On Severe Downslope Wind Due to the Mountain Wave-Induced Critical Level

by

Masahiko Aihara and Masanobu Hirasawa

Meteorological Research Institute, Tsukuba, Ibaraki 305, Japan

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Abstract

Numerical simulations of the evolution of the downslope wind were performed by means of the non-hydrostatic, compressible model including the effect of topography.

Result of numerical simulation with the famous Boulder windstorm-like profile as an upstream condition was examined in some detail. It was ascertained, in the example, that the violent wind on the leeside slope of the mountain, whose instantaneous wind speed rose well over 50 m/sec, was due to the amplification mechanism with the occurrence of wave breaking, or in other words, the mountain wave-induced critical level, particularly in the lower troposphere.

The characteristic features of the circulations in the troposphere as well as in the stratosphere are described briefly. The possibility of occurrence of such severe windstorms in Japan due to the mechanism mentioned above was also investigated.

1. Introduction

It is common that the mountain wave sometimes produces on the leeside slope a localized wind whose intensity is comparable to or even stronger than that of the moving meteorological disturbances, such as an extratropical cyclone or front.

The causes of downslope wind are commonly classified into two types. One is of mechanical origin, that is, an orographic distribution that strongly tends to accelerate the surface flow, such as a narrow gap in the mountain or valley and so on. For example, numerical simulations on the detailed structure of the air flow in periodic valleys were recently performed by Kimura and Manins (1987).

The other is the internal structure of the upper atmosphere itself. In this paper, we focus our attention on the latter origin.

The classical linear theory points out that a strong wind occurs over the leeside slope of a mountain even under the homogeneous atmospher-

ic condition (Queney, 1948). We will take up a similar problem later, that is, a mountain wave developed in a homogeneous atmosphere but based upon a quite different approach. Most of our concern consists in the investigation of the mechanism of the occurrence of severe downslope wind under more realistic atmospheric circumstances.

It is well known from observation that the strong downslope wind is closely related to the inversion layer in the lower troposphere and/or the presence of either linear or nonlinear critical level in the upper atmosphere. Here, the linear critical level implies that, there exists within the mean flow a level above and below which the wind direction reverses. On the contrary, the nonlinear critical level (NLCL) is defined by a local level of rather limited horizontal extent in the mean flow where the reversal of the wind direction takes place and is mainly due to the strong circulation associated with either a moving or a stationary meteorological disturbance.

It is a rather rare meteorological situation,

probably once a year or less, that the instantaneous surface wind speed over the leeside slope or at the base of a mountain well exceeds, say, 50 m sec^{-1} . Hereafter we will refer to an occasion as the extraordinary severe downslope wind or violent wind.

The mechanism of the severe downslope wind due to the optimum tuning of a partially reflected wave energy between the internal stable layer and the ground surface, proposed by Klemp and Lilly (1975), is highly convincing and supported by evidences based upon the statistical analysis of windstorms (Brinkmann, 1974).

Another explanation of the cause of the violent wind is the resonant amplification mechanism due to the multiple overreflection of the wave energy between the ground and the critical level in the shear flow where the Ri -number is less than $1/4$ and the unstable internal gravity waves are well maintained for a sufficiently long time. Detailed analyses of the amplification mechanism of the latter type have been made by various authors, especially Jones (1968), Davies and Peltier (1976), Lindzen and Rosenthal (1976), and Rosenthal and Lindzen (1983), based upon the linear theory.

However, direct observational evidence on this mechanism seems to be scarcely known. Probably, the Boulder windstorm that occurred at 11 Jan. 1972 is a typical example. According to the recent paper on the ALPEX (Hoinka, 1985), wind speeds exceeding 50 m sec^{-1} were recorded by anemometer measurement at the mountain observatory Zugspitze, 2963m high. This violent wind is currently explained as a result of the overreflection of the wave energy from NLCL in the troposphere whose existence is strongly supported by observation.

Studies on the famous Boulder windstorm event based upon the numerical simulation approach have been carried out on the various theoretical bases mentioned above, for example by Klemp and Lilly (1978), Peltier and Clark (1979) and Durran and Klemp (1983).

In this paper, we intend to follow the time evolution of a severe downslope wind in connection with either the presence or the absence of NLCL and the mountain wave-induced circulations both in the lower stratosphere and the troposphere. To avoid confusion, we define the lower

stratosphere as a portion of the atmosphere extending from the tropopause to a height of 25km.

Finally, the possibility of occurrence of a severe downslope windstorm of NLCL origin in Japan will be taken up and examined.

2. Numerical Model

Details of the non-hydrostatic, compressible model which we used in the present paper were already described in the authors' previous paper (1985, hereafter I.)

This time, however, some modifications were made mainly for the sake of computer time efficiency.

2.1 Pressure gradient term

In a finite-difference expression of the pressure gradient term both in the horizontal and in the vertical direction, we employed p itself instead of the vertically averaged pressure $\bar{p}^{\sigma\sigma}$ adopted in I. Thus, the relation between the vertical velocity w and ω , upward motion relative to the terrain following coordinate σ , is given as follows from the requirement of the identity of energy conversion term in the kinetic energy and thermodynamic equation :

$$\omega = w + (1 - \sigma) \overline{u \delta_x h^{\sigma x}},$$

where $h = h_0 - M(x)$, h_0 the height of the top boundary, $M(x)$ the topographic distribution and u the horizontal velocity. In addition, the computational boundary conditions, $\delta_\sigma u = 0$ and $\delta_\sigma m = 0$ at $\sigma = 0$, are necessary over the mountain region in order to preserve the total energy of the system.

2.2 Time integration scheme

In the previous paper I, we applied the Kurihara and Tripoli scheme (1976) to the time integration of the numerical model. The so-called time splitting method which has been proven to be effective in the numerical simulations of meso-scale meteorological phenomena (e. g., Klemp and Wilhelmson, 1978 ; Tatsumi, 1983 ; Horihata, 1986 and so on) was introduced in the present paper.

Acoustic wave relating high-frequency terms involved in the momentum, thermodynamic and continuity equations are separated from the remaining low-frequency terms and are integrated

successively with a short time increment $\Delta \tau$, while low-frequency terms are fixed temporarily.

After a small time interval $\Delta t = n \Delta \tau$, low-frequency terms are updated.

Severe restriction is usually imposed upon the length of $\Delta \tau$ for the sake of computational stability. This is mainly due to the adoption of a smaller vertical interval Δz than the horizontal gridsize Δx .

In order to lighten this restriction, we treated the terms relating to the vertical difference implicitly and all other terms explicitly in time.

Namely, for the time integration of high-frequency components, u -momentum, w -momentum, internal energy and continuity equation are written as follows :

$$\frac{\partial F}{\partial t} = -\delta_x (ph)^\tau - \delta_\sigma [(1-\sigma)\bar{p}^{\sigma x} \delta_x h]^\tau + N_x \quad (1a)$$

$$\frac{\partial \hat{G}}{\partial t} = -\delta_\sigma p^{\tau+1} + N_z \quad (1b)$$

$$\frac{\partial H}{\partial t} = -\frac{C_0^2}{\gamma-1} (\delta_x F + \delta_\sigma G')^{\tau+1} + N_h \quad (1c)$$

$$\frac{\partial m}{\partial t} = -(\delta_x F + \delta_\sigma G')^{\tau+1} + N_d, \quad (1d)$$

where $m = \rho h$, $F = m^* u$, $\hat{G} = m^\sigma w$, $H = mc_v T$ and $hG' = \hat{G} + (1-\sigma)\bar{p} \delta_x h^{\sigma x}$. C_0 means the sound wave speed and γ a ratio of specific heats. The equation of the state of gas is expressed by $ph = (\gamma-1)H$. The time tendency is replaced, for example, by

$$\frac{\partial F}{\partial t} = \frac{1}{\Delta \tau} (F^{\tau+1} - F^\tau)$$

Low-frequency terms, N_x , N_z , N_h and N_d , which include nonlinear advection, buoyancy, Rayleigh friction and nonlinear diffusion, are fixed during the time integration of high-frequency components.

The geopotential equation, not written here, is essentially of a low-frequency character.

An outline of the integration procedures will be given below.

First of all, $F^{\tau+1}$ is obtained from the momentum equation (1a), and is inserted into the righthand side of the thermodynamic equation (1c)

Eliminating $\hat{G}^{\tau+1}$ from the second and the third equation, one-dimensional Helmholtz equa-

tion concerning $H^{\tau+1}$ can be derived. Then, the Helmholtz equation is solved by means of the Lindzen-Kuo method without difficulty under the prescribed boundary conditions at the top and the bottom of the atmosphere.

Thermodynamic variable $H^{\tau+1}$ thus obtained is replaced in the righthand side of the second equation (1b) and we immediately know the spatial distributions of vertical momentum $\hat{G}^{\tau+1}$ at the new time step. After repeating this procedure n times, low-frequency terms are updated.

The time integration of low-frequency terms is carried out by use of the leap-frog scheme. The Asselin filter with the coefficient $\nu=0.1$ is applied in order to prevent the separation of prognostic variables at even and odd time steps and also to suppress the unwanted computational mode contained in the leap-frog integration scheme. The number of the integration step, e. g. $n=5$, was employed for all examples treated in this paper.

2.3 Boundary and other conditions

During the time integration of high-frequency component, radiation conditions are assigned at both the inflow and the outflow lateral boundary in order to reduce the influence of fast-propagating acoustic waves to the minimum.

As to the low-frequency components, the condition $\delta_x q = 0$ where q means the prognostic variable except u , was imposed at the inflow lateral boundary.

At the outflow boundary, on the other hand, radiation conditions are applied to all prognostic variables.

Both the thickness d and the maximum intensity of Rayleigh friction K_0 within the sponge layer settled in the upper portion of the atmosphere were selected to be optimum from the estimation of the linear theory. So that the amplitude reflectivity of the mountain wave from the top boundary is well suppressed below a few percent.

The so-called spatial noise, the wavelength of which is a two-grid interval, was eliminated by the application of Shapiro low-pass filter intermittently during the course of time integration. This is equivalent to the introduction of a biharmonic diffusion term into the prognostic equations.

The effect of energy dissipations due to the

development of the subgrid-scale turbulent motions is taken into account by use of the nonlinear diffusion terms as in I. The coefficient of nonlinear diffusion is dependent upon the local Ri -number and the deformation field. This coefficient was used in our numerical model 16 min. after the onset of time integration.

The barrier effect of topography is introduced into the numerical model through the so-called switch-on procedure. The time necessary to lift up the flat ground surface to the actual mountain height is taken to be 15 min.

3. Non-Linear Critical Level

The meso β -scale, forced mountain waves show a nearly hydrostatic character and are regarded as low-frequency internal gravity waves. The linear theory points out the important characteristics, known as Eliassen-Palm theorems, on the behaviour of the internal gravity waves superimposed upon the inviscid, adiabatic mean flow. Namely, the relation between energy flux and momentum flux due to the gravity wave is given as follows :

$$\begin{cases} \overline{p'w'} = -(U-c) \rho_0 \overline{u'w'} \\ (U-c) \frac{d}{dz} (\rho_0 \overline{u'w'}) = 0, \end{cases}$$

where U and ρ_0 express the mean flow and the mean density, respectively. Primed quantities belong to the gravity waves and c means the phase speed.

The second equation states that the momentum flux associated with the gravity wave is constant with respect to height except at critical levels. In other words, the intensity of the perturbed wind field u' and w' increases with respect to height inversely proportional to the square root of the mean density until the critical level is reached.

$$|u', w'| \propto \frac{1}{\sqrt{\rho_0}}$$

This suggests the possibility that the reversal of the direction of composite horizontal wind $u = U + u'$ may occur at some definite altitude above the mountain region. Namely, the occurrence of the wave breaking or wave-induced critical level is expected and is highly dependent upon the environmental atmospheric conditions in the upper

atmosphere and the shape of the underlying topography.

It is known from the linear theory that the amplitude distribution of the vertical motion due to the steady mountain wave in the Boussinesq atmosphere is subject to the Scorer equation. In the basic state where the atmosphere is isothermal and the flow is uniform, the Scorer equation takes the following form :

$$\frac{d^2 W}{dz^2} + (l^2 - k^2) W = 0, \quad (2)$$

here $W = \sqrt{\rho_0} \hat{w}$, $\hat{w}$ being the amplitude of the vertical motion, ρ_0 the mean density defined by $\rho_0 = \rho_s \exp(-z/H)$, H the scale height, k the horizontal wave-number. The Scorer parameter is given by $l^2 = N^2/U^2$, where N and U mean the Brunt-Vaisala frequency and the constant mean flow, respectively.

The profile of the mountain $M(x)$, which is symmetric with respect to x , can be expressed in terms of the Fourier cosine integral :

$$M(x) = h \int_0^\infty m(k) \cos(kx) dk, \quad (3)$$

where h is a maximum height of the mountain and $m(k)$ is a normalized spectral function.

The solution of the Scorer equation (2) satisfying the radiation condition at the top of the atmosphere and subject to the forced vertical motion due to topography at the ground surface is easily derived :

$$\begin{aligned} w'(x, z) &= \text{Re} \int_0^\infty \hat{w}(k, z) \exp(ikx) dk \\ &= -hU \exp(z/2H) \left[\int_0^l km(k) \sin(kx + \lambda z) dk \right. \\ &\quad \left. + \int_l^\infty km(k) \exp(-\mu z) \sin(kx) dk \right], \quad (4) \end{aligned}$$

where $\lambda^2 = l^2 - k^2$ and $\mu^2 = -\lambda^2$.

The perturbed horizontal wind $u'(x, z)$ is obtained from Eq.(4) through the continuity equation.

$$\begin{aligned} u'(x, z) &= -hU \exp(z/2H) \left[\int_0^l \left\{ \frac{1}{2H} \cos \right. \right. \\ &\quad \left. \left. (kx + \lambda z) - \lambda \sin(kx + \lambda z) \right\} \right. \\ &\quad \left. m(k) dk + \int_l^\infty \left(\frac{1}{2H} - \mu \right) m(k) \right. \\ &\quad \left. \exp(-\mu z) \cos(kx) dk \right] \end{aligned}$$

Noticing that the approximation made in the Boussinesq fluid, that is, the vertical variation of the mean density, $-(1/\rho_0)(d\rho_0/dz)$, is negligible compared with that of other mean quantities and considering the energy propagating mode ($\lambda^2 > 0$) only, we have

$$u'(x, z) = hU \exp(z/2H) \int_0^l \lambda m(k) \sin(kx + \lambda z) dk \\ \approx lhU \exp(z/2H) \int_0^l m(k) \sin(kx + lz) dk, \quad (5)$$

Thus, the spatial distributions of the minimal composite wind u are determined by the following relation :

$$kx + lz_c^{(n)} = \left(-\frac{3}{2} + 2n\right) \pi; n=0, 1, 2, \dots \quad (6)$$

The critical value of Fr -number Fr_c necessary to form the nonlinear critical level ($u=0$) at height $z_c^{(n)}$ is given by

$$Fr_c \exp(z_c^{(n)}/2H) = 1, \quad (7)$$

$$\text{where } Fr_c \equiv lh \int_0^l m(k) dk.$$

Strictly speaking, this corresponds to the inverse Froude number in the ordinary sense. It is evident that the critical Fr -number depends upon the shape of the mountain as well as the mean atmospheric condition l .

The height of NLCL z_c that appear above the mountain top ($kx=0$) is determined from Eq. (6) by the following eigenvalue :

$$lz_c^{(n)} = \left(-\frac{3}{2} + 2n\right) \pi, \quad (8)$$

Since the predominant wavelength of the forced mountain wave is approximately given by $L=2B$, B being a half-width of the symmetric mountain, those above the leeside mid-slope ($kx = \frac{\pi}{2}$) and the leeside foot ($kx = \pi$) of the mountain are respectively given by

$$lz_c^{(n)} = (1 + 2n) \pi \quad (9a)$$

$$lz_c^{(n)} = \left(\frac{1}{2} + 2n\right) \pi. \quad (9b)$$

For the homogeneous atmosphere where $U=20 \text{ m sec}^{-1}$ and $N=2 \times 10^{-2} \text{ sec}^{-1}$ are taken, height $z_c^{(n)}$ and the corresponding critical Fr -number above the top and the leeside mid-slope of

the mountain are easily calculated from Eqs. (7), (8) and (9a), and are summarized in Table 1.

4. Numerical Simulation of Mountain Waves in a Homogeneous Atmosphere

We carried out a numerical simulation of the mountain wave developed in an isothermal atmosphere having a constant mean flow.

The atmosphere treated is the same as described in the previous section,

$$\begin{cases} U=20 & \text{m sec}^{-1} \\ N=2 \times 10^{-2} & \text{sec}^{-1}, \end{cases}$$

where the Scorer parameter and the scale height are given by $l=10^{-3} \text{ m}^{-1}$ and $H=7 \text{ km}$, respectively.

We employed the cosine-square type topography as used in I. The half-width B of the mountain is taken to be 22.5 km. The normalized spectral function $m(k)$ of this topography is shown in the Appendix. It is understood that the Fr -number is simply given by a maximum height h and the mean state of the atmosphere l and is indifferent to the shape of the mountain. This is due to the fact that the integral of the spectral function is very close to unity as proved in the Appendix.

The size of the computational domain employed in the numerical simulation is defined by $M\Delta x$ and $N\Delta z$, respectively. In practice, $M=80$ and $N=68$ with $\Delta x=2.5 \text{ km}$ and $\Delta z=0.25 \text{ km}$ are taken, so that the horizontal and the vertical extent are 200 km and 17 km, respectively. The upper portion of the atmosphere above 8 km is occupied by the sponge layer.

The results of the numerical simulation with Fr -number 0.8 (i. e., $h=800 \text{ m}$) are shown below.

Table 1. Height of NLCL and the critical Froude number above the top and the leeside mid-slope of the mountain.

($U=20 \text{ m sec}^{-1}$, $N=2 \times 10^{-2} \text{ sec}^{-1}$)

Top			Leeside mid-slope		
n	Fr	Z_c	n	Fr	Z_c
0	0.7142	4.712 km	0	0.7990	3.142 km
1	0.4559	10.996	1	0.5101	9.425
2	0.2911	17.279	2	0.3256	15.708

The integration was carried out for 12 hr. The Fr -number exceeds the threshold value at $z_c^{(0)}$ slightly, while at $z_c^{(1)}$ moderately, as seen from Table 1. However, the growth of NLCL at height $z_c^{(1)}$ located far into the sponge layer is hardly expected.

Fairly good coincidences between the results of numerical simulation and the prediction by linear theory are anticipated, because the non-linearity of the surface boundary condition is negligible due to $Fr < 1$ (Smith, 1977).

Fig. 1 shows the time variation of the pressure drag D , the maximum surface wind speed over the slope of the mountain u_s and the maximum pressure difference Δp_s between the upstream and downstream grid points located at the same height within the mountain region.

The definition of the pressure drag or

surface drag is as follows :

$$D = \frac{1}{2S} \int_{-S}^S P \frac{dM}{dx} dx \quad \text{at } \sigma = 0. \quad (10)$$

where S is a half-width of the horizontal extent of the integration domain.

These three quantities are regarded as the proper measures of mountain wave activity. In this case, these quantities show rather slow development with the passage of time. The maximum pressure difference Δp_s and the surface wind speed u_s attain about 15 mb and 50 m sec⁻¹, respectively, at the maximum stage of the development. Although the linear theory predicts the maximum u_s to be $(1+Fr) U$, nonlinear model simulation produces a somewhat stronger downslope surface wind.

Fig. 1 Time evolution of the surface wave drag D , the maximum surface wind speed u_s and the maximum pressure difference across the mountain crest Δp_s . The background flow is homogeneous. $Fr=0.8$.

However, the curves shown in Fig. 1 do not indicate the tendency of resonant amplification.

In Fig. 2, fields of horizontal wind u and potential temperature θ at selected instances are shown. We can observe that the spatial distributions of maximal and minimal horizontal wind u are coincident with those predicted by linear analysis Eq. (6)

At 8 hr., NLCL was actually formed at $z_c^{(0)}$ above the downslope of the mountain and continued to exist until 12 hr. The downslope wind speed u_s is almost constant or slightly decays with

time after 8 hr., because of the onset of turbulent dissipations subsequent to the formation of NLCL at $z_c^{(0)}$. On the other hand, NLCL at $z_c^{(1)}$ was not observed during the entire period of numerical integration.

Contours of potential temperature θ at height $z_c^{(0)}$ and $z_c^{(1)}$ indicate relatively large amplitudes compared with those at other places. It is also seen that the isentropes at $z_c^{(0)}$ above the leeside slope at 8 hr. and 12 hr., respectively, show a supercritical gradient, that is, the over-turning of the contour.

Fig. 2 Field of isentropes and horizontal velocity at (a) 4, (b) 8 and (c) 12 hr. $Fr=0.8$.

In this example, well-developed internal gravity waves were not observed in the neighborhood of NLCL. One cause of this is the absence of shear kinetic energy of the basic flow necessary to enhance the wave growth, and another is the predominance of a strong turbulent mixing in the vicinity of NLCL where the stratification is convectively unstable.

When the unperturbed atmospheric conditions are subcritical, good agreement is commonly seen to exist between the results of numerical simulation and the prediction by linear theory. We can mention, as an example, a recent numerical study on the relationship between the heights of the bell-shaped topography and the surface wave drag produced (Hirasawa, et al., 1985).

5. Mountain Waves under the Presence of Strong Wind Shear and stable stratification

In the previous section, we studied the development of the mountain waves in an isothermal atmosphere having a constant basic flow. Although NLCL was actually formed at the height

estimated by linear theory, the amplitude of the mountain wave was rather small and almost constant. This is due to the absence of shear kinetic energy of the mean flow and the weakly supercritical state of the atmosphere treated.

Mean winter atmospheric conditions are, on the other hand, considerably different from those treated in the previous section. The westerly current in the middle latitudes normally possesses strong wind shears both in the vertical and horizontal directions, and a stable stratification. Under such atmospheric circumstances, a large amplitude mountain wave and an associated strong downslope wind are anticipated to develop.

The structure and behaviour of the unstable internal gravity waves in the shear flow having a critical level where the Ri -number is less than $1/4$, have been investigated by many authors as mentioned above. According to their conclusions, unstable gravity waves are apparently classified into two types. The waves that belong to the first class, similar to the classical Kelvin-Helmholtz wave, develop quite rapidly but are trapped within the shear zone of the basic flow and produce no effective net transfer of momentum or energy outside the shear zone.

On the other hand, the theory states that the second type of wave has a longer wavelength and indicates slower development compared with the first one, but reflects more energy from the critical level than incident energy, that is, the overreflection of wave energy.

The vertical profile of the latter type of wave indicates a peculiar characteristic, that is, the feature of resonant eigenmodes. Once NLCL is formed at some definite height due to the intense vertical circulation associated with the mountain wave and is well maintained by a long-lasting synoptic-scale environment, it is suggested that the overreflected wave energy from NLCL and its multiple reflections between NLCL and the ground surface intensify considerably the surface wind speed and finally result in an extraordinary severe windstorm.

The height of NLCL to be formed under the situation where the atmospheric properties slowly vary with height is estimated in the same manner as described in the previous section.

The Scorer equation is again given by Eq. (2),

$$\frac{d^2 W}{dz^2} + (l^2 - k^2) W = 0,$$

where $\lambda^2 = l^2 - k^2$ is assumed to be positive and a slowly varying function of height z . By use of WKB-method, we can obtain a solution like

$$W(z) = F(z) \exp \left\{ i \phi(z) \right\},$$

where the approximation, $\left| \frac{1}{F} \frac{d^2 F}{dz^2} \right| < \lambda^2$, holds.

The solution which satisfies the radiation condition at the top of the atmosphere and the topographically induced vertical motion at the ground surface is given by

$$w'(x, z) = -hU(0) \int_0^1 km(k) Q \sin(kx + \phi) dk,$$

$$\text{where } \begin{cases} \phi(z) = \int_0^z \lambda(z') dz' \approx \int_0^z l(z') dz' \\ Q(z) = \frac{F(z)}{F(0)} \sqrt{\frac{\rho_o(0)}{\rho_o(z)}} \end{cases}$$

From the continuity equation, the horizontal wind $u'(x, z)$ is derived. Due to the assumptions made both in the WKB-method and in the Boussinesq fluid, the following inequality is acceptable:

$$\left| \frac{1}{Q} \frac{dQ}{dz} \right| < \left| \frac{d\phi}{dz} \right|$$

Hence, corresponding to Eq. (5),

$$u'(x, z) \approx hU(0) \int_0^1 m(k) Q \frac{d\phi}{dz} \sin(kx + \phi) dk \quad (11)$$

The location of NLCL, therefore, is determined by the following relation similar to Eq. (6):

$$kx + \phi(z_c) = \left(\frac{3}{2} + 2n\right)\pi; n=0, 1, 2, \dots \quad (12)$$

Especially, above the top of the mountain, Eq. (12) reduces to

$$\phi(z_c) = \left(\frac{3}{2} + 2n\right)\pi \quad (13)$$

To the occurrence of NLCL in the upper atmosphere, it is further required that the Fr -number in the vicinity of NLCL should exceed some definite value, but it is difficult to estimate.

In this section, we will direct our attention to the structure of the mountain wave developed, especially, in the lower stratosphere and the

evolution of downslope wind under the atmospheric condition shown in Fig. 3. The maximum height of the mountain $h = 2$ km is taken at first. The linear theory predicts that, under such situations, NLCL will appear at the height $Z_c^{(0)} = 11.5$ km and $Z_c^{(1)} = 18$ km, that is, in the lower and the middle layers of the lower stratosphere, respectively.

It is readily seen from Fig. 3 that the vertical distribution of the Fr -number exceeds unity both in the vicinity of the ground surface and in the lower stratosphere.

The steady finite-amplitude theory applied, for example, to the semi-elliptical mountain by Huppert and Miles (1969) shows that the critical steepening of the streamline takes place in the leeside upper atmosphere if $Fr \geq Fr_c \sim 1$. However, it should be remarked that the theory is valid only for a special class of mean field satisfying the linearization condition of Long's original nonlinear equation.

Although the linear theory developed above is not applicable beyond the limit, i.e., $Fr = 1$, in the strict sense, it still offers a theoretical basis for the understanding of the results of numerical simulation under a wide variety of atmospheric condition.

The domain of numerical integration extends over 200 km in the horizontal and 25 km in the vertical direction. The grid size $\Delta x = 2.5$ km and the layer depth $\Delta z = 0.5$ km are adopted. The sponge layer is set above the height 16 km.

The unperturbed atmospheric condition shown in Fig. 3 has similar features to those of Grand Junction, the upstream-side upper air observation station at the Boulder windstorm event. However, it should be noticed that the inversion layer in the lower troposphere observed in the Grand Junction data is completely ignored in Fig. 3. Therefore, the possibility of the wave amplification due to the partial reflection of energy

Fig. 3 Vertical profile of horizontal wind and temperature. The Boulder windstorm-like situation. Corresponding Fr -number is also shown for the mountain height $h = 2$ km.

from the interfacial boundary is excluded from the first in our simulation. The importance of the inversion layer in the occurrence of severe wind was recently stressed by Durran (1986).

In Fig. 4, time evolutions of pressure drag D , maximum surface wind speed u_s and maximum surface pressure difference Δp_s are shown.

From the onset of numerical integration to about 8 hr., a tendency of gradual growth is seen in these quantities, namely, u_s is less than 40 m sec^{-1}

and Δp_s does not exceed 10 mb. All three quantities temporarily show a small amount of decay just after 8 hr. and then, grow rapidly, almost linearly with time. At 10 hr., the development of the mountain wave comes near to a mature stage, that is, the pressure drag attains 130 dyne cm^{-2} , u_s is well stronger than 50 m sec^{-1} and Δp_s develops as high as 18 mb. The degree of development of the mountain wave is almost the same as was observed at the Boulder windstorm (e.g. Lilly, 1978).

Fig. 4 Surface wave drag, the maximum surface wind speed and the maximum pressure difference as a function of time for the upstream conditions shown in Fig. 3. The mountain height $h = 2 \text{ km}$ is taken.

The structures of the mountain wave at selected times are shown in Fig. 5. From the beginning to 8 hr., it is remarkable that the horizontal wind at height $Z_c^{(1)}$ above the mountain top region continues to decay, while the localized jet stream is formed at about 15 km and accelerated steadily.

At 8 hr., we can observe the formation of NLCL at $Z_c^{(1)}$ and the deepening of the potential temperature trough above the leeside slope of the mountain. Thus, the initial flow field near height $Z_c^{(0)}$ above the mountain top region was considerably distorted by a marked development of the circulation in the middle layers of the lower stratosphere. Therefore, the formation of NLCL

at $Z_c^{(0)}$, anticipated from the linear theory, was entirely interrupted.

Flow patterns in the lower stratosphere at 8 hr. and 10 hr. do not indicate a wide difference but the localized jet stream is much accelerated. On the other hand, in the middle troposphere about 6 km above the leeside slope, the amplification of the wave advances steadily during the time interval from 8 hr. to 10 hr. Subsequent to the wave breaking near 6 km, the downslope wind grows rapidly. We can observe that, at 10 hr., wind speed u_s exceeds 50 m sec^{-1} . The amplitude of potential temperature, at that time, is estimated to be more than 5 km. Features of isentropic contours in the neighbourhood of the newly estab-

Fig. 5 As in Fig. 2, but at (a) 4, (b) 8 and (c) 10 hr. the boulder windstorm-like profile.

lished NLCL show close resemblances with the free surface of a single layer fluid at the hydraulic jump phenomenon (Smith, 1985).

The vertical motions at the time 10 hr. are also shown in Fig. 6. The maximum intensities are comparable with those of the aircraft measure-

Fig. 6 The vertical velocity field at 10hr. The contour interval is 2 m sec^{-1} .

ment at the Boulder windstorm event (Lilly, 1978).

The circulation system developed both in the lower stratosphere and in the troposphere during the numerical integration is schematically illustrated in Fig. 7. The time integration of nonlinear numerical model clearly showed that the pair of circulation systems I and II in Fig. 7 developed first and then NLCL was formed at height $Z_c^{(1)}$ rather than at $Z_c^{(0)}$. This is due to the exponential decrease of the mean density with height and is easily guessed from the second relation of the Eliassen-Palm theorems.

As time progresses, the finite amplitude

growth of the circulation cell II causes a considerable amount of distortion in the flow field near height $Z_c^{(0)}$ and prevents the occurrence of NLCL there. The lower branch of the circulation cell II provides a favourable atmospheric condition to induce the wave breaking in the mid-troposphere. Then, NLCL is formed about 6 km above the leeside slope of the mountain as shown in Fig. 5. Following NLCL formation, the down-slope wind speed increases quite rapidly. This is due to the multiple overreflection of wave energy between the newly established NLCL and the ground surface.

Fig. 7 Schematic illustration of the vertical circulation cells developed in the domain. Locations of NLCL anticipated from the linear theory are also indicated.

It is evident that the surrounding upper-air condition and the surface undulation strongly control the onset and subsequent development of the downslope wind. Among the factors characterizing the surface undulation, the most important is clearly the maximum height of the mountain, h . Based upon the reason mentioned above, 12 hr. numerical integration with the reduced mountain height $h=1$ km was performed under the upstream condition shown in Fig. 3.

In this example, we could not find any large amplitude growth of the wave and, consequently, the occurrence of NLCL within the computational domain during the entire course of time integration. This makes an outstanding contrast to the former example treated in this section. It was found that the maximum surface wind speed was weaker than 40 m sec^{-1} , the maximum pressure difference Δp_s was less than 9 mb and the amount of pressure drag produced was, at most, 30

dyne cm^{-2} , respectively.

The time variations of these quantities are

shown in Fig. 8.

Fig. 8 As in Fig. 4, except that the mountain height is reduced to $h = 1$ km.

6. Downslope Wind in the Winter Season in Japan

Numerical simulation on the growth of the extraordinary strong downslope wind treated in the previous section was carried out under upstream conditions similar to the famous Boulder windstorm that occurred on 11, Jan., 1972. It was clarified that the formation of NLCL in the middle layers of the lower stratosphere near height $Z_c^{(1)}$ acts as a trigger to induce a strong surface wind over the leeside slope of the mountain. Then, the newly established NLCL in the middle troposphere due to the nonlinear process further enhances the surface wind speed. It was recognized that the height of the mountain plays a decisive role, especially in the formation of NLCL at $Z_c^{(1)}$.

In fact, the numerical simulation carried out with the reduced mountain height did not induce an extraordinary severe downslope wind, at least for 12 hr. during the course of the numerical integration of the non-hydrostatic, compressible model.

In our country, we experienced in the past an extraordinary severe downslope wind in which the maximum surface wind speed was more than 50 m sec^{-1} (e.g., Arakawa, 1967). However, it is not an

easy task to pursue the details of the past windstorm which brought about serious damages. Usually, the windstorm takes place in a comparatively sparse data area. The detailed case study of the above-mentioned strong downslope wind that occurred in the Hidaka mountain range in Hokkaido, September 1958, has been done by several authors (e.g., Arakawa, 1960). The analyses on the aerological data available suggested the wave amplification mechanism due to the presence of a linear critical level in the lower troposphere as a cause of the violent wind. Numerical simulation along this line has been tried by, for example, Tomine (1984) and Ikawa (1984), and several aspects of the windstorm were fairly well reproduced.

In this section, we attempted a couple of numerical experiments in order to investigate the possibility of occurrence of a severe downslope windstorm in Japan due to the formation of NLCL in the upper atmosphere, instead of the linear critical level origin.

The Tohoku district was taken up as the object of study because the mountains in that region run almost parallel with the meridians so that the westerly current, predominant in the

winter season, blows nearly perpendicular to the mountain crest. The situation seems to be quite favourable to performing model simulation within a two-dimensional vertical cross section. To represent the unperturbed surrounding meteorological condition, we selected the observation station Akita, located on the upstream side of the Tohoku mountains and by the seaside of the Sea of Japan, which is comparatively warm in winter.

We employed the averaged data on a particular month of winter as an initial value. That is, the original 5 year (1981-1985) averaged aerological data on January (Japan Meteorological Agency, 1987) known at the standard pressure levels are interpolated, by use of the Aitken-Lagrange method, into the equidistant height levels located at every 500 m interval.

The vertical profiles of the west wind and the temperature thus constructed are shown in Fig. 9. That the surface wind speed is weak, i.e. only 4 m sec^{-1} , is to be noticed. The linear analysis suggests that NLCL may appear at the following three levels, that is, $Z_c^{(0)} = 3.5 \text{ km}$, $Z_c^{(1)} = 16.5 \text{ km}$ and $Z_c^{(2)} = 21.5 \text{ km}$.

Thus, in the mid-winter in the northern part of Honshu Island, there exists the possibility of occurrence of NLCL in the lower portion of the troposphere. In order to make the spurious reflection of wave energy from the top boundary less than a few percent, we set the bottom of the sponge layer at a height of 10 km, above which the wind speed and the temperature are constant at 43 m sec^{-1} and -50°C , respectively.

Hence, the possibility of occurrence of NLCL at $Z_c^{(2)}$ is completely eliminated, while that at $Z_c^{(1)}$ is weakened considerably, because of the introduction of uniform, high wind speed layers in the upper atmosphere.

As a comparison experiment, numerical simulations with the mountain height $h = 2 \text{ km}$ and 1 km were performed for 12 hr. under the modified upper-air conditions mentioned above. Here, it should be noticed that the average height and the horizontal extent of the mountain in the Tohoku district roughly correspond to 1 km and 50 km , respectively. Both the integration domain and the mesh size adopted remain the same as in the previous section.

Time variations of the characteristic quantities showing the wave activity for each case are

Fig. 9 Vertical profile of horizontal wind and temperature. Mean winter conditions at Akita, located on the windward side of the Tohoku mountains.

expressed in Figs. 10 a and 10 b. The case where the higher elevation $h = 2 \text{ km}$ is taken shows a rapid and large amplitude growth of the mountain wave due to the formation of NLCL at $Z_c^{(0)}$, and attains the mature stage at about 8 hr., while the time variations for the lower mountain show similar progress but only a weak development. It is seen from Fig. 10 that, in the former case, the surface drag reaches up to 150 dyne cm^{-2} , the maximum surface wind speed u_s rises as high as 60 m sec^{-1} and the maximum pressure difference Δp_s becomes 24 mb at the time of maximum development.

Structures of the thermal and wind field at 8 hr. are illustrated in Fig. 11. We can easily observe that the wave breaking actually takes

AKITA

Fig. 10 Evolution of the surface wave drag, the maximum surface wind and the maximum pressure difference with respect to time for the background state of the atmosphere shown in Fig. 9. Upper figure is for the mountain height $h = 2$ km and the lower for $h = 1$ km.

Fig. 11 Field of (a) horizontal wind and (b) isentropes at 8 hr.
Upper figure is for $h = 2$ km and the lower for $h = 1$ km.

place in the middle troposphere and an intense localized circulation develops in the lower stratosphere in the former case; on the contrary, in the latter example, both characteristics in the flow field are less pronounced. The amplitude of potential temperature contour near height $Z_c^{(0)}$ shows almost the same order of magnitude, say, 2 km or so, in both cases. However, in the former case, the overturning of the isentropic contour near $Z_c^{(0)}$ is clear, which means the occurrence of wave breaking.

7. Concluding Remarks

A very severe downslope windstorm whose maximum surface wind speed is more than 50 m sec^{-1} is obviously a rare meteorological event. Based upon observational and theoretical foundations, several amplification mechanisms on the severe wind have been proposed and discussed. Among others, the mechanism due to the optimum tuning of a partially reflected wave

energy between the ground surface and the inversion layer in the mid-troposphere, or the amplification mechanism due to the overreflection of wave energy from the critical level where the Ri -number is less than $1/4$ was proven to be most promising at the present time. Numerical simulations that include the mechanism mentioned above have successfully demonstrated the development of a violent wind over the leeside slope of the mountain.

The Boulder windstorm is probably one of the typical examples.

In this paper, the evolution of a violent wind of NLCL origin under a typical upstream condition has been investigated by means of numerical simulation. It is concluded that the resonant amplification mechanism due to the multiple overreflection of wave energy between the critical level and the ground surface is quite effective to intensify the surface wind over the downslope of the mountain.

On that occasion, we placed emphasis upon

the hydrostatic features of the mountain wave rather than their fine structures by use of a relatively coarse horizontal mesh size in the numerical model. The importance of NLCL formation in the middle layers rather than in the lower layers of the lower stratosphere due to the exponential decrease of the mean density with height was especially noticed. Numerical simulations carried out in the present study as well as other authors, show us that the environmental wind profiles and stratifications in the upper troposphere and lower stratosphere are crucial to induce NLCL and a resultant violent wind over the leeside slope of the mountain.

It was also recognized that, on the occasion of NLCL formation, the height of a mountain plays a decisive role and a long-term duration of the environmental synoptic conditions is indispensable.

Precise analysis of the Boulder windstorm event made by Lilly (1978) showed that the synoptic-scale wind field in the mid-troposphere surrounding the Colorado State was actually maintained, at least, for one day without indicating appreciable changes. However, it should be noticed that the occurrence and the long-term duration of the environmental synoptic-scale upper air conditions favourable to the genesis of NLCL do not take place so frequently in the real atmosphere.

Historical record on the maximum instantaneous wind speed measured at the meteorological observatory all over the Japan Islands discloses the fact that the severe wind, stronger than 40 m sec^{-1} , happened in the past at times (Tokyo Astronomical Observatory, 1987). However, its occurrence is rather concentrated upon a particular period of the year, that is, mainly from August to October. This strongly suggests that most of the severe winds are closely related to the passage of a mature typhoon in the neighborhood of or through the Japan Islands.

The possibility of occurrence of an extraordinary severe downslope wind due to the overreflection of wave energy from NLCL, on the other hand, seems to be quite small in our country. The reasons for this are the lower elevation of the topography of Japan and the prevailing westerly current in the lower stratosphere in winter as shown in Fig. 9. Both act to prevent the formation of NLCL in the upper atmosphere. In winter,

there exists the supply of a large amount of moisture and sensible heat from the warmer sea surface to the colder atmosphere through the planetary boundary layer. As a result, neutral or conditionally unstable stratification is commonly observed in the lower half of the troposphere over the coastal areas of the Sea of Japan. The effect of weaker stratification also tends to obstruct the occurrence of NLCL in the lower troposphere. The vertical mixing of momentum in the planetary boundary layer acts to prevent the unlimited growth of the surface wind speed. This effect cannot be overlooked, either.

In Japan, we have had neither experiences nor reliable documents on the extraordinary severe downslope wind event of NLCL origin, at least until now.

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The numerical calculations have been made with the use of HITAC S-810/10 installed in the Meteorological Research Institute.

Appendix

Normalized spectral function of the cosine square type topography

We adopted the topographic distribution defined below in the previous paper I as well as the present one.

$$M(x) = \begin{cases} h \cos^2(ax) & , \quad |x| < B \\ 0 & , \quad |x| \geq B \end{cases}$$

where $a = \pi/2B$ and $B = 22.5 \text{ km}$ are taken.

Since the function $M(x)$ is symmetric with respect to x , it is possible to express in terms of a Fourier cosine transform, that is,

$$M(x) = h \int_0^\infty m(k) \cos(kx) dk.$$

The normalized spectral function, $m(k)$, is given by

$$\left\{ \begin{array}{l} m(\alpha) = \frac{\pi}{\alpha(\pi^2 - \alpha^2)} \sin \alpha, \quad \alpha = kB \\ \int_0^\infty m(\alpha) d\alpha = 1 \dots\dots\dots (A) \end{array} \right.$$

The major contribution of $m(\alpha)$ to the integral (A) comes from the interval, $0 \leq \alpha \leq 2\pi$. The *Fr*-number defined in section 3 contains the integral of the spectral function $m(k)$ with respect to wavenumber k from 0 to the upper bound l . In terms of the notation used in this appendix, the upper bound corresponds to $\alpha = lB = 22.5 > 2\pi$, so that the integral becomes almost unity. Therefore, it is concluded that the *Fr*-number is free from the shape of the mountain.

References

- Aihara, M., and H. Okamura, 1985 : A numerical simulation of the finite-amplitude mountain waves using a meso-scale, non-hydrostatic compressible model. *Pap. Met. Geophys.* **36**, 119-135.
- Arakawa, S., 1960 : On the Hidaka-Shimokaze studies on the fallwind (Part 1). *J. Met. Res.*, **12**, 911-919.
- , 1969 : Climatological and dynamical studies on the local strong winds, mainly in Hokkaido, Japan. *Geophys. Magazine*, **34**, 359-425.
- Brinkmann, W. A. R., 1974 : Strong downslope winds at Boulder, Colorado. *Mon. Wea. Rev.*, **102**, 592-602.
- Clark, T. L. and W. R. Peltier, 1984 : Critical level reflection and the resonant growth of nonlinear mountain waves. *J. Atmos. Sci.*, **41**, 3122-3134.
- Davies, P. A., and W. R. Peltier, 1976 : Resonant parallel shear instability in the stably stratified planetary boundary layer. *ibid.*, **33**, 1287-1300.
- Durran, D. R., 1986 : Another look at downslope windstorms.
- Part I : The development of analogs to supercritical flow in an infinitely deep, continuously stratified fluid. *ibid.*, **43**, 2527-2543.
- and J. B. Klemp, 1983 : A compressible model for the simulation of moist mountain waves. *Mon. Wea. Rev.*, **111**, 2341-2361.
- Eliassen, A. and E. Palm, 1960 : On the transfer of energy in stationary mountain waves. *Geophys. Publ.*, **22**, 1-23.
- Hirasawa, M., M. Ikawa and K. Kurihara, 1985 : Numerical simulations of steady mountain waves using a non-hydrostatic, compressible model.
- Draft (in Japanese) vol. 48. The annual meeting of the Japan meteorological society.
- Hoinka, K. P., 1985 : Observation of the airflow over the Alps during foehn event. *Quart. J. Roy. Met. Soc.*, **111**, 199-225.
- Horiwata, Y., 1986 : Three-dimensional cloud model including topographic effect (2).
- linear stability of the finite-difference scheme——
- Draft (in Japanese) vol. 49. The annual meeting of the Japan meteorological society.
- Huppert, H. E. and J. W. Miles, 1969 : Lee waves in a stratified flow.
- Part 3. Semi-elliptical obstacle. *J. Fluid Mech.*, **35**, 481-496.
- Ikawa, M., 1984 : Numerical simulation on the Hidaka downslope wind. Draft (in Japanese) vol. 46. The annual meeting of the Japan meteorological society.
- Japan Meteorological Agency, 1987 : Aerological data of Japan, 5-year period averages (1981-1985) 204p.
- Jones, W. L., 1968 : Reflection and stability of waves in stably stratified fluids with shear flow : A numerical study.
- J. Fluid Mech.*, **34**, 609-624.
- Kimura, F. and P. Manins, 1987 : Blocking in periodic valleys. *Boundary Layer Meteor.* (Submitted).
- Klemp, J. B. and D. K. Lilly, 1975 : The dynamics of wave-induced downslope wind. *J. Atmos. Sci.*, **32**, 320-339.
- and ———, 1978 : Numerical simulation of hydrostatic mountain waves. *ibid.*, **35**, 78-107.
- and R. Wilhelmson, 1987 : The simulation of three-dimensional convective storm dynamics. *ibid.*, **35**, 1070-1096.
- Kurihara, Y. and G. J. Tripoli, 1976 : An iterative time integration scheme designed to preserve a low-frequency wave. *Mon. Wea. Rev.*, **104**, 761-764.
- Lilly, D. K., 1978 : A severe downslope windstorm and aircraft turbulence event induced by a mountain wave. *J. Atmos. Sci.*, **35**, 59-77.
- Lilly, D. K. and P. J. Kennedy, 1973 : Observations of a stationary mountain wave and its associated momentum flux and energy dissipation. *J. Atmos. Sci.*, **30**, 1135-1152.
- Lindzen, R. S. and H. L. Kuo, 1969 : A reliable method for the numerical integration of a large class of ordinary and partial differential equations.
- Mon. Wea. Rev.*, **97**, 732-734.
- and A. J. Rosenthal, 1976 : On the instability of Helmholtz velocity profiles in stably stratified fluid when a lower boundary is present. *J. Geophys. Res.*, **81**, 1516-1571.
- Orlanski, I., 1976 : A simple boundary condition for unbounded hyperbolic flows. *J. Comput. Phys.*, **21**,

- 251-269.
- Peltier, W. R. and T. L. Clark, 1979 : The evolution and stability of finite-amplitude mountain waves. Part II, Surface wave drag and severe downslope windstorms. *J. Atmos. Sci.*, **36**, 1498-1529.
- Queney, P., 1948 : The problem of air flow over mountains ; A summary of theoretical studies. *Bull. Amer. Meteor. Soc.*, **29**, 16-26.
- Rosenthal, A. J. and R. S. Lindzen, 1983 : Instabilities in a stratified fluid having one critical level. Part I, Results. *J. Atmos. Sci.*, **40**, 509-520. Part II ; Explanation of gravity wave instabilities using the concept of overreflection. *ibid.*, **40**, 521-529.
- Shapiro, R., 1970 : Smoothing, filtering and boundary effects. *Rev. Geophys. Space Phys.*, **8**, 359-387.
- Smith, R. B., 1977 : The steepening of hydrostatic mountain waves. *J. Atmos. Sci.*, **34**, 1634-1654.
- , 1985 : On severe downslope winds. *ibid.*, **42**, 2597-2603.
- Tatsumi, Y., 1983 : An economical explicit time integration scheme for a primitive model. *J. Meteor. Soc. Japan*, **61**, 269-288.
- Tokyo Astronomical Observatory, 1987 : *Rika-Nenpyo* (Chronological tables for Science, in Japanese). Maruzen Book Co. Ltd. p218.
- Tomine, K., 1984 : A numerical study on local depressions on the leeside of the Hidaka mountain range in Hokkaido. *ibid.*, **62**, 215-223.

非線型臨界層による山岳強風

相原正彦・平沢正信

山岳下流斜面上の強風発達に関する数値実験を、山岳効果を考慮した非静力学、圧縮系の数値モデルにより行った。

有名なボルダー山岳強風時の上流側の気象状態に似た条件下で行った数値実験の結果について、やゝ詳細に調べた。最大瞬間風速が 50 m sec^{-1} を越える山岳下流斜面の異常強風は、下部対流圏に発生した碎波、又は、山岳波により引き起こされた局所臨界層を原因とする増幅機構によるものであることが分った。また、対流圏、成層圏内に発達する循環の特徴についても述べる。

日本において上記の増幅機構による山岳下流強風の発生の可能性についても検討した。