On the Method of Computing Vertical Motion in the Atmosphere (1st Report)

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(Received May 24, 1950)

Abstract

Modifying the equation of potential temperature, two methods of computing vertical velocities are obtained. One of them is the single-station method using the data of pibal and sonde for a single aerological station only, and the other is the several-station method using the data of sonde for several stations. After making nomograms for simplicity in computations, the former method is applied in the case of cold front.

1. Introduction

It has been mentioned previously that small vertical velocity in the atmosphere has large influences on large scale weather phenomena, namely on cloudiness and precipitation, and there are many researches about them. According to J.E. Miller, the nature of vertical motions in the atmosphere is as follows:

"Two types of vertical motion can be recognized: turbulent, short-period motion with velocities up to several meters per second, occurring in eddies that are generally not more than several kilometers in diameter; and widespread, persistent motion with velocities of a few centimeters per second, occurring over areas as broad as a thousand kilometers".

"The first type is found in thermal currents, thunderstorms, and other types of turbulent motion, and its measurement by instrumental methods seems feasible. The second type of vertical motion is associated with large-scale weather systems such as fronts, cyclones and widespread areas of overcast cloudiness and precipitation. It does not seem likely that instrumental measurements of the second type of vertical motion would be practical, because the small velocities are well hidden by the large, superimposed turbulent motion".

Therefore, we must establish the method of computing largescale vertical motions indirectly using other weather elements that are more easily measured. The problem of the method of computation has been discussed by H.A. Panofsky, R.G. Fleagle, J.E. Miller, N. Arizumi and others. Moreover, in U.S.A. the isanabat chart is used for daily weather forecast practically as an auxiliary chart of weather maps.

For computing vertical velocities we have two independent methods, the adi-

abatic and the kinematic method. The former is based on the assumption that changes of state of atmospheric air are adiabatic, and the latter depends on the principle of conservation of mass as expressed by the equation of continuity. In this report, the adiabatic method of computing vertical velocity is obtained considering individual change of potential temperature, and its nomograms are proposed.

Moreover, as a practical example, vertical velocities in the vicinity of a cold front are computed, and the vertical structures, especially the vertical distribution of vertical velocities of this cold front are drawn.

2. The method of computing vertical velocity

The potential temperature θ is expressed by

$$(1) \quad \theta = T \left(\frac{P}{p} \right)^\kappa, \quad \kappa = \frac{c_p - c_v}{c_v}$$

where T is temperature, p pressure, P standard pressure and c_p , c_v the specific heat of constant pressure and constant volume respectively. Assuming the individual change of potential temperature, we obtain

$$(2) \quad \frac{d\theta}{dt} = \frac{\partial\theta}{\partial t} + u \frac{\partial\theta}{\partial x} + v \frac{\partial\theta}{\partial y} + w \frac{\partial\theta}{\partial z}$$

where x increases eastward, y northward, z upward, and u , v , w are corresponding velocity components. From (1) we have

$$(3) \quad \begin{cases} \frac{1}{\theta} \frac{\partial\theta}{\partial x} = \frac{1}{T} \frac{\partial T}{\partial x} - \frac{\kappa}{p} \frac{\partial p}{\partial x} \\ \frac{1}{\theta} \frac{\partial\theta}{\partial y} = \frac{1}{T} \frac{\partial T}{\partial y} - \frac{\kappa}{p} \frac{\partial p}{\partial y} \\ \frac{1}{\theta} \frac{\partial\theta}{\partial z} = \frac{1}{T} \left(\frac{\partial T}{\partial z} + \Gamma \right), \end{cases} \quad \Gamma = \frac{Ag}{c_p}$$

where Γ is the adiabatic lapse rate, A the equivalent heat of work and g the gravity of the earth. Putting these equations into (2)

$$w \frac{\theta}{T} \left(\frac{\partial T}{\partial z} + \Gamma \right) = \frac{d\theta}{dt} - \frac{\partial\theta}{\partial t} - \frac{\theta}{T} \mathbf{V}_2 \cdot \nabla_2 T - \frac{\theta \kappa}{p} \mathbf{V}_2 \cdot \nabla_2 p$$

where $\mathbf{V}_2$ is the horizontal wind vector, and $\nabla_2 T$, $\nabla_2 p$ are horizontal ascendant of temperature and pressure respectively. Assuming adiabatic change and geostrophic flow, that is, $\frac{d\theta}{dt} = 0$, $\mathbf{V}_2 \cdot \nabla_2 p = 0$, we have

$$(4) \quad w = \frac{-\mathbf{V}_2 \nabla_2 T - \frac{T}{\theta} \frac{\partial\theta}{\partial t}}{\frac{\partial T}{\partial z} + \Gamma}$$

The approximate thermal wind equation is represented by

$$(5) \quad \frac{2Q_2}{g} \times \frac{\partial \mathbf{V}_2}{\partial z} = - \frac{\nabla_2 T}{T}$$

where $\mathcal{Q}_2$ is vertical component of Corioris force. Substituting (5) into (4), we have

$$(6) \quad w = \frac{Tf|V_2| \left| \frac{\partial V_2}{\partial z} \right| \sin \beta - g \frac{T}{\theta} \frac{\partial \theta}{\partial t}}{g(\Gamma - \gamma)}$$

where $\gamma = -\frac{\partial T}{\partial z}$, $f = 2\omega \sin \varphi$, φ is the latitude, ω the angular velocity of the earth's rotation, and β is the angle between the vector V_2 and the vector $\frac{\partial V_2}{\partial z}$ and is measured anti-clockwise from the vector $\frac{\partial V_2}{\partial z}$ (see Fig.1).

In practice, we may put $T=270^\circ\text{C}$, $\varphi=36^\circ\text{N}$, therefore the vertical velocity w_h for the height h is given by

$$(7) \quad w_h = 0.239 \frac{v \frac{\partial v}{\partial z} \sin \beta}{1 - \gamma} - \frac{\left(\frac{p}{1000}\right)^{0.288}}{1 - \gamma} \times \frac{\partial \theta}{4.32}$$

Fig. 1 The measurement of the angle β .

where v and p are horizontal velocity and pressure for the level h , and

$$\begin{cases} \gamma = \frac{T_{h-500} - T_{h+500}}{1000} & ^\circ\text{C}/100 \text{ m} \\ \frac{\partial v}{\partial z} = \frac{v_{h+500} - v_{h-500}}{1000} & \text{m sec}^{-1}/100 \text{ m} \\ \frac{\partial \theta}{\partial t} = \frac{\theta_0 - \theta_{12}}{12 \times 60 \times 60} & ^\circ\text{C}/12h \end{cases}$$

where T_{h-500} , T_{h+500} , v_{h-500} , v_{h+500} , are temperatures and horizontal velocities at levels 500 m lower and higher respectively, than the level h , and θ_{12} , θ_0 are potential temperatures 12-hours before and at the present moment respectively. Fig.2 (I, II) and Fig.3 are the nomograms for the first and second term of the right side of the equation (7) respectively.

Now, in the case of computing vertical velocities using the equation (7), we must have observed values of wind, potential temperature and lapse-rate at a single station; that is to say, we must have observed data of pibal as well as of sonde. However, since the upper air observations are scanty in Japan, both of these data are not always available at the same time. Hence, we may modify the equation (7), so that the data of sonde alone may suffice.

From the assumption of geostrophic wind

$$(8) \quad V_2 = \frac{1}{\rho f} \text{grad}_2 p \times k$$

Fig. 2 (I) The nomogram for the computing vertical velocity.

Fig. 3 The nomogram for the computing vertical velocity.

where ρ is the density of the air and $\mathbf{k}$ is the unit vector of vertical axis. Substituting in (4)

$$(9) \quad w = \frac{\frac{1}{\rho f} |\text{grad}_2 p| |\text{grad}_2 T| \sin \beta' - \frac{T}{\theta} \frac{\partial \theta}{\partial t}}{\Gamma - r}$$

where β' is the angle between the vector $\text{grad}_2 p$ and $\text{grad}_2 T$, and measured from $\text{grad}_2 T$ anticlockwise. Furthermore, we may improve the equation (9) so as to use values on isobaric surface; that is to say, from the relation of a pressure gradient and a gradient of constant pressure surface,

Fig. 2 (I) The nomogram for the computing vertical velocity.

$$\frac{1}{\rho} \text{grad}_s p = g \text{grad}_p H$$

where $\text{grad}_p H$ represents a gradient of isobaric surface. A temperature gradient on an isobaric surface $\text{grad}_p T$ is given by

$$\text{grad}_p T = \text{grad}_s T \cos \sigma + \frac{\partial T}{\partial z} \sin \sigma$$

where σ is the angle between the isobaric surface and the horizontal level, and its magnitude is of order $1'$. Therefore we may take approximately

$$\text{grad}_p T = \text{grad}_s T$$

Moreover,

$$\frac{\partial \theta}{\partial t} = \left(\frac{\partial \theta}{\partial t} \right)_p + \left(\frac{\partial H}{\partial t} \right) \frac{\partial \theta}{\partial z}$$

$$\left(\frac{\partial \theta}{\partial t} \right)_p = \left(\frac{P}{p} \right)^\kappa \frac{\partial T}{\partial t}$$

where $\left(\frac{\partial \theta}{\partial t} \right)_p$ represents the potential temperature tendency on the isobaric surface.

Substituting these equations into (9), we obtain

$$(10) \quad w = \frac{\frac{g}{f} |\text{grad}_p H| |\text{grad}_p T| \sin \beta' - \frac{\partial T}{\partial t}}{\Gamma - \gamma} - \frac{T}{\theta} \frac{\partial H}{\partial t}$$

We may easily compose nomograms for the equation (9) and (10), for practical use.

3. The vertical structure of a cold front

By using the nomograms of Figs. 2 and 3, vertical velocities at Tokyo are computed in the case of cold front which passed Tokyo on Nov. 25 (1948). Fig. 4 reproduces the weather chart for Nov. 25 (0900 G.M.T.). And Fig. 5 shows the temperature isopleth of Tokyo for the time interval Nov. 23-26, where full lines represent isotherms for each 4°C , heavy broken lines inversions, and heavy dotted and broken lines surfaces of discontinuity and tropopause. From this figure, it is known that two warm fronts passed in front of the strong cold front. These were incidentals of the two depressions which were located off Genzan, Korea, and at the Tusima Straits in the previous day. These two depressions amalgamated together in the middle of the Japan Sea and moved eastward developing. The strong cold front seems to accompany this strong depression.

Fig. 6 shows the vertical distribution of vertical velocities computed by the nomograms of Figs. 2, and 3. In this figure, full lines are drawn for each 4 cm/sec, the numerals surrounded by them represent the maximum vertical velocities. Fig. 7 represents stream lines in the vertical cross section, where arrows show wind vectors. Horizontal components of these vectors represent east-west components of horizontal winds and vertical components show vertical velocities. In these

Fig. 4 The weather chart for Nov. 25 1948.

vectors vertical velocities are enlarged 200 times. From these figures it is seen that along the warm fronts, ascending currents exist, and also in front of the cold front the strong forced ascending currents exist, and the stream lines near the cold front are closed. At the rear of the cold front, descending currents prevail; however, when the secondary front has passed the air ascends again, and afterwards, these descending currents recover in the cold air except near the ground.

Fig. 5 The temperature isopleth of Tokyo between Nov. 23-26.

Fig. 6 The vertical distribution of vertical velocities.

Fig. 7 Stream lines in the vertical cross section.

4. Summary.

Modifying the equation of potential temperature, we obtained the method computing vertical velocities using sonde and pibal data of a single station, and the method using sonde data of several stations. As a practical example, the vertical distribution of vertical velocities and the stream lines in the vertical cross section of cold front is shown. In using the methods proposed in this reports, it must be remembered they are not applicable near the neutral of stability, because the denominators of the equation (6), (9), (10) approach to zero. This defect is unavoidable in all adiabatic methods, and according to H. A. Panofsky, errors of computation by the adiabatic method become greater, when the lapse-rates become greater than $0.8^{\circ}\text{C}/100\text{ m}$. Therefore, in this case, we must be satisfied with the estimation of the sign of vertical velocities only. Moreover, for the single station method we must use wind data which, however, show greater fluctuations than the other weather elements. Thus, it seems desirable to search for another method using no wind data.