“Some Statistical Considerations on Transhorizon Propagations of Microwaves—with Application to Radio Meteorology (I)”

by

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Abstract

Since sampling time of actual analysis or recording period of observation is of finite length, statistical values obtained are sample means, sample variances and so on, and not ones computed with the use of data over all members of the ensemble.

Taking this fact into consideration, the received fields in transhorizon propagation of microwaves are found to present, for a short sampling time, apparent coherency, which explains the results of Waterman's rapid beam-swinging experiments.

Furthermore, a method is presented to determine whether or not only radio scattering due to atmospheric eddies is dominant in transhorizon propagation.

Actual analyses seem to show that, in most cases, not only scatterings due to eddies but reflections due to wavy layers are dominant in over-sea transhorizon propagation of 3,000 MC/S and 1,300 MC/S, apart from other possible mechanisms which may be relevant to the propagations. In a few cases obtained in a well-mixed atmosphere, the most dominant seemed to be eddy scatterings. It may also be suggested that the sort of analyses made here would be useful in affording some measure of weather situation on the so-called meso-scale.

1. Introduction

There are mainly three representative theories in transhorizon propagations of microwaves, that is, radio scattering due to atmospheric eddies, reflection due to layers and earth diffraction (Joint Technical Advisory Committee, 1960). It seems that it has not yet been decided from observation which theory is the best.*

* In the integration of Maxwell's equations for transhorizon propagation of microwaves, the Stratton-Chu's formula based on the vector wave equation gives a solution, with the help of Born's approximation, which consists of three parts corresponding to the above three theories (Naito, 1962). STARAS and WHEELON (1959) gave only a special solution of the wave equation using Born's approximation. However, it would be better to mention here that the solution of wave equation of second order for electric or magnetic vector instead of the Hertz vector does not always satisfy Maxwell's equations of first order and that some virtual charge density should be introduced in order to avoid the dissatisfaction in this case with the use of Born's approximation (Naito, 1962). Apart from this problem, it is not possible theoretically to admit superiority to only one of the above theories, and it is our standpoint here that we should assume, for the time being, the coexistence of the three propagation mechanisms.

Of these three, the effects of eddies or layers have been considered statistically over the whole ensemble of all possible events. However, since the actual sampling time or observation duration is of finite length and the space with which we are mainly concerned is also of finite domain, most of statistical estimates obtained are sample mean, sample variance and so on. The part (1) of this study will deal with the effect of finite length of sampling time on the phenomena in transhorizon propagation chiefly in relation to scattering due to eddies.

2. Effect of finite length of sampling time

For simplicity, we shall not consider reflection due to random layers at present.

Electric field $E(t)$ at a receiving site is given, in the transhorizon propagation of microwaves, by

$$(1) \quad E(t) = E_0 + E_s(t),$$

where E_0 is the constant and coherent field caused by reflection due to persistent elevated layers, diffraction of the earth and/or super-refraction, $E_s(t)$ the field scattered by the atmospheric eddies, and t indicates time.

Let us denote by $\delta\epsilon(x, t)$ the deviation of the dielectric constant, ϵ , of the atmosphere from the ensemble average, and the eddies are expressed by the aggregations of $\delta\epsilon$, and E_s is given, as a first approximation, by

$$(2) \quad E_s(t) = \alpha \int_V \frac{\delta\epsilon(x, t)}{\epsilon} \exp(-ik \cdot x) dx,$$

where α is a constant vector, k the propagation vector, V the so-called scattering volume and x the position vector.

If we denote by $\langle \rangle_T$ the sample mean over the recording period or the sampling time, T and, for example,

$$(3) \quad \langle \mu(x, t) \rangle_T = \frac{1}{T} \int_{-T/2}^{T/2} \mu(x, t + \xi) d\xi,$$

where $\mu(x, t)$ will mean $\delta\epsilon(x, t)/\epsilon$ hereafter, then the fluctuation component in reference to the sample mean is

$$(4) \quad \mu'_T(x, t + \xi) = \mu(x, t + \xi) - \langle \mu(x, t) \rangle_T.$$

For a process stationary in time,

$$(5) \quad \overline{\langle \mu(x, t) \rangle_T} = \lim_{T \rightarrow \infty} \langle \mu(x, t) \rangle_T = 0,$$

and

$$(6) \quad \lim_{T \rightarrow \infty} \mu'_T(x, t + \xi) = \mu(x, t + \xi),$$

where the upper bar indicates the ensemble average.

Considering the relation (4), (1) is rewritten as

$$(7) \quad E(t + \xi) = E_0 + \langle E_s(t) \rangle_T + E'_{s,T}(t + \xi)$$

$$\begin{aligned}
&= E_0 + \alpha \int_V \langle \mu(x, t) \rangle_T \exp(-ik \cdot x) dx \\
&\quad + \alpha \int_V \mu'_T(x, t + \xi) \exp(-ik \cdot x) dx
\end{aligned}$$

and consequently, averaging the energy flow over the duration and then over the ensemble, we have

$$\begin{aligned}
(8) \quad \overline{\langle |E(t)|^2 \rangle_T} &= \overline{|E_0 + \langle E_s(t) \rangle_T|^2} + \overline{\langle |E'_{s,T}(t)|^2 \rangle_T} \\
&= |E_0 + \alpha \int_V \langle \mu(x, t) \rangle_T \exp(-ik \cdot x) dx|^2 \\
&\quad + |\alpha|^2 \int_V \int_V \overline{\langle \mu'_T(x+y, t) \mu'_T(x, t) \rangle_T} \exp(-ik \cdot y) dx dy
\end{aligned}$$

where the first term in the right-hand side of the above equation is obviously a coherent field and the second term an incoherent one.

The apparently coherent field $\langle E_s(t) \rangle_T$ resulting from the averaging over T is a significant effect of finite length of the sampling time.

In addition, we had better notice that, for a stationary process,

$$(9) \quad \overline{\langle |E(t)|^2 \rangle_T} = \lim_{T \rightarrow \infty} \langle |E(t)|^2 \rangle_T,$$

which means together with (8) that the sum of coherent and incoherent power is independent of the sampling time on an average.

3. Coherent components

The coherent term in (8) becomes

$$|E_0|^2 + \overline{|\langle E_s(t) \rangle_T|^2},$$

the second term of which is the variance of the sample mean and does not vanish as long as T is finite.

Since $\langle \mu \rangle_T$ may be assumed to vanish outside the integration domain V , we have, from (7),

$$\begin{aligned}
(10) \quad \langle E_s(t) \rangle_T &= \alpha \iiint_{-\infty}^{+\infty} \langle \mu(x, y, z) \rangle_T \exp\{-i(k_1 x + k_2 y + k_3 z)\} dx dy dz \\
&= \alpha \iint dx dy \exp\{-i(k_1 x + k_2 y)\} \int \frac{dz}{ik_3} \frac{\partial \langle \mu \rangle_T}{\partial z} \exp(-ik_3 z)
\end{aligned}$$

after integration by parts with respect to z , and the procedure in calculation is essentially the same as that of SIEGERT and GOLDSTEIN in KERR's book (1951).

It is shown from (10) that the large contributions come from the regions of large gradient of $\langle \mu \rangle_T$.

In case there are n layers of large gradient of $\langle \mu \rangle_T$, eq. (10) may be expressed by

$$(11) \quad \langle \underline{E}_s(t) \rangle_T = \sum_{j=1}^n \alpha \iint dx dy \exp\{-i(k_1 x + k_2 y)\} \int \frac{1}{ik_3} \left\{ \frac{\partial \langle \mu \rangle_T}{\partial z} \right\}_j \exp(-ik_3 z) dz.$$

If, moreover, $\langle \mu \rangle_T$ does not vary inside each of the layers, that is, $\langle \mu \rangle_T$ is constant between $z_{1j}(x, y)$ and $z_{2j}(x, y)$ ($j=1, 2, \dots, n$) and if $\langle \mu \rangle_T$ is negligibly small outside the layers, it follows from eq. (11) that

$$(12) \quad \begin{aligned} \langle \underline{E}_s(t) \rangle_T &= \sum_{j=1}^n \alpha \iint dx dy \exp\{-i(k_1 x + k_2 y)\} \\ &\quad \times \int_{z_{1j}(x, y)}^{z_{2j}(x, y)} dz \frac{\langle \mu \rangle_{Tj}}{ik_3} \{\delta(z - z_{1j}) - \delta(z - z_{2j})\} \exp(-ik_3 z) \\ &= \sum_{j=1}^n \alpha \frac{\langle \mu \rangle_{Tj}}{ik_3} \iint \exp\{-i(k_1 x + k_2 y)\} [\exp(-ik_3 z_{1j}) - \exp(-ik_3 z_{2j})] dx dy \\ &= \sum_{j=1}^n \alpha \frac{\langle \mu \rangle_{Tj}}{ik_3} \iint_{S_j} \exp\{-i(k_1 x + k_2 y + k_3 z)\} \cos \gamma dS_j, \end{aligned}$$

where $\delta(z)$ is Dirac's delta function, dS_j the surface element of the j -th layer, and γ the angle between z -axis and the normal of dS_j .

It is seen from (12) that, in case of constant $\langle \mu \rangle_{Tj}$ in the j -th layer, the boundary surface of the layer acts as a reflecting surface, while its inner portion has no effects as a whole. This is a typical feature of coherency.

Furthermore, it is found from (5) that the apparently coherent field, $\langle \underline{E}_s(t) \rangle_T$, vanishes as T approaches infinity. In other words, the apparently coherent field is dominant for the short sampling time and it shows reflections by large eddies specified by $\langle \mu(x) \rangle_{Tj}$.

For a stationary process, we have, thus,

$$(13) \quad \lim_{T \rightarrow \infty} \overline{|\underline{E}_0 + \langle \underline{E}_s(t) \rangle_T|^2} = |\underline{E}_0|^2.$$

It should be mentioned here that the apparently coherent field was visualized by the rapid beam-swinging experiments in transhorizon propagations made by A.T. WATERMAN (1958). So far most of the beam-swinging experiments have been analyzed with a long sampling time, so that the results have not shown the apparently coherent field but only the incoherent field which results in the beam spread. WATERMAN's short sampling time showed, however, little beam spread, which means the dominant existence of a coherent field, that is, the apparently coherent field in most cases. It is natural that the superposition of the apparently coherent fields in WATERMAN's experiment should result in the beam spread or the incoherent field.

4. Incoherent components

For stationary fluctuations we have

$$(14) \quad \overline{\langle \mu'_T(\underline{x} + \underline{y}, t) \mu'_T(\underline{x}, t) \rangle_T} = \overline{\langle \{\mu'_T(\underline{x})\}^2 \rangle_T} \rho_T(\underline{y}, \underline{x}),$$

where ρ_T is a spatial correlation of μ'_T with separation vector $\underline{y}$ at the position indicated by $\underline{x}$, and $\overline{\langle (\mu'_T)^2 \rangle_T}$ an intensity of fluctuations based on T .

Thus, the incoherent term of (8) becomes

$$(15) \quad \overline{\langle |E'_{s,T}|^2 \rangle_T} = |\alpha|^2 \int_V \int_V \overline{\langle \{\mu'_T(\underline{x})\}^2 \rangle_T} \rho_T(\underline{y}, \underline{x}) \exp(-i\mathbf{k} \cdot \underline{y}) d\underline{x} d\underline{y},$$

and, if the fluctuations are homogeneous in space,

$$(16) \quad \overline{\langle |E'_{s,T}|^2 \rangle_T} = |\alpha|^2 V \overline{\langle \mu'^2_T \rangle_T} \int_V \rho_T(\underline{y}) \exp(-i\mathbf{k} \cdot \underline{y}) d\underline{y}.$$

It is obvious from (6) that, as T increases, $\overline{\langle |E'_{s,T}|^2 \rangle_T}$ increases and finally becomes the scattered power which has been discussed by many authors since BOOKER and GORDON (1950) paid attention to it.

Let us consider the properties of $\overline{\langle (\mu'_T)^2 \rangle_T}$ to some extent. In statistical words, $\overline{\langle (\mu'_T)^2 \rangle_T}$ is the sample variance which increases with the sample size T . Physically it is considered as the following. As is obvious from (3) and (4), clouds of large scales which do not appreciably move or change for the duration T are not regarded as fluctuating components but as mean trends for the duration, while only those of small scales contribute to the intensity $\overline{\langle (\mu'_T)^2 \rangle_T}$. As T increases, larger clouds come to be regarded as fluctuating components, so that $\overline{\langle (\mu'_T)^2 \rangle_T}$ increases to be $\overline{\mu'^2}$ and the incoherent term thus increases with T , while the coherent term decreases with T , as stated in the previous section.

Furthermore it should be mentioned that the ensemble average of coherent power, $|\langle \tilde{E}_s(t) \rangle_T|^2$, is considered as the variance of the sample mean of a two-dimensional vector with zero mean which decreases with the sample size necessarily, as is well known in statistics, and, on the other hand, the ensemble average of incoherent power, $\overline{\langle |E'_{s,T}|^2 \rangle_T}$, is considered as the sample variance of a two-dimensional vector with zero mean which necessarily increases with the sample size.

Fig. 1 illustrates the facts described in the preceding and present sections.

5. Received power and sampling time

Received power p is written as

$$(17) \quad p = \langle p \rangle_T + p'_T,$$

where $\langle p \rangle_T$ is the sample mean and p'_T the deviation from the mean. In case p is stationary in time, it follows that

$$(18) \quad \overline{p^2} = \overline{\langle p \rangle_T^2} + \overline{(p'_T)^2}$$

Fig. 1. Variations of the apparently coherent component and the incoherent component with the sampling time T . (1) indicates the apparently coherent component $|\langle \tilde{E}_s \rangle_T|^2$, (2) the incoherent component $\overline{\langle |E'_{s,T}|^2 \rangle_T}$, (3) the coherent component $|\tilde{E}_0 + \langle \tilde{E}_s \rangle_T|^2$ in case persistent coherent field $\tilde{E}_0$ exists, and (4) total power.

* Similar situations have been recognized on turbulent flows by many workers on atmospheric turbulence (for example YAYAMOTO and SHIOTANI, 1948 and DEACON, 1955).

The two quantities in the right-hand side of (18) are both dependent on T , but the sum of them is independent of T . Consequently $\overline{\langle p \rangle_T^2}$ decreases as T increases, since $(p'_T)^2$ is the sample variance increasing with T .

Now let us put

$$(19) \quad \left. \begin{aligned} |E_0|^2 &= p_0, \\ |E_s|^2 &= p_s, \end{aligned} \right\}$$

where these quantities are proportional to power and the proportional constants will be, for simplicity, assumed to be unity hereafter. Then we have

$$(20) \quad \overline{|E_0 + E_s|^2} = p_0 + p_s = \bar{p}.$$

And it is obtained that, as $T \rightarrow 0$,

$$(21) \quad \left. \begin{aligned} \overline{\langle p \rangle_T^2} &\rightarrow \overline{\langle p \rangle_0^2} = \bar{p}^2 = \overline{|E_0 + E_s|^4} = p_0^2 + 4p_0p_s + 2p_s^2, \\ \overline{(p'_T)^2} &\rightarrow 0, \end{aligned} \right\}$$

and that,

as $T \rightarrow \infty$,

$$(22) \quad \left. \begin{aligned} \overline{\langle p \rangle_T^2} &\rightarrow \overline{\langle p \rangle_\infty^2} = (\bar{p})^2 = (|E_0|^2 + |E_s|^2)^2 = (p_0 + p_s)^2, \\ \overline{(p'_T)^2} &\rightarrow \bar{p}'^2 = \overline{|E_0 + E_s|^4} - (|E_0|^2 + |E_s|^2)^2 = 2p_0p_s + p_s^2, \end{aligned} \right\}$$

where use is made of the fact that E_s is a Rayleigh-distributed vector.

Without E_0 , it follows that, as $T \rightarrow 0$,

$$(23) \quad \left. \begin{aligned} \overline{\langle p \rangle_T^2} &\rightarrow 2p_s^2, \\ \overline{(p'_T)^2} &\rightarrow 0, \end{aligned} \right\}$$

and that, as $T \rightarrow \infty$,

$$(24) \quad \left. \begin{aligned} \overline{\langle p \rangle_T^2} &\rightarrow p_s^2, \\ \overline{(p'_T)^2} &\rightarrow p_s^2. \end{aligned} \right\}$$

The variations with the sampling time, described above, are illustrated in Fig. 2.

Fig. 2. Variations of $\overline{\langle p \rangle_T^2}$ and $\overline{(p'_T)^2}$ with the sampling time T .

(a) is the case with E_0 and (b) the case without E_0 .

It will be interesting to notice that, without E_0 , both $\overline{\langle p \rangle_T^2}$ and $\overline{(p'_T)^2}$ tend to the same value as p_s^2 with the increasing T , and that, in other words, if they do not tend to the same value, it means the presence of E_0 . However, it should be mentioned again here that the above arguments hold only if the random process here dealt with is stationary in time and E_0 is zero or constant.

Nevertheless, the above fact may be useful. In case we get such a situation as shown in Fig. 2(b), only scattering due to eddies is concluded to be dominant in transhorizon propagation at that time. When aerological soundings show that the atmosphere is well mixed up with no persistent elevated layers, and if we get such a situation as shown in Fig. 2(a) at that time, an experimental verification will be made on the presence of the earth diffraction which seems to be rather disregarded in transhorizon propagation at present.

6. Examples of analyses

As stated before, there seem to be no experiments except those of WATERMAN to give direct evidence of the apparent coherency, and analyses were made here only in the way described in section 5.

High speed recordings from which instantaneous values can be read off at considerably small time intervals are needed in order to make calculations for the present purpose. In 1958, the Electrical Communication Laboratories, Tokyo, carried out a series of experiments on over-sea 300-km propagations beyond the horizon, in which two-minute fast recordings were obtained for some two weeks in winter, although there lacked observations several times. The two-minute records were not long enough to give sufficient basis for definite conclusions, but were useful to some extent in affording examples.

The records were analyzed by subdividing the traces into a number of equal portions (say, of ten seconds each). Each portion being treated as a separate record, a number of values of $\langle p \rangle_T$ and p'_T were found by following the definitions shown in (3) and (4), and the corresponding values of $\overline{\langle p \rangle_T^2}$ and $\overline{\langle (p'_T)^2 \rangle_T}$ were obtained. By assuming that the mean of these was approximately equal to the ensemble average, calculations were made for the values of $\overline{\langle p \rangle_T^2}$ and $\overline{(p'_T)^2}$. This was repeated for other intervals, that is, for various sampling time T . Fig. 3 shows typical examples of the results thus obtained, in which, it should be noted, the power p here dealt with should not be logarithmic (db-value) but antilogarithmic.

In Fig. 3, (a) and (b) are nearly the same as those of Fig. 2, but (c) is quite different. The undulating tendency in the case (c) would suggest the presence of undulating components in the received power due to reflections from layers of quasi-periodic structure, which will be described in detail in part (II) of this study.

The weather symbols added in the figure may be interesting. In the case (a), the atmosphere with which we are concerned could be stated to be well mixed up, judging from the usual weather map.

The curves in Fig. 3 seem to change slowly or rapidly as the weather changes. However, we will not go into details here about this matter.

Most of the cases analysed here were of the type (c), which would mean that

Fig. 3. Examples of variations of $\langle p \rangle_T^2$ and $\langle p'_T \rangle^2$ with the sampling time T . Cloud cover is at a midpoint of the propagation path.

the reflection due to wavy layers as well as the scattering due to eddies should be taken into consideration in transhorizon propagations.

7. Conclusions

Our main conclusions are the following:

(1) In case the sampling time is short, the received fields are found to present apparent coherency which explains the results of WATERMAN's beam experiments.

(2) A method is presented to determine whether or not only eddy scattering is dominant in transhorizon propagations, and whether or not eddy-scattering fields and a constant field are both dominant.

(3) Actual analyses seem to show that, in most cases, not only scattering due to eddies but reflection due to wavy layers are dominant in over-sea transhorizon propagations of 3,000 MC/S and 1,300 MC/S, apart from other possible mechanisms which may be relevant to the propagations. In a few cases with a well-mixed atmosphere, the most dominant seemed to be eddy scattering. It may also be suggested that the sort of analyses made here would be useful in indicating some measure of the weather situation on the so-called meso-scale. Details on the reflection due to wavy layers will be reserved for part (II) of this study.

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極超短波見通し外伝搬に関する統計的考察—電波気象学への応用 (I)

内 藤 恵 吉

実際のデータ解析における抽出時間長, また観測時間は有限であるので, これらを基にして得られる統計値は, 標本平均, 標本分散等であつて, 可能性のあるすべての事象にわたつての統計値ではない。

この事実を考慮して極超短波の見通し外伝搬の受信界を処理すると, 短い抽出時間のときには, 見掛け上の可干渉性が得られる。これによつて, WATERMAN の有名な "rapid beam-swinging experiment" の結果が説明される。

また, 大気渦による電波散乱のみが, 見通し外伝搬に有力であるかどうかを決定する, 一つの方法も示されている。

以上の考察に基づいて, 3,000 MC/S と 1,300 MC/S を用いての海上見通し外伝搬データを解析してみると, 多くの場合, 渦による散乱だけでなく, 波状の層からの反射も大いに寄与していると考えられる。他の伝搬機構については, ここでは論じない。よく混合している大気の場合に得られた僅かの場合には, 解析の結果, 渦による散乱が最も有力のようにみえる。また, ここでなされた解析は, いわゆるメソスケールの天気状態を示す, ある種の尺度を与えるのに役立つであろう。