_{2i}, \mu_{0j}) + \frac{1}{2\mu_{0j}} w_j \tilde{R}_T^*(\mu_{2j}, \mu_{0j}) \quad (29)$$

where $\tilde{R}_s$ and $\tilde{R}_T$ are computed by eqs. (27) and (28), respectively, whereas $\tilde{R}_T^*$ is given by

$$\tilde{R}_T^*(\mu_{2j}, \mu_{0j}) = \left\{ 2\mu_{0j} - \sum_{i=1}^{n_1} w_i \tilde{R}_s(\mu_{1i}, \mu_{0j}) - \sum_{i=1, i \neq j}^{n_2} w_i \tilde{R}_T(\mu_{2i}, \mu_{0j}) \right\} / w_j \quad (30)$$

To examine the usefulness of this modification in the atmosphere-ocean system, we computed the albedo and the azimuth averaged radiance of the upwelling radiation at the top with a modified system (15, 24) and a non-modified system (100, 140) (Fig. 8). To evaluate the radiance and the degree of polarization, the equation of radiative transfer is

Fig. 7. Azimuth averaged radiance ($\pi F=1$).

Fig. 7a indicates the case where the incident radiation is transmitted from the atmosphere into the ocean, whereas in Fig. 7b the transmission is reverse. The symbols θ_0 and θ represent the incident and the reflection or refraction angle, respectively. $n=1.335$ at $\lambda=0.5\mu\text{m}$. $v=2\text{m/sec}$.

Fig. 8. (a) albedo and (b) azimuth averaged radiance at the top of the atmosphere ($\pi F=1$).

The symbols θ_0 and θ represent the solar zenith and the observation angles, respectively. $\lambda=0.5\mu\text{m}$ and $v=2\text{m/sec}$.

expanded into the Fourier series about an azimuthal difference. This computation would require a great deal of computer time and memory space. Therefore, we computed the albedo and the radiance averaged over the azimuth angle which corresponds to the first component of the Fourier series expansion in

the scalar form. The values of both the albedo and the azimuth averaged radiance in the modified system (15, 24) agree with those in the system (100, 140). The detail results are also shown at Table 1. This suggests that the method using eq. (30) is useful for the computation of the radiation in the atmosphere-ocean sys-

Table 1. Relative differences(%) between the values computed by the modified system (15, 24) and non-modified system (100, 140).

(a) albedo

solar zenith angle (°)						
6.3	22.5	38.2	53.1	66.5	77.6	
−0.27	−0.31	−0.28	−0.10	0.05	0.12	
(49.26	38.97	20.71	9.38	6.47	4.14) *	

* As a reference, the relative differences (%) between the two non-modified systems (15, 24) and (100, 140) are also shown.

(b) azimuth averaged radiance

observation angle (°)	solar zenith angle (°)					
	6.3	22.5	38.2	53.1	66.5	77.6
6.3	-0.75	-1.58	-1.84	-1.56	-1.15	-0.72
22.5	-0.04	0.15	-0.10	0.04	0.14	0.15
38.2	-0.94	-0.89	-0.63	-0.61	-0.41	-0.18
53.1	0.54	0.44	0.44	0.60	0.49	0.34
66.5	-0.39	-0.51	-0.45	-0.18	0.13	0.16
77.6	-0.02	-0.16	-0.14	-0.04	0.13	0.27

Fig. 9. Example of reflected radiation directed towards the opposite hemisphere.

Fig. 10. Example of multiple reflections and refractions by the ocean surface.

tem even for rather small-size discrete coordinate systems.

It should be noted that there exist regions where the $\sum \rho + t < 1$ when θ_0 is between 70° and 90° in Fig. 6. This may be partly due to the fact that the computation of radiations whose incident and reflected directions are in the opposite hemispheres to each other (Fig. 9) is not included in eqs. (23) or (25). Furthermore, multiple reflections and refractions, such as shown in Fig. 10, are not considered in the present computations. Fortunately, the effects of these errors are negligible for the computation of radiation in the entire atmosphere-ocean

system.

Finally, we show the albedo, the radiance and the degree of polarization at the top of the atmosphere-ocean system denoted by (A+S+O; $\times$), with a modified system (15, 24) (Fig. 11). To evaluate the effect of the ocean surface and the hydrosols in the ocean, the results for the atmosphere-surface system (A+S: Δ), where the radiation into the ocean is assumed to be completely absorbed and for the atmosphere only (A; $\circ$) are also shown. Fig. 11a shows the albedo where θ_0 is the solar zenith angles. Three figures of Fig. 11b show the radiance for the solar zenith angles (θ_0) 66.46° ,

38.23° and 6.28° from top to bottom, respectively, where the abscissa denotes the nadir angle of observations (θ) with the solar horizon on the left ($\phi=0^\circ$), nadir at the center, and the antisolar horizon on the right ($\phi=180^\circ$). The corresponding degrees of polarization are shown in Fig. 11c. The values of the albedo and the radiance increase in the order of A, A+S, A+S+O by the effect of the ocean surface and the hydrosols in the ocean. Radiance is relatively high at large nadir angles of observations due to the atmospheric scattering, and strong sunglints are noted at the specular directions. The effect of the hydrosols is strong in the region of the small nadir angle of observations (θ), and decreases as θ approaches to the horizon or the sunglint region. As for the degree of polarization, one or two maximal values appear for large solar zenith angles (38.23° and 66.46°). One is located in the direction $\sim 90^\circ$ away from the solar incident direction, mainly from the atmospheric scattering ($P_{\max}^{90}$). The other maximum degree of polarization ($P_{\max}^s$) appears near the specular direction in the case of A+S and A+S+O, resulting from the radiation reflected by the surface; $P_{\max}^s$ appears when θ_0 is near the Brewster angle (53.3°), where the reflected radiation is most polarized. For $\theta_0=66.46^\circ$, $P_{\max}^s$ appears separately from $P_{\max}^{90}$, whereas for $\theta_0=38.23^\circ$, $P_{\max}^s$ does not appear separately from $P_{\max}^{90}$. It should be noted that for $\theta_0=6.28^\circ$, $P_{\max}^s$ does not

appear because of the small degree of polarization of radiation reflected by the surface. The degree of polarization decreases by the effect of hydrosols, because the upward radiation traveling diffusely through the air-ocean boundary disturbs the radiation field in the atmosphere.

6. Conclusion

Upon computing the radiation in the atmosphere-ocean system, large quadrature points such as (100, 140) in the discrete coordinates must be adopted to obtain sufficient accuracy in the numerical results, because the radiation refracted by the ocean surface should be simulated appropriately, which becomes very large with a sharp peak at the direction connected with the incident direction by Snell's law. Computation with such a large system, however, requires a great deal of computer time and memory space. To avoid this difficulty, we have presented the method of modification of the refraction matrix so that the law of energy conservation at the surface might be satisfied. Using the present method, satisfactory accuracy on computations has been obtained even in the case of rather small quadrature points in discrete coordinates such as (15, 24). This method makes it possible to compute the radiation in the entire atmosphere-ocean system including the polarization effect with a moderate size computer. The effects of

Fig. 11. (a) albedo, (b) radiance and (c) degree of linear polarization at the top of the atmosphere ($\pi F=1$). The symbols θ_0 and θ represent the solar zenith and the observation angles, respectively. $\lambda=0.5\mu\text{m}$ and $v=2\text{m/sec}$. $(n_1, n_2)=(15, 24)$. Principal plane is shown in Figs. (b) and (c).

multiple reflections and refractions as well as that of the white cap which is formed by innumerable bubbles are not included in the present computation technique. In order to simulate the ocean surface more accurately, the

development of a technique to include these effects is desired for the reflection and refraction matrices of radiation.

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Fig. 11. continued

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APPENDIX. Effective Area of the Facet.

In order to estimate the effect of the adjacent regions to the facet, we set the plane π parallel to the facet and planes corresponding to the adjacent regions as shown in Figs. a and b. Without loss of generality, the incident direction of radiation I_0 can be parallel to the Z - Y plane. The direction cosines of I_0 are

$$(a_1, b_1, c_1) = (0, -\sin \theta_0, -\cos \theta_0) \quad (\text{a-1})$$

(a)

(b)

Fig. app. (a) Effective area of the facet and (b) the corresponding plane figure seen downwards along the Z -axis. The symbols I_0 , α and β denote the incident radiation, the azimuth and the tilt of the facet, respectively. The oblique lines represent the shadow area.

where θ_0 is the zenith angle of I_0 . The direction cosines of the normal of plane π are

$$(a_0, b_0, c_0) = (\sin \alpha \sin \beta, \cos \alpha \sin \beta, \cos \beta) \quad (\text{a-2})$$

where α and β are the azimuth angle and tilt of plane π . The coordinates of the point P are

$$(x_1, y_1, z_1) = (\sin \alpha \cot \beta, \cos \alpha \cot \beta, 1) \quad (\text{a-3})$$

assuming the height of the point P to be 1. With the aid of elementary geometry, the equation of a plane is expressed as

$$a_0(x - x_0) + b_0(y - y_0) + c_0(z - z_0) = 0 \quad (\text{a-4})$$

where (a_0, b_0, c_0) are the direction cosines of the normal to the plane, and (x_0, y_0, z_0) are the coordinates of a point on it. The equation of a straight line is expressed, by using parameter t , as

$$\begin{cases} x = x_1 + ta_1 \\ y = y_1 + tb_1 \\ z = z_1 + tc_1 \end{cases} \quad (\text{a-5})$$

where (a_1, b_1, c_1) are the direction cosines of the line and (x_1, y_1, z_1) are the coordinates of a point on it. Substitution of eq. (a-5) into eq. (a-4) yields the parameter t which gives the point of the intersection of the plane and the straight line:

$$t = \frac{a_0(x_0 - x_1) + b_0(y_0 - y_1) + c_0(z_0 - z_1)}{a_0a_1 + b_0b_1 + c_0c_1} \quad (\text{a-6})$$

Considering that the origin $O((x_0, y_0, z_0) = (0, 0, 0))$ lies on the plane π , eqs. (a-1), (a-2), (a-3) and (a-6) yield the parameter t which gives the point Q of the intersection of the plane π and the line I_0 ,

$$t = 2 \cos \beta / (\cos \alpha \sin \beta \sin \theta_0 + \cos \beta \cos \theta_0). \quad (\text{a-7})$$

The denominator can be transformed into $\cos \chi_i$ with the aid of Fig. 3b. Therefore, the Z -coordinate of the point Q is

$$z_Q = 1 - 2 \cos \theta_0 \cos \beta / \cos \chi_i \quad (\text{a-8})$$

and

$$r = 1 - z_Q = 2 \cos \theta_0 \cos \beta / \cos \chi_i. \quad (\text{a-9})$$

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大気-海洋系モデルにおける海面からの放射の計算精度

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可視から近赤外域の波長での放射伝達のシミュレーションを行う場合には、海面での反射、屈折の取り扱いが計算精度上重要である。本論文では計算方法として doubling-adding 法を用いる場合の反射・屈折 matrix を等方的 Cox-Munk 海面モデルを用いて表し、その計算精度を海面でのエネルギー保存に注目して検証した。ただしこの海面モデルでは多重反射・多重屈折の効果は無視してある。波長としては比較的海水の透明度の高い 0.5 μm を用いた。検証方法としては大気中、海水中における quadrature point の数を様々に変化させて、それらの結果を比較する方法で行った。その結果屈折 matrix に若干の補正を加えると比較的 point 数の少ない (15, 24) の場合にも満足する結果が得られた。このことは多大な計算時間と記憶容量が必要な偏光度の計算を含むシミュレーションでは特に重要である。また太陽の天頂角が大きい場合に生ずる shadow の問題を解決する方法を示し、多重反射、多重屈折等による誤差についても若干の検討を行った。最後に大気上端でのアルベド、上向き放射輝度・偏光度の計算結果の例を示した。