3-D Mountain Waves by the Lokal-Modell of DWD and the MRI Mesoscale Nonhydrostatic Model

by

Kazuo Saito

Meteorological Research Institute, 1-1 Nagamine, Tsukuba, Ibaraki 305-0052, Japan

Gunter Doms, Ulrich Schaettler and Juergen Steppeler

Deutscher Wetterdienst, Frankfurterstr. 135, 63067 Offenbach, Germany

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Abstract

A brief comparison between the Deutscher Wetterdienst's next generation nonhydrostatic regional forecast model (the Lokal-Modell of DWD) and the Meteorological Research Institute mesoscale nonhydrostatic model (MRI-NHM) is performed by computing the numerical solutions of the 3-dimensional mountain waves over an isolated circular mountain. For linear cases with a free-slip lower boundary condition, both models well reproduce the characteristics of the Smith's (1980) analytic solutions for the 3-dimensional nonhydrostatic mountain waves. Analysis of the CPU time of the two models with the CRAY C-98 of DWD shows the tendency that the Lokal-Modell (time-splitting, horizontally explicit model) is relatively efficient in lower horizontal resolution while MRI-NHM (3D-implicit model) becomes more efficient at higher horizontal resolution. This is because the number of small time step integrations of the horizontally explicit model increases when the horizontal resolution becomes higher. The CPU time for one iterative procedure in the direct method for the three dimensional elliptic pressure equation in MRI-NHM is 2.1~2.6 times of that for one small time step integration in the Lokal-Modell. A rough yardstick which determines the comparative efficiency of the two models is given by the relative magnitude between the Courant-number for sound wave speed and the number of iterative procedures in the elliptic pressure equation solver in the 3D implicit model.

1. Introduction

Several nonhydrostatic models have been developed in recent years at various forecast centers, institutes, and universities in the world. In Germany, the Deutscher Wetterdienst (DWD) has been developing a regional nonhydrostatic model (the Lokal-Modell) for the operational forecast purpose and is about to start daily probational runs. In Japan, the Meteorological Research Institute (MRI) has developed a nonhydrostatic mesoscale model (MRI-NHM) and applied it to many scientific simulations including forecast experiments.

Both models were recently developed or modified, and their computational accuracy and/or efficiency is not yet so clear.

Model intercomparison is an effective way to check and modify the numerical models. For example, COMPARE (Comparison of Mesoscale Prediction and Research Experiment) is an international program designed to develop mesoscale models through intercomparison. One of the subjects of GCSS (GEWEX Cloud System Study) is also to improve the cloud resolving models (CRMs) by model comparison.

In this paper, we report a brief comparison between the Lokal-Modell of DWD and MRI-NHM. Three dimensional mountain waves over an

isolated circular mountain are selected as the basic event of the comparison. Computational efficiency of the two models in various horizontal resolutions and different lower boundary conditions is also compared. The purposes of this work are following two points.

1) Verification of the models :

Verification of the model solution against the analytic solution of the mountain wave is a basic, and indispensable test for developing accurate non-hydrostatic models. Two dimensional mountain waves are often employed for the verification for the reason of easiness to get the analytic solution, but here we use 3-dimensional mountain waves. Both the Lokal-Modell and MRI-NHM are developed or modified recently, while neither model has been verified enough by this basic check. As for MRI-NHM, Ikawa and Saito (1991) once verified their model against the Smith's (1980) analytic solution of 3-D mountain waves, however the model was anelastic version with the Bousinesq approximation at that time. Saito and Kato (1996) have conducted preliminary verification of the newly developed fully compressible version of MRI-NHM, though they did not check for nonhydrostatic situations. Although the model was elastic, the 'anelastic start-up procedure' was used to initiate the model's pressure.

In this study, we verify the two models against the analytic solution of 3-D mountain waves, using several conditions including nonhydrostatic situation. The experimental conditions (e.g., stability, vertical grid stretching, *etc.*) are set as similar as possible on the two models. The anelastic start-up procedure used in Saito and Kato (1996) is not used in MRI-NHM.

2) Comparison of the numerical schemes of the Lokal-Modell and MRI-NHM in terms of the computational efficiency :

Current elastic nonhydrostatic models are classified into two groups by the treatment of sound waves¹; the *time-splitting model* and the *3D-implicit model*. The time-splitting models use two time steps, where the sound waves-relating terms are evaluated on a shorter time step and the low-frequency terms are computed on longer time

steps. Usually, sound waves are treated explicitly for horizontal directions while implicitly for vertical (E-HE-VI scheme; e.g., Klemp and Wilhelmson, 1978). On the other hand, the 3D-implicit models treat sound waves implicitly both horizontally and vertically (E-HI-VI scheme; e.g., Tapp and White, 1976). The short time step is not used and pressure is prognosticated by solving a 3-dimensional Helmholtz-type tendency equation.

Ikawa (1988) once tested the computational efficiency of the two schemes using his quasi-compressible model and showed that the 3D-implicit model was 2.7 times faster than the time-splitting, horizontally explicit model. However, his comparison was made in 2-dimensional model with only one set of time step (12 s) and horizontal resolution (1200 m), and the model was not fully optimized, as he admitted. Skamarock and Klemp (1994) showed the advantage of the Klemp-Wilhelmson type time-splitting model, but they neglected to mention the 3D-implicit model. Evaluating the advantages and disadvantages of the two numerical schemes is something of a hot topic in current nonhydrostatic modelling.

In this study, we compared the two models' CPU time by running the models with almost the same experimental situations using the Cray C98 of DWD. Although we could not check the efficiency of the two models for so many cases, our results still provide useful information on the computational schemes of the nonhydrostatic model.

2. Numerical Models

2.1 Lokal-Modell

The Lokal-Modell is a nonhydrostatic model currently under development at the Department of Research of DWD as a next-generation limited area prediction model. The model has been developed for the purpose of operational numerical weather prediction, while it is also designed for various scientific applications from the meso- β scale (horizontal grid spacing 3~15 km) to the micro- α scale (horizontal grid spacing 50~500 m) meteorological phenomena. The prototype version of the model was made with reference to the formulation of the

¹ Currently, there is confusion in terminology of nonhydrostatic model classification. The Klemp-Wilhelmson type models are sometimes called *split-explicit model* in terms of time-splitting and explicit treatment of sound waves for horizontally. 3D-implicit models are sometimes called *semi-implicit model* since usually for horizontal directions only the horizontal components of the sound waves in chain rules are treated implicitly when the model includes orography. On the other hand, Skamarock and Klemp (1994) called their Klemp-Wilhelmson type models the 'semi-implicit KW model' or 'split-implicit model' to distinguish it from the time-splitting explicit model and the simple explicit model where the sound waves are treated explicitly in both horizontal and vertical directions (E-HE-VE scheme). In this paper, we classified the models by the horizontal treatment of sound waves, presuming horizontally explicit models use a time-splitting scheme.

NCAR-MM5 (Dudhia, 1993; Grell *et al.*, 1994), but some new features have been implemented. A generalized terrain-following stretched coordinate is used vertically, while rotated spherical coordinates are adopted horizontally. Metrical terms can be adjusted to represent tangential Cartesian geometry with constant or zero Coriolis parameters. Prognostic variables are physical components of wind velocity, perturbation pressure, temperature, specific humidity, cloud water content, and (optionally) cloud ice component, while total density and precipitation fluxes of rain and snow are the diagnostic variables. The Klemp-Wilhelmson type time splitting integration scheme is employed, where the sound waves are treated explicitly horizontally while implicitly vertically.

The model code is written in FORTRAN 90, and the Message Passing Interface software is used for message passing on distributed memory machines. Description of the latest version of the model is presented in Doms and Schaettler (1997).

2.2 MRI-NHM

The MRI mesoscale nonhydrostatic model (MRI-NHM) is a general purpose numerical model developed at the Forecast Research Department of

MRI. The effort to develop a cloud resolving nonhydrostatic model was initially started by Ikawa (1988), and the first version was completed by Ikawa and Saito (1991). The model was modified by Saito (1994) so that it can be used as a nested mesoscale model. Recently, basic equations were changed by Saito (1997) from anelastic to fully compressible with a map factor. Historically, the model was developed as a research tool, while the latest version has the performance of a regional numerical prediction model. Mass-virtual potential temperature is introduced to make the state equation concise, and fall-out of precipitable water substances (rain, snow and graupel) is considered in the mass continuity equation in flux form. The buoyancy term is directly evaluated from the density perturbation. A 3-D implicit time integration scheme which treats sound waves implicitly both vertically and horizontally is used. Turbulent closure model level 2.5 which predicts the turbulent kinetic energy determines the diffusion coefficients. A preliminary report of recent model modifications is presented in Saito and Kato (1996), and a comprehensive description of the model will be given in Saito and Kato (1998).

For a specification of the dynamical part of the

Table 1. Specification of the Lokal-Modell and MRI-NHM

Category	Lokal-Modell	MRI-NHM
Basic equations	Fully compressible ¹	$\lll\lll^2$
Prognostic variables	u', v', w', p', T^3	$\rho u, \rho v, \rho w, p', \theta, E^4$
Coordinates (Hor.)	Rotated spherical coordinates	Polar-stereo graphical coordinates
(Ver.)	Terrain-following	$\lll\lll$
Grid structures (Hor.)	Arakawa C	$\lll\lll$
(Ver.)	Lorenz-type	$\lll\lll$
Advection	Advective form	Flux form
(Accuracy)	Second order	$\lll\lll\lll^5$
Time discretization	Second-order leapfrog with Asselin's time filter	$\lll\lll$
	First order forward-backward for Δt	-----
Sound waves (Hor.)	Explicit	Semi-implicit
(Ver.)	Implicit	Implicit
Gravity waves	Implicit	Explicit
Pressure equation	1-D Helmholtz	3-D Helmholtz
Subgrid turbulence	Mellor & Yamada level 2	Deardorff level 2.5
Upper boundary	Rigid-lid with Rayleigh damping	$\lll\lll$
Lower boundary	Free-slip ⁶	$\lll\lll$
(Pressure equation)	Linear extrapolation	Kinematic condition
Lateral Boundary	Davies-type absorbing layer	Orlanski-type radiation condition
Numerical smoothing	4-th order linear horizontal diffusion	$\lll\lll$
	Divergence-type acoustic filter	-----
Coding	FORTRAN 90	FORTRAN 77
Code parallelization	Message Passing Interface	-----

¹Diabatic heating term may be neglected in the pressure equation

²Fall-out of water substances is considered for wet model

³ q_v, q_c , and (optionally) q_i , for wet model

⁴ q_v, q_c, q_r and (optionally) q_i, q_s, q_g, N_s, N_g for wet model

⁵Horizontally 3rd or 4-th order (Saito, 1998) & the modified advection scheme (Kato, 1998a)

⁶Surface layer parameterization by the similarity theory for non-slip cases

two models, see Table 1.

3. Linear analytic solution of 3-D mountain waves

In this Section, we refer to the analytic solution of 3-D mountain waves. The solution is based on Smith (1980), while the effect of wave amplification due to decrease of the reference density is considered.

For the steady state, stratified Bousinesq fluid, the vertical displacement of streamline $\delta(x, y, z)$ of linear mountain waves is given as

$$\frac{\partial^2}{\partial x^2} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \delta + \frac{N^2}{U^2} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \delta = 0, \quad (1)$$

where U is the environmental wind speed and N the Brunt-Vaisala frequency:

$$N^2 = -\frac{g}{\bar{\theta}} \frac{d\bar{\theta}}{dz}. \quad (2)$$

Here, $\bar{\theta}$ is the potential temperature of environmental atmosphere.

With constant N and U , the solution of Eq. (1) is obtained by using double Fourier transform analysis (Smith, 1980):

$$\delta(x, y, z) = \iint_{-\infty}^{\infty} H(k, l) e^{imz} e^{i(kx+ly)} dk dl, \quad (3)$$

where $H(k, l)$ is the double Fourier transform of the orography height $h(x, y)$ defined as

$$H(k, l) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} h(x, y) e^{-i(kx+ly)} dx dy. \quad (4)$$

Here, m in Eq. (3) is calculated by the horizontal wave number vector (k, l) as

$$m^2 = \frac{k^2 + l^2}{k^2} \left(\frac{N^2}{U^2} - k^2 \right). \quad (5)$$

In case of the upper radiation condition, the positive imaginary root of Eq. (5) is chosen for $k^2 > N^2/U^2$, while the sign of m is chosen to be the same as that of k for $k^2 < N^2/U^2$. Once δ is obtained, vertical wind component w is given by the following linear relation for steady flow:

$$w = U \frac{\partial \delta}{\partial x}. \quad (6)$$

Smith (1980) computed the field of w for the mountain wave over an isolated 3-D mountain with circular contours:

$$Z_s(x, y) = \frac{h_m}{\left\{ 1 + \left(\frac{x}{a} \right)^2 + \left(\frac{y}{a} \right)^2 \right\}^{\frac{1}{2}}}, \quad (7)$$

by using the discrete complex Fourier transform. According to his work, Saito and Ikawa (1991) computed the analytic solution of the 3-D mountain wave, and used the solution for the verification of their anelastic nonhydrostatic model with the Bousinesq approximation (Ikawa and Saito, 1991).

In this study, in order to express the amplification effect of the mountain waves due to decrease of the density of reference atmosphere $\bar{\rho}$, we multiply the amplitude of w -field by the following amplification factor:

$$\left(\frac{\bar{\rho}}{\bar{\rho}_0} \right)^{-\frac{1}{2}} = \left[e^{-sz} \left\{ 1 - \frac{g}{c_p \theta_0 S} (1 - e^{-sz}) \right\}^{\frac{c_p}{R}} \right]^{-\frac{1}{2}}. \quad (8)$$

Here, S is the static stability ($= N^2/g$), g the gravity acceleration, c_p the specific heat of air at constant pressure, R the gas-constant, and subscripts 0 express the values at the ground surface. Eq. (8) can be obtained by the following relation for a constant stability non-Bousinesq atmosphere:

$$\bar{\theta} = \theta_0 e^{sz}. \quad (9)$$

In computing the analytic solution, the mountain top is set to $h_m = 100$ m, and its shape is discretized on a grid mesh with $\Delta x = \Delta y = a/3$. The mesh size $(Nx, Ny) = (128, 128)$ is used for Section 4.2, while for Section 4.3, the domain size for the x -direction is enlarged to $(Nx, Ny) = (256, 128)$ because in a nonhydrostatic case, the mountain wave extends leeward and the analytic solution is contaminated due to the periodic nature of Fourier transform if the domain size is too small. For the environmental atmospheric state, $U = 8$ ms⁻¹ and $N = 0.01$ s⁻¹ are chosen as in Saito and Ikawa (1991).

4. Comparison between numerical and analytic solutions

4.1 Design of numerical experiments

Numerical solutions by the two models are computed with the following conditions:

- 1) Horizontal grid interval $\Delta x = \Delta y = a/3$, and model domain $(Nx, Ny) = (62, 42)$.
- 2) 32 vertical levels with stretched grid interval $\Delta z = 40$ m ~ 1200 m (Fig. 1) for MRI-NHM. The w -levels are located in middle of the pressure levels, which are the same levels of the potential temperature and horizontal wind components. In case of the Lokal-Modell, w -levels are set on the same height levels as of MRI-NHM (Fig. 1), while the pressure levels are defined in middle of the w -levels.
- 3) Rayleigh damping layers to imitate the unbounded upper boundary radiation condition are

Fig. 1. Vertical grid staggering of MRI-NHM. Solid line shows the levels where the pressure, horizontal wind components (momentum) and the potential temperature are computed, while broken lines show the levels where vertical wind is calculated.

imposed on the upper five levels.

- 4) Constant temperature/potential temperature lapse rates $dT/dz = -6.85$ K/km and $d\theta/dz = 3.0$ K/km are given for the Lokal-Modell and MRI-NHM, respectively. These lapse rates correspond to atmospheric stability $N = 0.01$ s⁻¹.

The experiments are conducted with three different horizontal resolutions ($\Delta x = 10$ km, 2 km and 400 m). The time step and lower boundary condition are also changed as summarized in Table 2 and Table 3.

4.2 Almost hydrostatic case

First, we show the results for the case of $\Delta x = 2$ km. In this case, the horizontal scale of the mountain is $a = 6$ km, and the product of the Scorer number N/U and a is 7.5, which means the horizontal scale of the mountain is much larger than the horizontal wave length of the buoyancy oscillation, and the nonhydrostatic effect is small. Figure 2 shows the vertical wind w obtained by the nonhydrostatic analytic solution (2a and 2b), the Lokal

-Modell (2c and 2d), and MRI-NHM (2e and 2f). In these figures, each scale on the horizontal axes represents the horizontal grid, and the mountain top is located at $x = y = 21.5$. The scale of vertical axes of Figs. 2a, 2c and 2d is 400 m. As shown in these figures, both models simulate the characteristics of the analytic solution of the mountain wave generally well. The amplitude of simulated mountain waves for the both models is smaller than that of the analytic solution in upper layers. This underestimation of mountain waves near the upper boundaries is primarily attributed to the Rayleigh damping layer imposed near the model top (above $z = 12$ km). Another reason may be the finite difference approximation in the numerical models. Figure 3 shows the mountain wave by MRI-NHM, when the horizontal advection terms are computed using the optional 4-th order accuracy scheme instead of the original second-order scheme. The simulated mountain wave is stronger in upper layers compared to Figs. 2e and 2f, and approaches the analytic solution. Some differences between the solutions of the Lokal-Modell and MRI-NHM can be seen near the lateral boundaries. These come from the contrast of the lateral boundary conditions of the two models; MRI-NHM employs the Orlanski-type radiation condition, while the Lokal-Modell uses the Davies-type absorbing layer. Except for this, the solutions by the two models resemble each other.

4.3 Nonhydrostatic case

Next, we show the results for $\Delta x = 0.4$ km. In this case, the horizontal scale a is 1.2 km. The product of the Scorer number and a is 1.5, which means that the nonhydrostatic effect is significant. Figure 4 shows vertical wind w obtained by the nonhydrostatic analytic solution, the Lokal-Modell, and MRI-NHM. Again, both the numerical models simulate the characteristics of the analytic solution of the nonhydrostatic mountain wave very well. Slight differences near the lateral boundaries come from the contrast of the lateral boundary conditions of the two models. The radiation condition of MRI-NHM works very well and the mountain wave propagates leeward through the right-side lateral boundary. In the Lokal-Modell, although the amplitude of lee wave tends to decrease near the lateral boundaries where the Davies-type absorbing layer is imposed, the inside solution is not affected by the boundary condition.

4.4 Low resolution case

Additional experiments with $\Delta x = 10$ km are

Fig. 2. 3-D mountain waves by the nonhydrostatic analytic solution, Lokal-Modell and MRI-NHM. For the case of $a=6$ km ($\Delta x=2$ km). **a)** Vertical cross-section of w through the mountaintop by the analytic solution. The contour interval is 1 cm s^{-1} . The graduations on the vertical and horizontal axes correspond to 400 m and 2 km, respectively. **b)** Horizontal cross-section of w at $z=2.44$ km by the analytic solution. The graduations on the x - and y -axes correspond to 2 km, **c)** Same as in a), but by the Lokal-Modell at $t=3$ h. **d)** Same as in b), but by the Lokal-Modell at $t=3$ h. **e), f)** Same as in c), d) but by MRI-NHM.

performed as the test of lower horizontal resolution case. In this case, the horizontal scale of the mountain is $a=30$ km, and the product of the Scorer number N/U and a is 36, which means that the nonhydrostatic effect can be neglected. Figure 5 shows the mountain waves simulated by the Lokal-Modell and MRI-NHM. In this case, the analytic solution is not shown because the mountain wave pattern is almost the same as in the case of $\Delta x=2$

km except that the amplitude is $1/5$. Numerical solutions at $t=4$ hr are shown in this figure, but the amplitude of the simulated mountain waves is very weak in upper layers. This is because the phase speed of vertical propagation of the mountain wave is small in this case and vertical propagation of the mountain waves is insufficient at this time. Figures 5b and 5d show the horizontal cross-sections of w at $z=0.74$ km, which correspond to Fig. 6b. Mountain

Fig. 3. Same as in Figs. 2 e) and f) but for the case where 4-th order advection scheme is used.

Fig. 4. Same as in Fig. 2 but for the case of $a=1.2\text{ km}$ ($\Delta x=0.4\text{ km}$). The contour interval is 5 cm s^{-1} . The scales on the x - and y - axes correspond to 0.4 km . Numerical model solutions are for the results at $t=2\text{ h}$.

Fig. 5. 3-D mountain waves by the Lokal-Modell and MRI-NHM. For the case of $a = 30$ km ($\Delta x = 10$ km). **a)** Vertical cross-section of w through the mountaintop by the Lokal-Modell at $t = 4$ h. The contour interval is 0.2 cm s^{-1} . The graduations on the vertical and horizontal axes correspond to 400 m and 10 km, respectively. **b)** Horizontal cross-section of w at $z = 2.44$ km by the Lokal-Modell at $t = 4$ h. The graduations on the x - and y -axes correspond to 10 km, **c)** Same as in **a)**, but by MRI-NHM. **d)** Same as in **b)**, but by MRI-NHM.

waves by the Lokal-Modell (Fig. 5b) are slightly weak compared to the analytic solution and MRI-NHM (Fig. 5c). The reason of this difference is not clear. It may come from differences of vertical grid staggering and lower boundary conditions in the pressure equations of the two models (kinematic condition for MRI-NHM; see Eq. (68) in Saito (1997), linear extrapolation for the Lokal-Modell). Differences of the numerical smoothing between the two models may also affect the propagation of the mountain waves in this transient stage.

4.5 Non-slip case

Comparative experiments are also performed with a non-slip lower boundary condition. In both models, surface momentum fluxes are given by the Monin and Obukhov's similarity law, and the roughness length is assumed to be 10 cm. Since no analytic solution exists for the mountain wave for non-slip condition, we presumed constant wind $U = 8 \text{ ms}^{-1}$ as the environmental field for simplicity.

The analytic solution for $a = 6$ km ($\Delta x = 2$ km)

is shown in Figs. 6a and 6b, where the horizontal cross-section is for at $z = 0.74$ km. Figures 6c-6f show the corresponding numerical solutions by the Lokal-Modell and MRI-NHM. Considerable differences between the solutions are seen near the lateral boundaries. In the Lokal-Modell, a boundary layer develops in the Davies-type absorbing layer and the maximum upward velocity is around at $x = 6$, corresponding to the width of the absorbing layer. The maximum downward velocity is also seen at $x = 56$, corresponding to the lee-side absorbing layer. In case of MRI-NHM, upward and downward motions are seen just in the vicinity of the lateral boundaries.

The amplitude of the mountain wave by the Lokal-Modell (Figs. 6d) is weaker than that by MRI-NHM (Fig. 6f). This shows a similar tendency as with the free-slip condition (Figs. 5b and 5d), while in this case the absorbing layer around the lateral boundaries may also affect the solution of the Lokal-Modell. Another difference between Figs. 6c and 6e is the magnitude of w at the lowest level. This is

The non-slip experiments were conducted for the cases of $\Delta x = 400$ m and 10 km as in Table 2 and Table 3, but figures are not shown.

Table 2 and Table 3 show the total CPU time of the comparative experiments with the two models by Cray C-98 of DWD. CRAY-C98 is 8-CPU version of the vector supercomputer CRAY-C90, whose peak performance is estimated as 12 GFLOPs. In experiments, four CPUs were used for the Lokal

Table 2. Total CPU time for various experiments with the Lokal-model by Cray-C98. Δt is the time step and $\Delta \tau$ means the short time step in the Lokal Modell. N_s is the number of small time step integrations in one large leap-frog time integration; $N_s = 2\Delta t / \Delta \tau$.

Δx (km)	Δt (s)	$\Delta \tau$ (s)	N_s	Lower Boundary	hour	CPU time (s)	Figure
10	30	15	4	non-slip	4	246.3	5a, 5b
10	60	17.1	7	free-slip	4	144.2	
10	60	17.1	7	non-slip	4	149.9	
2	15	3.3	9	free-slip	4	588.4	2c, 2d
2	15	3.3	9	non-slip	4	587.3	
2	30	3.5	17	free-slip	4	357.0	
2	30	3.5	17	non-slip	3	320.3	6c, 6d
2	60	3.6	33	free-slip	4	324.5	4c, 4d
0.4	15	0.7	41	free-slip	2	681.7	
0.4	15	0.7	41	non-slip	2	770.3	

Table 3. Same as in Table 2 but for MRI-NHM. Order is the accuracy in computing advection term, and N_i is the number of iterative procedures to solve the pressure equation.

Δx (km)	Δt (s)	Order	N_i	Lower Boundary	hour	CPU time (s)	Figure
10	30	2nd	1	non-slip	4	275.0	5c, 5d
10	60	2nd	3	free-slip	4	173.1	
10	60	2nd	1	non-slip	4	137.7	
2	30	2nd	3	free-slip	4	341.6	2e, 2f
2	30	4-th	3	free-slip	3	267.0	3a, 3b
2	30	2nd	1	non-slip	3	208.6	6e, 6f
2	60	2nd	3	free-slip	4	171.9	4e, 4f
0.4	15	2nd	3	free-slip	2	344.8	
0.4	15	2nd	1	non-slip	2	275.0	

-Modell and only one CPU was used for MRI-NHM, and the CPU times in Table 2 are the total of the four CPUs. In Table 2, N_s is the number of small time step integrations in one large leap-frog time integration; $N_s = 2\Delta t / \Delta \tau$. The maximum value of $\Delta \tau$ is automatically determined by the following stability condition:

$$\Delta \tau \leq \frac{\Delta x}{\sqrt{2}C_s}, \quad (10)$$

where C_s is the sound wave speed. Consequently, the CPU time in the Lokal-Modell depends on not only Δt but also the horizontal resolution. On the other hand, in case of MRI-NHM, the CPU time depends on the lower boundary condition because the Helmholtz-type pressure tendency equation solver requires iterative procedures in principle if the simulation includes orography and surface friction is excluded². In Table 3, N_i is the number of iterative procedures. In same lower boundary con-

ditions, the CPU time of MRI-NHM is simply proportional to the inverse of the time step ($1/\Delta t$), and independent of the horizontal resolution. In Table 3, the CPU time by an extra experiment with 4-th order advection scheme corresponding to Figure 3 is added for reference.

Table 4 summarizes the estimated CPU time for one-hour simulation with the two models. In this table the CPU times of MRI-NHM for $\Delta t = 15$ s cases are estimated assuming that the CPU time is proportional to $1/\Delta t$. As shown in this table, the CPU times of the Lokal-Modell increase in higher resolution cases because N_s tends to increase. Generally speaking, the Lokal-Modell is slightly more efficient in cases of $\Delta x = 10$ km, while MRI-NHM is more efficient in cases of higher horizontal resolution or a larger time step. Simple estimation of the CPU time for one small time step integration in the Lokal-Modell is about 0.050~0.062 s for $\Delta t = 15$ s cases, 0.058 s for $\Delta t = 30$ s cases and 0.056 s for $\Delta t = 60$ s cases. On the other hand, similar estimation of the CPU time for one iterative procedure of the direct method in MRI-NHM is about 0.133 s for $\Delta t = 30$ s cases and 0.148 s for $\Delta t = 60$ s cases if CPU time of the physical process for the boundary layer is negligible. That is, the CPU time

² Actually, in this study, where the mountain height is only 100 m, MRI-NHM provides virtually the same results without instability even if the iterative procedure is omitted. We used $N_i = 3$ for all free-slip cases for comparison, though $N_i = 1$ is enough to simulate the steady state linear mountain wave.

Table 4. Estimated CPU time by Cray-C98 for one-hour simulation with the Lokal-Modell and MRI-NHM (2nd order).

Δx (km)	Δt (s)	Courant Number	Lower Boundary	CPU time (s) Lokal-Modell	CPU time (s) MRI-NHM	Ratio of CPU time
10	30	1.02	non-slip	61.6 ($N_s=4$)	68.8 ($N_i=1$)	0.90
10	60	2.04	free-slip	36.1 ($N_s=7$)	43.3 ($N_i=3$)	0.83
10	60	2.04	non-slip	37.5 ($N_s=7$)	34.4 ($N_i=1$)	1.09
2	15	2.55	free-slip	147.1 ($N_s=9$)	170.8 ($N_i=3$)	0.85
2	15	2.55	non-slip	146.8 ($N_s=9$)	139.0 ($N_i=1$)	1.06
2	30	5.10	free-slip	89.3 ($N_s=17$)	85.4 ($N_i=3$)	1.04
2	30	5.10	non-slip	106.8 ($N_s=17$)	69.5 ($N_i=1$)	1.53
2	60	10.2	free-slip	81.1 ($N_s=33$)	42.9 ($N_i=3$)	1.89
0.4	15	12.75	free-slip	340.9 ($N_s=41$)	172.4 ($N_i=3$)	1.97
0.4	15	12.75	non-slip	385.2 ($N_s=41$)	137.5 ($N_i=1$)	2.80

for one iterative procedure of the direct method in MRI-NHM is 2.1~2.6 times of that for one small time step integration in the Lokal-Modell³. Therefore, a rough yardstick where the Lokal-Modell becomes more efficient than MRI-NHM may be given as

$$C_s = \frac{\Delta t}{\Delta x} \leq N_i, \quad (11)$$

where the left-hand side is the Courant-number for sound wave speed.

The CPU time for one large time step excluding small time step integration of the Lokal-Modell is apparently smaller than that excluding the iterative procedure in MRI-NHM. This may be attributed to the differences of the optimization for the CRAY computer and/or number of prognostic variables (turbulent energy is prognosticated in MRI-NHM) between the two models.

6. Summary and concluding remarks

A brief comparison between the Deutscher Wetterdienst's next-generation nonhydrostatic regional forecast model (the Lokal-Modell of DWD) and the Meteorological Research Institute mesoscale nonhydrostatic model (MRI-NHM) is performed. The 3-dimensional mountain waves over an isolated circular mountain are used as the compara-

tive instance. For linear cases with a free-slip lower boundary condition, both models reproduce well the characteristics of the Smith's (1980) analytic solutions of the 3-dimensional nonhydrostatic mountain waves.

The CPU times of the two models for different horizontal resolution and time step were analyzed. The Lokal-Modell (time-splitting, horizontally explicit model) is relatively efficient in lower horizontal resolution while MRI-NHM (3D-implicit model) becomes more efficient in higher horizontal resolution. The CPU time for one iterative procedure in the direct method of MRI-NHM is 2.1~2.6 times of that for one small time step integration in the Lokal-Modell. The relative efficiency of the two models may be estimated by the Courant-number for sound wave speed. When the Courant-number is larger than the number of iterative procedures in MRI-NHM, the CPU time of the Lokal-Modell relatively increases because the short time step is restricted by the sound wave speed and the number of small time step integrations increases.

In our case, the differences of the efficiency between the two models are not so large in the practical mesoscale resolution ($\Delta x = 2 \sim 10$ km). For future MPP machines, parallelization efficiency of the 3-D Helmholtz solver may be an issue of the 3-D implicit model as MRI-NHM. Dependency of the efficiency on the model size, and computational robustness of the models are also important factors, while they are subject to future study.

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³ This result is not consistent with Ikawa's (1988) suggestion that the CPU time required for one small time step integration in E-HE-VI scheme is almost comparable to that required for one iterative application in E-HI-VI scheme. In his case, a vertical 1-D Helmholtz equation solver used in E-HE-VI scheme was not optimized for vector processor. We also should note that the CPU times in the vector processor sometimes considerably depend on the vectorization technique. Pressure solver in MRI-NHM could be made faster by loop enlarging in case of HITAC S3800 of MRI (Kato, 1998b).

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ドイツ気象局非静力学局地予報モデルと気象研究所 非静力学メソスケールモデルによる 3 次元山岳波

斉 藤 和 雄・G. Doms*・U. Schaettler*・J. Steppeler*

孤立峰に対する 3 次元山岳波を例に、ドイツ気象局の次世代非静力学局地予報モデル (Lokal-Modell) と、気象研究所非静力学メソスケールモデル (MRI-NHM) との簡単な比較を行った。摩擦を含まない下部境界条件を用いたケースでは、両モデルとも 3 次元山岳波の非静力学線形解析解 (Smith, 1980) の特徴をよく表現した。ドイツ気象局計算機システム (CRAY C-98) を用いた CPU 時間の相対的な比較では、水平分解能が低い時には Lokal-Modell (タイムスプリット法を用い音波を水平方向には陽に扱うモデル) が、水平分解能が高い時には MRI-NHM (音波を 3 方向ともインプリシットに扱うモデル) がより効率的になる傾向を示した。これは音波を水平方向には陽に扱うモデルでは、高分解能時には音速による制限のため小さなタイムステップによる積分回数が増加するためである。MRI-NHM の 1 回の 3 次元楕円方程式直接解法に要する CPU 時間は、Lokal-Modell の 1 回あたりの小さなタイムステップによる積分に要する CPU 時間の 2 倍強で、音速に対するクーラン数と楕円方程式直接解法の繰り返し計算の回数の大小関係が、どちらのモデルがより効率的になるかを決める大まかな目安となる。

* ドイツ気象局