Meso-scale Disturbance Observed in the Vicinity of a Cold Vortex Center
— With special regards to gravity waves —

by

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Abstract
A considerable amount of snowfall was observed on January 20 and February 2, 1964 when the westerly troughs had passed over Japan. This was related to the meso-scale disturbance which developed in the vicinity of the cold dome rather than to the synoptic cyclone in front of the westerly trough.

The meso-scale disturbance observed on February 2 associated a system of line echo and moved SSE-wards with a velocity of 100 km/hr and a period of 3 hrs. This phenomenon is well explained by the concept of internal gravity wave.

1. Introduction
Among synoptic meteorologists, it has been well known that heavy snowfalls in the Hokuriku District (the Japan Sea coastal region of central Japan) is closely related to the cold vortex in the midtroposphere (e.g. FUKUDA, 1960). A cold vortex is obviously a kind of well-developed westerly trough which is generally associated with the upward motion field and precipitation within the region of a surface cyclone towards the east, as is known theoretically and synoptically. While the principal snowfall in the Japan Sea coastal region is usually observed rather in the central part of an upper cold vortex where downward motion would be expected as to the ordinary atmospheric model. It is thus suggested that the combined effect of the upper cold air and the heating from the very warm sea surface makes the air column unstable when the cold vortex covers the Japan Sea, resulting in a predominant convective activity and probably meso-scale disturbances. (MATSUMOTO et al. 1965).

In this paper, analyses are made on two cases of snowfall related to the westerly disturbances observed on January 20 and February 2, 1964, when the second year's observation was carried on in the Hokuriku District as a part of the Heavy Snow Storm Project. Although it was a year of comparatively less snow coverage, a considerable amount of snowfall was observed for either cases. Since an aerophotographic observation was also made on February 2 (MATSUMOTO and NINOMIYA, 1965a), analyses on convective clouds were incorporated into meso-analysis.

There have been so many studies on meso-scale disturbances which are characterized by a remarkable jump in pressure and/or temperature and/or wind field.
FUJITA's model (1959) of pressure rise due to cold air production associated with rain seems to be applicable to many cases. In the situation which will be described here, however, we could not find any noticeable temperature change, and the gravity wave originating on the cold dome boundary would provide a good understanding.

Fig. 1 shows the observation network in the Hokuriku District which was used for the meso-analysis mentioned in Section 3.

Fig. 1. Distribution of weather stations in Hokuriku District.

2. Cold vortex and surface disturbances

The analytical fact that cyclones develop in the forward region of westerly troughs is fully explained theoretically by baroclinic instability. As is shown in Fig. 2, a well-developed cyclone is found about 800 km east of the cold vortex under consideration. In this figure, the surface pressure field at 1200 LST February 2, 1964 and the cold dome contour on 500 mb (cf. MATSUMOTO and NINOMIYA, 1965b) are shown together with the indication of location of the cyclone. Besides this synoptic scale cyclone which is considered to be related to baroclinic instability, a smaller scale cyclone is found over the western coast of Japan. The latter stayed almost at the same locality until the upper cold vortex had passed over Japan, and it could be regarded as a secondary depression caused probably by orographic effect.

It should be remarked that cumulus development and convective precipitation are distributed inside the cold dome as is shown in Fig. 3. A system of line echo is observed in the vicinity of the cold vortex center and is represented by cross symbols in Fig. 2. It brought a considerable snowfall amounting to 4 mm/hr on its arrival. The line of small circles which is also entered in Fig. 2 indicates the minimum pressure of the surface pressure disturbance described in Section 3.

The circumstance mentioned above is verified by the time change of pressure
Fig. 2. Surface pressure distribution at 1200 LST February 2, 1964. Cold dome boundary at 500 mb level at 0900 LST is shown by a broken line. Cross symbols indicate a line echo and small circles pressure minimum line of meso system.

Fig. 3. Weather distribution at 1200 LST and cold dome contour at 0900 LST February 2.
and temperature at the upper level and surface pressure. Two examples are given in Fig. 4. The upper figure is for January 17-21 and the lower figure for January 30-February 4, 1964. 500 mb pressure and temperature and surface pressure at Wajima are shown there. Double arrows indicating the transit time of cyclones closest to Wajima show clearly that cyclones preceded towards the east of cold vortices. As to the latter case, low pressure continued until the passage of the upper cold vortex and then a steep pressure rise took place, showing that a local depression stayed on the windward side of the Japan Islands.

In the lowermost part of Fig. 4 is shown the 3 hourly precipitation at Ōno (see Fig. 1), which clearly shows two groups of precipitation, one associated with the baroclinic instability cyclone and the other caused by static instability in the cold dome center. Generally speaking, the latter plays an important role in the snowfall in the Hokuriku District.

The details of the behavior of line echo which was observed in the cold dome center on February 2 will be discussed elsewhere (MATSUMOTO and NINOMIYA, 1965a) in relation to aerophotographic observations.

3. Analyses of meso-scale disturbances

There have been many analyses showing that the size and moving velocity of the precipitating area are different from those of synoptic scale phenomena. Now let us look at the structure of pressure and temperature disturbances which are directly related to precipitation.

Meso-analyses by FUJITA (1955, 1959), OSAWA (1958, 1960), USHIJIMA (1959, 1960) MIYAZAWA (1961), SENSHU (1961) and others deal mostly with violent phenomena in which sudden changes of meteorological elements are observed. A sudden pressure rise and temperature drop are easily detected by self-recording observation, and the amount of change is almost comparable to that of synoptic deviations, so that it may be directly used for analysis without applying any operations to observed values.
In our examples, on the other hand, the temperature change is hardly noticeable and, moreover, the pressure change is less significant than the pressure gradient in a synoptic scale or the semi-diurnal variation of the pressure field. Therefore we have to introduce some operation on the original materials in order to pick up meso-scale disturbances. Actually we applied 5 terms running mean to the self-recorded materials and discussed the deviation fields.

Fig. 5. 2.5 hrs. running mean surface pressure at Aikawa is shown by thinner solid line (the scale is given on the right). Deviations from 2.5 hrs. running means are given by heavy full line (Aikawa) and broken line (Takada), the scale of which is given on the left.

In Fig. 5 are shown 2.5 hr running mean pressure at Aikawa (thin solid line), deviation pressures at Aikawa (heavy solid line) and at Takada (broken line) from their 2.5 hr running means. On the running mean curve, we can point out that the pressure started to rise at 1300 LST February 2 indicating the influence of a continental high and the predominance of semi-diurnal pressure variation. On the deviation curve, we can find a period of about 3 hrs. and a time lag of about 1 hr. between Aikawa and Takada (see Fig. 1).

The filtering effect of 5 terms running mean is shown on Fig. 6. It is seen from this figure that the amplitude of 3 hrs. period fluctuation (6 terms period) is reduced by 20% for 2.5 hr running mean. Since the amplitude appearing on the deviation curve in Fig. 5 is thus 80% of the original fluctuation, the amplitude of pressure change is estimated to be 0.7 mb. The phase velocity, on the other hand, is of the order of 100 km/hr because the distance between Aikawa and Takada is about 100 km.

Time sections of surface weather elements observed in the Hokuriku District are given in Fig. 7 and Fig. 8. The upper chart of Fig. 7 shows the hourly readings
Fig. 7. Upper chart: the time section of surface weather elements. Heavy solid lines show pressure, thinner solid lines precipitation in mm hr$^{-1}$ and berbs wind (a full berb is 2 m sec$^{-1}$). Lower chart: the time section of deviations from 5 hrs. running mean. Solid lines show pressure deviation in every 0.2 mb (negative deviations are stippled) and berbs wind deviation.

The 30 min precipitation is shown in this figure by broken lines. The characteristic feature of precipitation for both cases is that the principal precipitation took place in the transition between minimum pressure and final pressure rise.
Fig. 9. The divergence of surface wind and precipitation in the area (Aikawa-Niigata-Nagaoka-Takada) of $0.48 \times 10^4$ km$^2$. The ordinate for divergence is equal to the estimated precipitation. The divergence of the surface wind field is shown in Fig. 9. Aikawa, Niigata, Nagaoka and Takada were selected in the coastal area for divergence computation where it is considered to be free from orographic effect. The area is $0.48 \times 10^4$ km$^2$, which is one order smaller than that of the aerological network to be used for the discussion of synoptic scale phenomenon. The precipitation average over 4 stations is also entered in Fig. 9.

The reason why convergence was obtained throughout this period is probably because the north-westerly monsoon is weakened by the downstream mountain effect. A 3 hrs. period is superposed on this larger scale convergence field. The amplitude of divergence is found from Fig. 9 to be of the order of $10^{-4}$ sec$^{-1}$. It is seen that precipitation field changes correspondingly. The relation between precipitation and divergence field of the meso-scale phenomena is studied by many authors, e.g. SYONO et al. (1959), SENSHU (1961), MIYAZAWA (1962) and RIKITAKE (1964). It has been pointed
out that the divergence of the surface wind field is $2-3 \times 10^{-4} \text{ sec}^{-1}$ for the network smaller than $10^4 \text{ km}^2$ and has a good relationship with the amount of precipitation.

The estimation of the amount of precipitation by means of surface wind divergence is much easier in winter than in summer because the water vapor content in winter is small and the major part of it is concentrated in the lowermost part of the troposphere. As a first approximation, the moisture directly related to precipitation seems to be limited to the layer lower than 700 mb. Furthermore, the lower atmosphere over the Hokuriku District is almost always saturated in winter and thus the specific humidity $q$ does not change very much both in time and space. Under those circumstances, the amount of precipitation is given by $-q D \Delta p$, where $D$ is the divergence of surface wind and $\Delta p$ is the depth of layer, since the horizontal divergence of moisture flux $F \cdot v q$ is approximated by $q D$ and the vertical divergence of moisture flux $\partial \omega q / \partial p$ vanishes when integrated over $\Delta p$. Another method of estimating precipitation by using the so-called condensation function $F$ gives similar results. The ordinate of Fig. 9 is taken to indicate the estimated precipitation. It is seen that the estimated and observed amount of precipitation is about 3 mm/hr at most, which is one order less than that in summer.

Let us now consider the spatial distribution of disturbance having a period of 3 hrs. which is detected by taking deviation from running mean. Fig. 10 shows the deviation field at every 30 min. Displacements of the positive or negative pressure deviation region in one hour are given by double arrows in this figure. The moving velocity of the system is estimated to be 90 km/hr. The direction of displacement of a large negative deviation region coincides with the principal wind direction. At the same time, we can also find a small positive deviation region moving parallel to the coastal line. A similar situation is observed in the movement of echo cells which is shown by thinner arrows. The echo cell in a remarkable line echo (hatched in Fig. 10) moved with the movement of the air, but there existed other echo cells moving parallel to the coastal line.

The underlined numbers given to the stations in Fig. 10 show precipitation during the preceding 30 min. It is seen that the maximum precipitation is observed when the line echo indicated by hatches arrived at Takada. This line echo closely connected with precipitation is located in the intermediate region which follows the negative pressure deviation and precedes positive pressure deviation. SENSU (1961) and NINOMIYA (1965) also showed similar analyses. In FUTERA's case (1959), on the other hand, the precipitation is found within a high pressure area named “meso high” which is explained as resulting from cooling caused by evaporation from raindrops. There are many investigations dealing with the same kind of phenomena (e.g. OSAWA, 1958; USHIJIMA, 1959). Most of these cases were observed in summer or warm season. Our case, however, falls within the winter season. Because of low temperature and high relative humidity, the cooling effect by evaporation is not considered to be of primary importance. Therefore some other mechanism should be taken into account.

The authors (1965a) studied in detail the structure of the system of line echo and its relation to the aerophotographic observation of clouds and found that the cloud height was more than 5000 m for the higher clouds and about 3000-4000 m on the average over the area under consideration on February 2, 1964.
Fig. 10. A half-hourly distribution of pressure deviation from 2.5 hr running mean. Negative deviations are indicated by broken lines. Double arrows show the displacement of max. or min. deviation area within 1 hr. Thin arrows show the displacement of sample echo cells within 30 min. Shaded area indicates the principal line echo. The underlined numbers show precipitation in mm during the preceding 30 min.
4. Meso-scale disturbance in relation to the gravity wave

Summarizing the observational facts mentioned above and assuming harmonic disturbances of surface pressure $p$, divergence $D$ and the height of the stable layer in the mid-troposphere $H$ with their amplitudes $A_p$, $A_D$ and $A_H$ respectively, i.e.

$$p, D, H \propto A_p, A_D, A_H e^{i(\frac{2\pi x}{L} - \frac{2\pi}{T} t)}$$

where $L$ is the wave length and $T$ is the period,

we have

$$A_p = 0.7 \text{ mb},$$
$$A_D = 10^{-4} \text{ sec}^{-1},$$
$$C = 90 \text{ km/hr},$$

where $C = L/T$ is the phase velocity.

The rawin-sonde observation at Wajima, 1500 LST Februray 2, clearly shows the existence of a stable layer at about 500 mb level (Fig. 11), which is in good agreement with the height of cloud obtained by airplane observation. Therefore it would be reasonable to assume

$$H = 5000 \text{ m},$$
$$A_H = 1000 \text{ m},$$

approximately. The potential temperature difference between the air above the boundary layer and the air under it $\Delta T = T_1 - T$ causes the density difference $\rho = \rho - \rho_0$, which reduces the effect of gravitational force $g$ to

$$g_1 = \frac{\Delta \rho}{\rho} g = \frac{\Delta T}{T_1} g = 3 \frac{240}{g},$$

(see Fig. 12).

Fig. 11. Temperature distribution at 1500 LST Feb. 2, 1964 observed by sounding at Wajima (solid line). Broken lines are moist adiabats and dashed line indicates the temperature change by virtual displacement of stable layer.

The gravity wave originating on an intersurface is treated as follows. For the sake of simplicity let us consider it as a one-dimensional problem. Then the equation of motion and that of continuity are written as
where $u$ is the horizontal velocity and $v$ is the vertical velocity at the top of the lower layer. Eqs. (1) and (2) give the propagation velocity of the gravity wave

$$C-u = \pm \sqrt{g_1 H}$$

Eq. (1) is almost equivalent to the divergence equation which is to be used in case of a 2-dimensional problem. Now let us try to make some quantitative considerations by the aid of (1), (2) and (3). The hydrostatic relation which is introduced in the right hand side of Eq. (1) gives

$$\frac{1}{\rho} A_p = g_1 A_H$$

by which we obtain $A_H = 500$ m for the values given above. This shows a possibility of coordinating the aerophotographic knowledge with the physical interpretation of gravity waves. Comparing the above with Fujita's model (1959) and similar ones (Osawa, 1958; Ushijima, 1959), we find that the amplitude of pressure change for them is two or three times as large as ours. This must be due to a much larger $g_1$, caused by larger temperature difference and to a much larger $A_H$.

Next, let us consider the propagation velocity. Taking into account the fact that $D$ is given by $\partial u/\partial x$, Eq. (1) provides us with the amplitude relation between divergence field and pressure field,

$$\left( \frac{L}{T} - u \right) A_D = \frac{2\pi}{\rho} A_p$$

From Eq. (5) we obtain the wave length

$$L = \frac{T u}{2} + \sqrt{\left( \frac{T u}{2} \right)^2 + \frac{2\pi T A_p}{\rho A_D}}$$

Assuming that $u = 10$ m/sec., we then obtain

$L = 260$ km and, therefore, $C = 87$ km/hr.

We can also obtain the phase velocity directly from Eq. (3) as follows:

$$C-u = 2.5 \times 10^8 \text{C.G.S.} = 90 \text{ km/hr.}$$

Both of these estimated values are in good agreement with the observed phase velocity of the pressure system.

Tepper (1950) and Wagner (1962) introduced the concept of gravity wave to the propagation of meso-scale disturbance. As a matter of fact there are many analyses
listed in Table 1 showing the same order of propagation velocity as the gravity wave.

The velocity of the internal gravity wave to be expected in a continuous fluid without an intersurface is given by

\[ C-u = \pm \sqrt{\frac{\partial p}{\rho} \frac{\delta \theta}{\theta}} \]

in place of (3), where \( \partial p \) has the value of 800/\( \pi \) mb in case of the disturbance having the simplest node along the vertical and \( \delta \theta \) is the potential temperature difference within the layer of \( \partial p \). Applying the temperature distribution given in Fig. 11, we obtain

\[ C-u = 2.8 \times 10^3 \text{ C.G.S.} \]

which is again quite similar to the value given above.

Finally, the phase relationship will be touched upon briefly. Figs. 7, 8, 10 indicate that the precipitation is observed in the area between the maximum pressure and the preceding minimum pressure. Since the precipitation area should evidently coincide with the convergence area, Eq. (1) requires that there should be a phase difference of \( \pi/2 \) between pressure field and divergence (precipitation) field. Generally speaking, the phase velocity of gravity wave \( C \) is larger than the substantial velocity \( u(C>u) \), and, therefore, the convergent area is located ahead of the maximum pressure, which is again in agreement with the observation.

The characteristic features of various meso-scale phenomena which have since been reported will be listed in Table 1. In many cases, meso-scale disturbances have the character of a solitary wave rather than a periodic wave; namely some of them are defined as “pressure jump” or “pressure surge”. On such occasions, the amount of pressure rise and the maximum convergence are to be equivalent to twice of \( A_p \) and \( A_D \) respectively.

<table>
<thead>
<tr>
<th>Author</th>
<th>Date of Occur.</th>
<th>Pressure Rise</th>
<th>Max. Conv. (area)</th>
<th>Precip.</th>
<th>Scale</th>
<th>Phase Vel.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tepper (1950)</td>
<td>May</td>
<td>2.3</td>
<td>( \times 10^{-4} )</td>
<td>1.2</td>
<td>400</td>
<td>75</td>
</tr>
<tr>
<td>Fujita (1955)</td>
<td>June</td>
<td>5.0</td>
<td>( \times 10^4 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Osawa (1958)</td>
<td>Jan.</td>
<td>3.0</td>
<td>km²</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fujita (1959)</td>
<td>June</td>
<td>4.0</td>
<td>( \times 10^{-4} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Syono et al. (1959)</td>
<td>July</td>
<td>2 (0.57)</td>
<td></td>
<td>40</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>Ushijima (1959)</td>
<td>June, July</td>
<td>5.0</td>
<td>&gt;10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ushijima (1960)</td>
<td>April</td>
<td>2.0</td>
<td>0.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Osawa (1960)</td>
<td>July</td>
<td>180</td>
<td>20</td>
<td>40-100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shinshu (1961)</td>
<td>Jan.</td>
<td>2.0</td>
<td>60</td>
<td>100</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>Miyazawa (1961)</td>
<td>Jan.</td>
<td>3</td>
<td>(0.04)</td>
<td>3</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>Miyazawa (1962)</td>
<td>Dec.</td>
<td>4 (0.97)</td>
<td>150</td>
<td>5-10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wagner (1962)</td>
<td>April</td>
<td>4.0</td>
<td></td>
<td></td>
<td>100-150</td>
<td></td>
</tr>
</tbody>
</table>
5. Remarks

The analyses reported in this paper are based on the materials from the Heavy Snow Storm Project observation in 1964. Although the winter of 1964 was unusually calm one with a relatively small amount of snowfall, the cases studied here seem to contain sufficient information relating to heavy snowfall. Synoptic studies and aerophotographic studies on the same occasion will be reported elsewhere.

As conclusion to this meso-scale analysis, the physical characters as a gravity wave have been elucidated. However, gravity waves are originally stable phenomena, and the mechanism of initiation and development should be studied in future.

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寒冷渦中心部に観測されたメソスケール擾乱
—重力波的考察—

松本誠一・二宮 浚三

昭和39年1月20日および2月20日には上層トラフが通過し、これに伴って北陸地方にかなりの降雪がみられた。主要な降雪は寒冷渦の中心部にみられる中規模擾乱と関係し、顕著な線状エコーと共に移動した。

気圧の半日週期、低気圧周辺部の大きな気圧傾度などを消去して振幅の小さい中規模擾乱をとり出してその性質を調べるために、2.5時間移動平均よりの偏差値につき解析した。

気圧偏差値は3時間週期をもち、時速90kmで移動した。その振幅、発散場などの関係は、コールドドームを境とする界面に発生する重力波としてよく説明がつくようである。

本報告は北陸豪雪研究の一端をなすものである。