Studies of the Distribution of the Ultraviolet Sky Radiation (II)*

—on the Role of the Secondary Scattering—

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Abstract

Regarding the calculation of the secondary scattering of the solar radiation from the clear sky, a modification of the theoretical formula of A. Hammad and S. Chapman (Phil. Mag. 1939) was done considering the effect of polarization caused by primary scattering. A quantitative treatment became possible by the new formula in the vicinity of the zenith, that is, in the range from 0° to 45° of the zenith angle.

1. Introduction

Regarding the distribution measurement of the intensity of scattered sunlight from the clear sky, a few properties, for example, the position of the minimum point, can be explained by the primary scattering. But, in order to make a more quantitative treatment of the distribution, it is necessary to estimate the contribution of the secondary scattering. As to this problem, there are papers written by L. V. King,[1] H. P. Berlage,[2] A. Hammad & S. Chapman[3] and T. Sato.[4] But they are not sufficient, because in the first two papers the secondary scattering is not treated rigorously, and in the third paper, although the treatment is rigorous, the polarization caused by primary scattering is neglected, and in the last paper the problem is solved only by numerical integration and no analytical formula is given.

In this paper the author intends to give a modification of the formula of A. Hammad and S. Chapman by considering the effect of polarization caused by the primary scattering to attain a reasonable result.

2. The theory of A. Hammad and S. Chapman will be briefly explained below

The optical position is defined by the azimuthal angle φ measured from the direction of the sun in clockwise, the zenith distance θ , and the optical level m or the depth $\bar{m}$ that is the fractional height or depth and is defined by $m = \int_0^h \rho dh / M$ or $\bar{m} = \int_h^\infty \rho dh / M$, where ρ is the atmospheric density per unit length, h is the height of a point in the atmosphere, and M is the total mass of the atmosphere perpendicular to the earth per unit area, that is, $M = \int_0^\infty \rho dh$. According to Hammad and

Chapman the quantity of the secondary emission at any optical level m , to any direction (θ, φ) is given as follows.

$$(1) \quad E_2(z, \theta, \varphi, m) = \frac{9\sigma s}{256\pi^2} I_\infty \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^1 e^{-\bar{c}m \sec z - c(m \wedge m') \sec \theta'} \times (1 + \cos^2 \psi') (1 + \cos^2 \chi) \sec \theta' dm' d\theta' d\varphi'$$

where,

z : the sun's zenith distance

σ : the scattering coefficient per unit length of the air at normal temperature and pressure

s : the total scattering coefficient ($=\sigma M$)

c : the coefficient of extinction ($=s+a$)

a : the total absorption coefficient In our case the assumption, $a=0$ is adopted.

ψ' : the angle between the direction of incidence and that of emission by primary scattering

χ : the angle between the direction of incidence and that of emission by secondary scattering

I_∞ : the intensity of the solar radiation outside the earth's atmosphere In this paper in which we are interested only in the relative value, we take this as a unit strength.

θ', φ', m' are relating to primary scattering.

θ, φ, m are relating to secondary scattering. And next relations hold,

$$\cos \psi' = \cos z \cos \theta' + \sin z \sin \theta' \cos \varphi'$$

$$\cos \chi = \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\varphi - \varphi')$$

The physical meaning of the above formula is as follows (Ref. Fig. 1). We suppose that the direct sunlight comes down from the direction $(z, 0)$ and receives

Fig. 1 Schematic diagram of the scattering of the sunlight.

primary scattering at M_1 (level m') and the primary scattered light goes along the direction (θ', φ') with the angular deviation ψ' , then at M_2 (level m) this light beam receives secondary scattering, the scattered light going along the direction (θ, φ) with the angular deviation χ which we observe at O on the earth's surface. At this situation we seek first of all the quantity of secondary emission at M_2 (m) along the direction (θ, φ) , which originates from the primary scattering at all points of the space. First the direct sunlight receives extinction until it arrives at M_1 by the amount $e^{-\bar{c}m' \sec z}$, next it is scattered to the direction (θ', φ') and the intensity becomes $\frac{3\sigma}{16\pi} I_\infty e^{-\bar{c}m' \sec z} (1 + \cos^2 \psi')$ according to the Rayleigh scattering and farther the scattered light receives extinction by the amount $e^{-c(m \wedge m') \sec z}$, then again being scattered at M_2 (m), the intensity becoming

$$\frac{9\sigma^2}{256\pi^2} I_{\infty} e^{-c\bar{m}' \sec z - c(m-m') \sec \theta'} (1 + \cos^2 \psi') (1 + \cos^2 \chi).$$

Now considering that the unit volume of space is $M \sec \theta' \sin \theta' dm' d\theta' d\varphi'$ with regard to primary scattering, we integrate the intensity about M_1 over the whole space, thus getting at the formula (1).

A. Hammad and S. Chapman carried out the integral calculation rigorously. Noting that φ' is contained only in $1 + \cos^2 \psi'$ and $1 + \cos^2 \chi$, the integration of those terms with respect to φ' is shown as follows.

$$(2) \left\{ \begin{array}{l} \frac{1}{\pi} \int_0^{2\pi} (1 + \cos^2 \psi') (1 + \cos^2 \chi) d\varphi' \\ \quad = A_0(z, \theta, \varphi) + A_1(z, \theta, \varphi) \cos^2 \theta' + A_2(z, \theta, \varphi) \cos^4 \theta' \\ \text{where} \\ A_s(z, \theta, \varphi) = \sum_{r=0,1,2} A_{sr}(z, \theta) \cos r\varphi \\ A_{sr}: \end{array} \right.$$

$\begin{array}{c} r' \\ s \end{array}$	0	1	2
0	$2\left(1 + \frac{1}{2} \sin^2 z\right)\left(1 + \frac{1}{2} \sin^2 \theta\right)$	0	$\frac{1}{4} \sin^2 z \sin^2 \theta$
1	$4 - 2 \sin^2 z - 2 \sin^2 \theta - 3 \sin^2 z \sin^2 \theta$	$\sin 2z \sin 2\theta$	$-\frac{1}{2} \sin^2 z \sin^2 \theta$
2	$2\left(1 - \frac{3}{2} \sin^2 z\right)\left(1 - \frac{3}{2} \sin^2 \theta\right)$	$-\sin 2z \sin 2\theta$	$\frac{1}{4} \sin^2 z \sin^2 \theta$

Then using the above results the integrated formula of E_2 with respect to θ' , m' becomes as follows:

$$(3a) \quad E_2(c, z, m) = \frac{9\sigma s}{256\pi} I_{\infty} \{A_0(z, \theta, \varphi) L_1(c, z, m) + A_1(z, \theta, \varphi) L_3(c, z, m) + A_2(z, \theta, \varphi) L_5(c, z, m)\}$$

$$(3b) \quad \text{where } L_n(c, z, m) = \frac{1}{c} e^{-c\bar{m} \sec z} \left(\int_0^{cm} e^{-t \sec z} E j_n(t) dt + \int_0^{\bar{cm}} e^{t \sec z} E j_n(t) dt \right)$$

$$(3c) \quad E j_n(t) = \int_1^{\infty} e^{-tx} x^{-n} dx \quad (n=1, 3, 5)$$

In the above calculation the transformation $x = \sec z$ and $t = c(m - m')$ or $t = c(\bar{m} - \bar{m}')$ is done.

Finally integrating function E_2 with respect to m from 0 to 1 we get at the last formula giving the intensity of the secondary scattered light along the direction (θ, φ) which we observe on the earth, giving the following result.

$$(4a) \quad Rd_2 = \frac{9\sigma^2}{256\pi} I_{\infty} \sec \theta \{A_0(z, \theta, \varphi) Ld_1(c, z, \theta) + A_1(z, \theta, \varphi) Ld_3(c, z, \theta) + A_2(z, \theta, \varphi) Ld_5(c, z, \theta)\}$$

$$(4b) \quad \text{where } Ld_n(c, z, \theta) = e^{-c \sec \theta} \int_0^1 e^{c\bar{m} \sec \theta} L_n(c, z, m) d\bar{m}$$

This is the theory first given by A. Hammad and S. Chapman. Their scheme was

to simplify the above formula mathematically and go farther to the calculation of the higher order. But as Dr. Sato pointed out, in the case of sky radiation the polarizing effect of primary scattering is essential and therefore we cannot repeat the scattering law of unpolarized light twice. Now, the term $(1 + \cos^2 \psi')(1 + \cos^2 \chi)$ must be corrected.

3. Correction of the theory of A. Hammad and S. Chapman

We start upon more fundamental standpoint of the law of Rayleigh scattering considering the two linearly polarized light individually.

Fig. 2 Diagram to illustrate the correct Rayleigh scattering of the second order.

In the rectangular coordinates (X, Y, Z) (Ref. Fig. 2), suppose the two kinds of linearly polarized light beam advance along the X axis towards the origin. The planes of polarization are perpendicular to each other, one being the (X, Z) plane and the other the (X, Y) plane. The intensity is $1/2$ each, the sum being 1. We suppose that these light beams receive scattering at the origin O and the scattered lights advance toward the direction (α, β, r) , then again receiving the secondary scattering at O' , and are emitted to the direction (l, m, n) , which is the direction observed. Here by the notations $(\alpha, \beta, r, l, m, n)$ we mean direction cosines. Taking r as the traveling distance of the primary scattered light and R as that of the secondary one, and applying the law of Rayleigh scattering to each polarized light, we get the following formula representing the final light intensity observed.

$$(5) \quad \frac{1}{2} \left(\frac{3\sigma}{8\pi} \right)^2 \frac{1}{R^2 r^2} (\sin^2 \omega \sin^2 \Omega + \sin^2 \omega' \sin^2 \Omega'),$$

where, ω, ω' denote the the angles between the direction of oscillation of the first light beam and that of propagation of the primary scattered light, for each of the polarized lights, and Ω, Ω' denote the angles between the direction of oscillation of the primary scattered light and that of propagation of the secondary one, again for each of the polarized lights. Then, the following relations hold, obeying the law of physical optics,

$$(6) \quad \begin{cases} \cos \omega = r, & \cos \omega' = \beta \\ \cos \Omega = \frac{1}{\sqrt{1-r^2}} \{l\alpha r + m\beta r - n(\alpha^2 + \beta^2)\}, \\ \cos \Omega' = \frac{1}{\sqrt{1-\beta^2}} \{l\alpha\beta - m(\alpha^2 + r^2) + n\beta r\}. \end{cases}$$

Now our problem is to introduce the formula (5) in our integral calculation in place of $\left(\frac{3\sigma}{16\pi} \right)^2 \frac{1}{R^2 r^2} (1 + \cos^2 \psi')(1 + \cos^2 \chi)$, which was the fundamental of H.C. formula [H.C. is the abbreviated notation of Hammad and Chapman]. In our problem

of sky light, the direction X is that of the sun, that is, $(z, 0)$ direction in our (θ, φ) system. Taking this into consideration, we get the following equation of transformation with respect to $\alpha, \beta, \gamma, l, m, n$.

$$(7) \quad \left\{ \begin{array}{l} \alpha = \sin \theta' \cos \varphi' \sin z + \cos \theta' \cos z \\ \beta = \sin \theta' \sin \varphi' \\ \gamma = \cos \theta' \sin z - \sin \theta' \cos \varphi' \cos z \\ l = \sin \theta \cos \varphi \sin z + \cos \theta \cos z \\ m = \sin \theta \sin \varphi \\ n = \cos \theta \sin z - \sin \theta \cos \varphi \cos z \end{array} \right.$$

Using (6) and (7), we can represent the terms of (5) in the following way.

$$\begin{aligned} & \sin^2 \omega \sin^2 \Omega + \sin^2 \omega' \sin^2 \Omega' \\ &= (1-r^2) \left[1 - \left\{ \frac{l\alpha\gamma + m\beta\gamma - n(\alpha^2 + \beta^2)}{\sqrt{1-r^2}} \right\}^2 \right] \\ & \quad + (1-\beta^2) \left[1 - \left\{ \frac{l\alpha\beta - m(\alpha^2 + \gamma^2) + n\beta\gamma}{\sqrt{1-\beta^2}} \right\}^2 \right] \\ &= \{(1-r^2) + (1-\beta^2)\} - \{l\alpha\gamma + m\beta\gamma - n(1-r^2)\}^2 - \{l\alpha\beta - m(1-\beta^2) + n\beta\gamma\}^2 \\ &= 1 + \alpha^2 - (r\gamma - n)^2 - (\beta\gamma - m)^2 \quad [\text{where } q = l\alpha + m\beta + n\gamma] \end{aligned}$$

Before integrating, we proceed further, showing the difference between the H.C. formula and ours.

$$\begin{aligned} &= 1 + \alpha^2 - q^2(r^2 + \beta^2) + 2q(n\gamma + m\beta) - (n^2 + m^2) \\ &= 1 + \alpha^2 - q^2(1 + \alpha^2) - 2q\alpha l - (1 - l^2) \\ &= (1 + \alpha^2)(1 + q^2) - (1 - l^2 + 2q\alpha l) \end{aligned}$$

Now considering that $\alpha = \cos \psi'$, $q = \cos X$ and our modification is a change from $(1 + \cos^2 \psi')(1 + \cos^2 X)$ to $2(\sin^2 \omega \sin^2 \Omega + \sin^2 \omega' \sin^2 \Omega')$, we can see that by the following integration we have only to add the correction term $-(1 - l^2 + 2q\alpha l)$ and multiply the result by 2. After carrying out the transformation from l, α, q to θ', φ' and integrating with respect to φ' , the result of the integration of the correction term becomes as follows:

$$(8) \quad \left\{ \begin{array}{l} \frac{1}{\pi} \int_0^{2\pi} (1 - l^2 + 2q l \alpha) d\varphi' = A_0 + A_1 \cos^2 \theta' \\ \text{where} \\ D_s = \sum_{r=0,1,2} D_{sr} \cos r\varphi \\ D_{sr}: \end{array} \right.$$

$\begin{array}{c c} r & s \end{array}$	0	1	2
0	$2(\sin^2 \theta + \sin^2 z - \sin^2 \theta \sin^2 z)$	$-\frac{1}{2} \sin 2\theta \sin 2z$	0
1	$4 - 4 \sin^2 z - 4 \sin^2 \theta + 3 \sin^2 \theta \sin^2 z$	$\frac{1}{2} \sin 2\theta \sin 2z$	$-\sin^2 \theta \sin^2 z$

Our final result is to replace A_0, A_1 by $A_0 - A_0, A_1 - A_1$. After doing this, following formula is obtained

$$\left\{ \begin{array}{l} \frac{1}{\pi} \int_0^{2\pi} (\sin^2 \omega \sin^2 \Omega + \sin^2 \omega' \sin^2 \Omega') d\varphi' = A_0^*(z, \theta, \varphi) + A_1^*(z, \theta, \varphi) \cos^2 \theta' \\ \quad + A_2^*(z, \theta, \varphi) \cos^4 \theta' \end{array} \right.$$

where

$$A_s^*(z, \theta, \varphi) = \sum_{r=0,1,2} A_{sr}^*(z, \theta) \cos r\varphi$$

(9) A_{sr}^* :

$s \backslash r$	0	1	2
0	$2 - \sin^2\theta - \sin^2z + \frac{5}{2} \sin^2\theta \sin^2z$	$\frac{1}{2} \sin 2\theta \sin 2z$	$\frac{1}{4} \sin^2\theta \sin^2z$
1	$2 \sin^2\theta + 2 \sin^2z - 6 \sin^2\theta \sin^2z$	$\frac{1}{2} \sin 2\theta \sin 2z$	$\frac{1}{2} \sin^2\theta \sin^2z$
2	$2 - 3 \sin^2\theta - 3 \sin^2z + \frac{9}{2} \sin^2\theta \sin^2z$	$-\sin 2\theta \sin 2z$	$\frac{1}{4} \sin^2\theta \sin^2z$

There is no change in other terms of the formula of the secondary scattered light, which is written in the following way,

$$(10a) \quad Rd_s^* = -\frac{9s^2}{128\pi} I_\infty \sec\theta \{A_0^*(z, \theta, \varphi) Ld_1(c, z, \theta) + A_1^*(z, \theta, \varphi) Ld_1(c, z, \theta) + A_2^*(z, \theta, \varphi) Ld_5(c, z, \theta)\},$$

$$(10b) \quad \text{where} \quad Ld_n(c, z, \theta) = e^{-c \sec\theta} \int_0^1 e^{cm \sec\theta} I_n(c, z, \theta) d\bar{m}.$$

4. Comparison of the theory with observations

The next task is the evaluation of the above formula and the comparison with observations.

First, A^* 's are trigonometric functions of z , θ , φ and are easily computed numerically. Ld_n contains the integration with respect to θ' , m' and m . As to $Ej_1(t)$, the first of the terms including the integration with respect to θ' , it reduces to the wellknown function $E_i(-t)$ by the following relation.

$$Ej_1(t) = \int_1^\infty e^{-tx} x^{-1} dx = \int_t^\infty e^{-x} x^{-1} dx = -E_i(-t).$$

We can find its numerical values in any mathematical table. [5] The function Ej_3 and Ej_5 can be deduced from Ej_1 by the following recurrence formula.

$$(n-1)Ej_n(t) = e^{-t} - tEj_{n-1}(t).$$

The curves of the three functions are shown in Fig. 3.

As to the integration with respect to m' , the evaluation can be done by replacing the integration of continuous function with the summation of the discontinuous step functions where we divided the space into ten equal parts with respect to m' . That is, in place of the equation

Fig. 3 Curves of $Ej_n(t)$.

(3 b), we use the following equation.

$$L_n(c, z, m_i) = \frac{1}{c} e^{-\bar{m}_i \sec z} \left(\sum_{r=1}^i e^{-t_r \sec z} E j_n(t_r) \Delta t + \sum_{r=1}^{10-i} e^{t_r \sec z} E j_n(t_r) \Delta t \right),$$

where, $i=0, 1, 2, \dots, 10, \Delta t=0.1c$.

We take the mean value of t in each division as the value of t_r . The last integration with respect to m can be evaluated by just the same method, giving the following equation in place of (4 b) or (10 b).

$$Ld_n(c, z, \theta) = e^{-c \sec \theta} \sum_{i=1}^{10} e^{\bar{m}_i \sec \theta} L_n(c, z, m_i) \Delta m,$$

where, $\Delta m=0.1$.

After these calculations we can evaluate the formula (10 a) and know the quantity of the sky radiation due to secondary scattering along any direction, on any extinction coefficient and at any altitude of the sun.

Numerical calculation was done by the extinction coefficient $c=0.357$ and 0.693 or the transmission coefficient $p=0.7$ and 0.5 , at $z=30^\circ$, respectively. As to the direction the values $\theta=0^\circ, 15^\circ, 30^\circ, 45^\circ$ and 60° ; $\varphi=\pi$ were adopted, that is, the quantity in the vertical plane through the sun. The curves of L_n are shown in Fig. 4. The numerical values are shown in Tab. I. After adding the quantity due to primary scattering, we get the total intensity, each from the directions which are to be compared with observed data. In Tab. II are shown the quantity due to primary scattering and the sum of both the primary and the

Fig. 4 Curves of $L_n(c, z, m)$ by $z=30^\circ$.

Table I Numerical calculation of the scattered sunlight of the second order. (by the theory of this author)

θ	0°	15°	30°	45°	60°	
$2A_0^*$	3.500	3.025	2.588	2.321	2.246	
$2A_1^*$	1.000	0.651	0.527	0.759	1.152	
$2A_2^*$	2.500	3.120	3.088	2.420	1.272	
$p=0.7$	Ld_1	1.413	1.405	1.373	1.320	1.195
	Ld_3	0.278	0.276	0.270	0.259	0.236
	Ld_5	0.143	0.142	0.139	0.134	0.123
	Rd_2^*	7.95×10^{-3}	7.19×10^{-3}	6.72×10^{-3}	7.22×10^{-3}	8.87×10^{-3}
$p=0.5$	Ld_1	0.693	0.685	0.659	0.607	0.515
	Ld_3	0.169	0.168	0.162	0.150	0.127
	Ld_5	0.0902	0.0896	0.0862	0.0800	0.0680
	Rd_2^*	1.51×10^{-2}	1.37×10^{-2}	1.27×10^{-2}	1.30×10^{-2}	1.49×10^{-2}

Table II Numerical calculation of the scattered sunlight of the first order. (by $z=30^\circ$, $\varphi=\pi$)

θ	0°	15°	30°	45°	60°
$p=0.7 \begin{cases} Rd_1 \\ Rd_{1+2}^* \end{cases}$	2.58×10^{-2} 3.38×10^{-2}	2.28×10^{-2} 3.00×10^{-2}	2.04×10^{-2} 2.71×10^{-2}	2.00×10^{-2} 2.72×10^{-2}	2.43×10^{-2} 3.32×10^{-2}
$p=0.5 \begin{cases} Rd_1 \\ Rd_{1+2}^* \end{cases}$	3.44×10^{-2} 4.96×10^{-2}	2.98×10^{-2} 4.35×10^{-2}	2.68×10^{-2} 3.95×10^{-2}	2.58×10^{-2} 3.89×10^{-2}	2.81×10^{-2} 4.30×10^{-2}

$$\text{where } Rd_1 = \frac{3s}{16\pi} \sec \theta (1 + \cos^2 \psi) e^{-c \sec \theta} \frac{e^{c(\sec \theta - \sec z)} - 1}{c(\sec \theta - \sec z)}$$

Table III Relative intensities of the scattered sunlight by $z=30^\circ$ (Theoretical and Observed)

θ	0°	15°	30°	45°	60°
$p=0.7$	1.24	1.10	1.00	1.00	1.22
$p=0.5$	1.27	1.12	1.02	1.00	1.11
Observed	1.26	—	1.01	1.00	1.03

secondary one. In Tab. III are shown the relative intensities of both the theoretical and the observed, taking the values of $\theta=45^\circ$ as a unit in each case. The observed values are those which were measured by the Sb-Cs photoelectric tube and its amplifier set with a glass filter for ultraviolet, the spectral sensitivity lying in the range of 3600 Å or so. Measurement [6] was done by the author and his collaborator at Rebus Island on May 12, 1948. The values computed by the formula (4a), are shown in Tab. IV and Tab. V for comparison. A good agreement between theory and observation can be obtained in our corrected theory when we take $p=0.6$ or so. The deviation at $\theta=60^\circ$ shows the limit of the theory which is on the assumption of plane stratified atmosphere, neglecting the spherical shape of the earth. At any rate satisfactory explanation can be obtained by this theory. On the contrary, as to the values calculated by the formula of A. Hammad and S. Chapman there is no hope of agreement with the observation. It is due to the fact

Table III Numerical calculation of the scattered sunlight of the second order. (by the theory of A. Hammad & S. Chapman)

θ	0°	15°	30°	45°	60°
A_0	2.250	2.329	2.549	2.841	3.142
A_1	3.500	2.875	2.235	1.196	0.600
A_2	1.250	1.560	1.544	1.210	0.636
$Rd_2 \begin{cases} p=0.7 \\ p=0.5 \end{cases}$	5.65×10^{-3} 1.22×10^{-2}	6.34×10^{-3} 1.23×10^{-2}	7.35×10^{-3} 1.35×10^{-2}	8.54×10^{-3} 1.52×10^{-2}	1.13×10^{-2} 1.86×10^{-2}

Table V Relative intensities of the scattered sunlight. (Calculated by the theory of A. Hammad & S. Chapman)

θ	0°	15°	30°	45°	60°
p					
0.7	1.11	1.02	0.97	1.00	1.26
0.5	1.13	1.03	0.98	1.00	1.14

that in their theory the secondary scattering increases with θ while in our theory it decreases.

5. Discussion

Let us make some considerations on the meaning of our calculation. We are now chiefly interested in the variation with θ . And so let us examine the variation of Rd_2^* of (10a) or Rd_2 of (4a) with θ . Among the terms containing θ , the first are A 's which were introduced by integrating the terms of scattering law with respect to φ' from 0 to 2π , the second is $\sec \theta$ which shows the variation of the length of the path with θ and the third is $e^{-\sec \theta}$ which shows the variation of the quantity of absorption with θ . The second and the third terms can be seen in the formula of primary scattering too, and only the first terms are characteristic to secondary scattering. Further the second and the third terms have maximum or minimum values at the zenith and their variation is small compared with that of A^* 's or A 's in the vicinity of the zenith. Besides, among A^* 's or A 's, it is remarkable that the A_1^* or A_1 plays the most important role of all other A 's. It is because E_{j_1} is larger than E_{j_3} and E_{j_5} , and consequently Ld_1 becomes the largest of the three in the absolute value. This fact is reasonable from the standpoint of the stratified structure of the atmosphere, for E_{j_1} is the horizontal component of the integrated values. We can see why our modification gave so much change in the result in spite of our comparatively small formal change which is limited in the value of A_0 and A_1 . A_{00} of A_0 in H.C. formula increases with θ whereas A_{00}^* of A_0^* in the corrected theory decreases. A_{01} of A_1 , the coefficient of $\cos \varphi$ in H.C. formula vanishes, whereas in our formula this does not vanish and gives asymmetrical property with respect to the zenith in the vertical plane through the sun. And all of these play the essential role in order to get sufficient explanation of the experiment. In conclusion we can see the directional characteristics of secondary scattering in the horizontal component of the integrated functions of the scattering laws with respect to the azimuth from 0 to 2π , and sufficient agreement between the theory and observation of the distribution of the near ultraviolet sky radiation can be obtained only by considering the effect of polarization due to primary scattering.

The author has the hope of studying the atmospheric structure by the measurement of the distribution, to which these studies will be of use. The eclipse effect and the effect of the absorption layer are now being studied.

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* This paper is the continuation of the report of the same title of Ref. [6] which was written by the author and his collaborator and will be mentioned as the paper I.