The Spectrum of the Local Momentum Transfer

by

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Abstract

The spectrum analysis of momentum transfer has been made using the observed winds over the four stations at Sapporo, Tateno, Naze and Clark at the four isobaric levels of 850-, 500-, 300- and 100-mb for the year 1961. It is found that in the middle latitudes the northward momentum transfer is largely accomplished by the eddies with periods of about 3 days and about 6 days, whereas in the lower latitudes the transfer due to fluctuations with periods shorter than 4~5 days is entirely negligible.

1. Introduction

Since the time of STARR and WHITE (1951), it has been known that there is a constant northward flow of momentum from the source regions in the tropics to the sink at middle latitudes. They also showed that this northward momentum transfer is mostly associated with the migratory troughs and wedges which are generally oriented along northeast-southwest direction. Based upon this consideration, it is to be expected that the maximum northward transfer is found in regions of the spectrum corresponding to a period of about 4 to 6 days which is approximately the mean interval between successive passages of troughs moving eastward. So far as the present authors know, ESTOQUE (1955) is the first to show from the spectrum analysis of momentum transfer that the northward transfer is accomplished largely by the eddies with a period of about 4 days. However, his analysis was made using the observations over only one station, Greensboro (36°N, 80°W), for a single level of 850-mb. We have, in this paper, extended the analysis to include observations at upper levels as well as at other latitudes. Our main concern is about how the spectra of the momentum transfer differ with the different heights and with the different latitudes.

2. Analysis for the spectrum

Exact determination of the spectrum would require a perfectly-measured, infinitely-long piece of records, and would require infinitely detailed computations. Both of these requirements are, of course, impractical. A somewhat heuristic approach will be employed here by adopting a method introduced by BLACKMAN and TUKEY (1959) for the case of the equally spaced records.

Let us denote the latitudinal and meridional components of the wind by u and v

respectively. Suppose we have prepared sufficiently long record of observations of u and v over a station. The assumption regarding the length of the interval is necessary in order that sufficiently large number of fluctuations are included in the record in question, and one may obtain reasonable accurate information about the statistical characteristics of the fluctuations. Thus the average values, $\bar{u}$ and $\bar{v}$, evaluated over the interval considered, are assumed to be nearly equal to the corresponding averages over infinitely long periods of time.

Suppose that the observational data are prepared at equally spaced time interval Δt , *i.e.*

$$(2.1) \quad \begin{aligned} u(n\Delta t) &= u_n \\ v(n\Delta t) &= v_n \end{aligned} \quad n=1, 2, \dots, M,$$

where M denotes the total number of the data used. Now the averages may be estimated by

$$(2.2) \quad \bar{u} = \frac{1}{M} \sum_{n=1}^M u_n, \quad \bar{v} = \frac{1}{M} \sum_{n=1}^M v_n$$

Denoting the deviations from the means by the primed quantities

$$(2.3) \quad u'_n = u_n - \bar{u}, \quad v'_n = v_n - \bar{v}$$

the average of the momentum transfer due to eddies may be obtained to be

$$(2.4) \quad \overline{u'v'} = \frac{1}{M} \sum_{n=1}^M u'_n v'_n$$

This mean eddy momentum transfer is composed of many contributions due to fluctuations. An attempt will now be made to decompose the mean eddy transfer, $\overline{u'v'}$, into its spectral components.

BLACKMAN and TUKEY developed their analysis through the use of cross-correlations and Fourier transforms. We shall first estimate the cross-correlations by

$$(2.5) \quad \left. \begin{aligned} C(k) &= \frac{1}{M-k} \sum_{n=1}^{M-k} u'_n v'_{n+k} \\ C(-k) &= \frac{1}{M-k} \sum_{n=1}^{M-k} u'_{n+k} v'_n \end{aligned} \right\} \quad k=0, 1, \dots, h,$$

where h represents the largest lag considered. It is usually undesirable to use lags longer than a moderate fraction (perhaps 5 or 10 per cent) of the length of the record, *i.e.*, $h \leq 0.1 M$. Since we are going to use radiosonde soundings obtained once a day in the one year time interval, M is set to be equal to 365 and hence h is assigned to be $h=40$. It is here worthy to note again that the time interval of one year must be assumed to be long enough to include a large number of fluctuations, and hence both $C(k)$ and $C(-k)$ measured by (2.5) are regarded as an excellent approximation of the actual value of the cross-correlation with lag k evaluated for an infinitely long record of the original fluctuations.

Next the average cross-correlation may be obtained in the form

$$(2.6) \quad \bar{C}(k) = \frac{1}{z} [C(k) + C(-k)]$$

Applying a discrete finite cosine series transform to the sequence $\bar{C}(0), \bar{C}(1) \dots, \bar{C}(h)$, we find

$$(2.7) \quad S\left(\frac{r}{2\Delta t h}\right) = \Delta t \left[\bar{C}(0) + 2 \sum_{k=1}^{h-1} \bar{C}(k) \cos \frac{\pi}{h} rk + \bar{C}(h) \cos \pi r \right],$$

$$r=0, 1, 2, \dots, h.$$

$S(r/2\Delta t h)$ is the amplitude-spectrum for the frequency $f=r/2\Delta t h$. To avoid needless repetition, however, the adjective "amplitude" will be dropped in the subsequent pages when referring to the amplitude-spectrum. It is very convenient to express the spectrum in terms of the period, rather than using the frequency. This is

$$(2.8) \quad p = \frac{2\Delta t h}{r}$$

Therefore $r=40$ corresponds to a period of 2 days, $\dots$, and $r=2$ is equivalent to a period of 40 days.

In order to minimize a possible smearing effect, it is customary to perform "hamming" over $S(r/2\Delta t h)$ obtained by (2.7),

$$(2.9) \quad \left\{ \begin{array}{l} \hat{S}\left(\frac{0}{2\Delta t h}\right) = 0.54 S\left(\frac{0}{2\Delta t h}\right) + 0.46 S\left(\frac{1}{2\Delta t h}\right), \\ \hat{S}\left(\frac{r}{2\Delta t h}\right) = 0.23 S\left(\frac{r-1}{2\Delta t h}\right) + 0.54 S\left(\frac{r}{2\Delta t h}\right) + 0.23 S\left(\frac{r+1}{2\Delta t h}\right), \\ \quad \quad \quad r=1, 2, \dots, h-1, \\ \hat{S}\left(\frac{h}{2\Delta t h}\right) = 0.46 S\left(\frac{h-1}{2\Delta t h}\right) + 0.54 S\left(\frac{h}{2\Delta t h}\right) \end{array} \right.$$

We shall measure (2.9) at several observation stations in an effort to find the effective eddies in transporting momentum.

3. The spectrum of the local momentum transfer

The measurements have been made using the observations on 1200 GMT over four stations at Sapporo (43°N, 141°E), Tateno (36°N, 140°E), Naze (28°N, 129°E) and Clark (15°N, 121°E) for the year 1961. The observed winds were used at the four isobaric levels of 850-, 500-, 300-, and 100-mb.

Before starting the computations, it was decided to remove the possible smearing effect of the annual cycles of u and v by estimating their components using harmonic analysis. These components, together with the corresponding annual means, were subsequently subtracted from the original observations to obtain the fluctuations u' and v' .

Fig. 1. The spectrum of momentum transfer at Sapporo (43°N , 141°E) obtained by (2.9) for the 850-mb level (top-left), the 500-mb level (top-right), the 300-mb level (bottom-left), and the 100-mb level (bottom-right). The unit is in $\text{m}^2 \cdot \text{sec}^{-2} \cdot \text{day}$.

Fig. 1 demonstrates the spectra of momentum transfer at Sapporo (43°N , 141°E). The most striking characteristic of the spectra at the 300-mb level, where the northward momentum transfer is generally maximum as was pointed out by STARR and WHITE (1951), is the extreme importance of the fluctuations with a period about 6 days. A high peak with $p=6$ days is also in clear existence at the 500- and 100-mb levels, whereas at the 850-mb level no corresponding peak can be found. Another significance in Fig. 1 is the appearance of the large contribution due to the fluctuations with a period of about 3 days. This contribution seems to be most important at the 500-mb level and less important at the other levels, at the highest level $p=100$ -mb in particular. Probably, the high peaks both at $p=6$ days and at $p=3$ days would be the expected contributions of the migratory troughs and wedges.

Concerning this, there arises the question why the spectra at the 850-mb level have no significant peak, notwithstanding that successive passages of frontal disturbances and the associated anticyclones are encountered at this level.

Based upon observations at only one station, Greensboro (36°N , 80°W), and at a single level of the 850-mb level, ESTOQUE (1955) found that the momentum transfer is largely accomplished by the fluctuations with a period of about 4 days, and the contribution due to eddies with periods larger than 6~7 days is negligibly small. Thus, so far as the spectrum at the 850-mb level is concerned, there is a great difference between the two results, the one obtained by ESTOQUE and that by the present authors. We think, therefore, that it would be premature to offer any definite conclusion about the nature of momentum transfer process in the atmosphere before we perform much more extensive analyses of other cases.

Another interesting feature noted in Fig. 1 is the existence of a negative peak with a period of about 15 days at almost all levels. A negative sign indicates a contribution to the southward momentum transfer. It is also revealed in Fig. 1 that the total amount of momentum transfer due to eddies with periods larger than 20 days is appreciably large. This may possibly be peculiar to the particular set of

Fig. 2. The spectrum of momentum transfer at Tateno (28°N , 140°E). The others are the same as in Fig. 1.

Fig. 3. The spectrum of momentum transfer at Naze (28°N, 129°E). Refer to Fig. 1 for the further explanations.

Fig. 4. The spectrum of momentum transfer at Clark (15°N, 121°E). Refer to Fig. 1 for the further explanations.

data analyzed or due to the unreliability of results at the longer periods. We shall here emphasize again that the results of the spectrum analysis for longer periods are not entirely reliable so that these should be interpreted with caution.

We shall now take a look at Fig. 2, showing the spectra obtained at Tateno (36°N , 140°E). The dominant role of the eddies with periods about 6~8 days is clearly seen at almost all levels, except the 850-mb level. The fluctuations with a period of about 3 days are only predominant at the 500-mb level.

Figs. 3 and 4 have been presented here to offer information about the spectra of momentum transfer in the lower latitudes. First of all, we note that the total amount of transfers due to eddies with periods shorter than 4~5 days is entirely negligible for this particular set of data. The spectra are somewhat large around the period 6~8 days. Generally speaking, the spectrum in the lower latitudes does not show distinct characteristic as compared with that in the middle latitudes shown in Figs. 1 and 2.

It is certainly not appropriate to draw any definite conclusions about the nature of the momentum transfer process from the very limited application of the spectrum analysis presented above. It would appear, however, that the main features of both vertical and latitudinal dependencies of the momentum transfer spectra are revealed by this study, namely:

- (1) in the middle latitudes the northward momentum transfer is mainly accomplished by the eddies with marked periods of about 6 days and 3 days, whereas in the lower latitudes the short-period fluctuations with periods shorter than 4~5 days are insignificant in transporting momentum.
- (2) as for the vertical dependency, it may be said that the dominant period is shorter in the lower atmosphere and longer in the upper atmosphere.

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運動量輸送のスペクトル解析

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1961年一カ年について札幌、館野、名瀬、クラークの四地点で850 mb, 500 mb, 300 mb 及び 100 mb の運動量輸送 uv を毎日計算し、そのスペクトル解析を行なった。スペクトルの緯度による相違、高度による相違をしらべるのが目的である。札幌、館野では3日と6日周期が卓越する。3日周期は500 mb で最も明瞭に現われ300 mb でもみられるが100 mb では全く現れない。6日周期は300 mb で最も顕著で500 mb, 100 mb でも明瞭である。

低緯度の状況をみようと名瀬、クラークを選んだ。3日周期は全然現われず、6~8日周期のみがみられる。大体において緯度が低くなるほど卓越週期が長くなる傾向がある。