Some Considerations on the Atmospheric Internal Boundary Layer over the Ground Surface

by

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Abstract

According to BRADLEY, velocity changes do not agree fairly well with the PANOFSKY and TOWNSEND theory in the rough-smooth direction. This problem is reexamined by the use of BRADLEY's data. It is concluded that the velocity profiles observed agree fairly well with those predicted by the PANOFSKY and TOWNSEND theory also in this case by a reasonable choice of roughness parameters.

1. Introduction

Experimental studies of the roughness transition in the atmosphere have been performed by KUTZBACH (1961), STEARNS (1964), STEARNS and LETTAU (1963) and BRADLEY (1968). On the other hand, three theoretical approaches to the problem have come from ELLIOTT (1958), TAYLOR (1962) and PANOFSKY and TOWNSEND (1963).

According to BRADLEY, the theoretically predicted velocity profiles downwind of rough-smooth transition were found to have smaller values than those actually observed, as the fetch over the smooth surface increased. That is, the flow got acceleration over the predicted values. BRADLEY has not discussed the mechanism of the acceleration. Is acceleration in this sense really present? We examined this problem by the use of BRADLEY's data, which were obtained under neutral or near neutral conditions.

2. Models of the atmospheric internal boundary layer

A schematic diagram of the internal boundary layer is shown in Fig. 1. The transition (from rough to smooth in this case) takes place at A. The internal boundary layer (region 2) develops downwind of A. The familiar logarithmic velocity profile given by

$$\frac{U}{v_{*1}} = \frac{1}{\kappa} \ln \frac{z}{z_{01}} \quad (2.1)$$

Fig. 1. Schematic representation of the transition region.

is supposed to hold upwind of A, and outside the boundary layer interface AB (ELLIOTT, PANOFKY and TOWNSEND, TAYLOR), and this unmodified part of the surface boundary layer is called region 1 in this paper, where

- u : mean horizontal velocity at a height of z ,
- v_{*1} : friction velocity,
- z_{01} : roughness parameter,
- κ : von Karman's constant.

The following two velocity profiles have been proposed in region 2.

- 1) The velocity profile

$$\frac{u}{v_{*2}} = \frac{1}{\kappa} \ln \frac{z}{z_{02}} \quad (2.2)$$

is proposed by ELLIOTT, where

- u : mean horizontal velocity at a height of z ,
- v_{*2} : friction velocity,
- z_{02} : roughness parameter.

In this model, velocity is continuous but stress is not at the interface AB.

- 2) The velocity profile

$$\frac{u}{v_{*1}} = \frac{1}{\kappa} \left[(1-S) \ln \frac{z}{z_{02}} + S \frac{z}{\delta'} \right] \quad (2.3)$$

is proposed by PANOFKY and TOWNSEND and this is derived from the relation

$$\frac{\partial u}{\partial z} = \frac{v_*}{\kappa z}$$

which holds for the equilibrium flow, on the assumption that

$$v_* = v_{*1} \left[(1-S) + S \frac{z}{\delta'} \right]$$

where

$$S = \frac{v_{*1} - v_{*2}}{v_{*1}}$$

δ' : height of the modified region defined by the PANOFKY and TOWNSEND theory (PANOFKY-TOWNSEND interface height).

In this case, friction velocity varies linearly with height and is continuous at the

interface AB.

TAYLOR has proposed the velocity profile

$$\frac{u}{v_{*2}} = \frac{1}{\kappa} \ln \frac{z}{z_{02}} \quad (2.4)$$

for the region far leeward of the transition, but this case will not be treated here.

3. Height of the modified region

The height of the modified region is a function of fetch over the smooth surface, x . The interface height defined by the ELLIOTT theory δ and that defined by the PANOFSKY and TOWNSEND theory δ' will be estimated as follows:

1) ELLIOTT interface height δ

ELLIOTT supposed the velocity profile (2.1) for region 1 and (2.2) for region 2. Therefore, if $U = u = u_\delta$ at $z = \delta$, at a certain fetch, and u_δ is eliminated by using equations (2.1) and (2.2), the relation

$$\frac{v_{*2}}{v_{*1}} = \frac{\ln \delta - \ln z_{01}}{\ln \delta - \ln z_{02}}$$

may be obtained. Then, the relation

$$\log \delta = \frac{\log z_{01} - R \log z_{02}}{1 - R} \quad (3.1)$$

can be derived by the substitution of R for v_{*2}/v_{*1} . Therefore, δ is finally given as

$$\delta = \left(\frac{z_{01}}{z_{02}^R} \right)^{\frac{1}{1-R}} \quad (3.2)$$

It will be seen from the above relation that δ is the function of z_{01} , z_{02} , v_{*1} and v_{*2} .

Fig. 2. Height of the modified region downwind of rough-smooth (spikes-tarmac) transition.

The lower curve in Fig. 2 shows the relation between x and δ calculated by equation (3.1) using z_{01} , z_{02} , v_{*1} and v_{*2} at each fetch, which are obtained from the set of velocity profiles observed by BRADLEY. It will be seen from the figure that the relation

$$\delta = ax^{\frac{4}{5}} \quad (3.3)$$

holds for x more than 4.0 m. The experimental constant a may be determined by the figure as

$$a = 0.264$$

Therefore, the empirical formula (3.3) can be written as

$$\delta = 0.264 x^{\frac{4}{5}} \quad (\text{cm}) \quad (3.4)$$

The experimental constant a is not a universal one and depends on roughness. The set of observations in the case of the spikes-tarmac transition were used here.

2) PANOFSKY-TOWNSEND interface height δ' .

PANOFSKY and TOWNSEND supposed the velocity profile (2.1) for region 1 and (2.3) for region 2. Then, if $U = u = u_{\delta'}$ at $z = \delta'$, at a certain fetch, the relation

$$\ln \delta' = \frac{\ln z_{01} - R \ln z_{02} + (1-R)}{1-R} \quad (3.5)$$

may be obtained from equations (2.1) and (2.3) in the same way as mentioned above. The relation (3.5) can also be written as

$$\delta' = e \left(\frac{z_{01}}{z_{02}^R} \right)^{\frac{1}{1-R}} \quad (3.6)$$

Fig. 3. Height of the modified region downwind of rough-smooth (grass-tarmac) transition.

Therefore, it will be seen from a comparison of the above relation with (3.2) that the PANOFSKY-TOWNSEND interface height δ' is e ($=2.72$) times the ELLIOTT interface height δ . The upper curve in Fig. 2 shows the relation between δ' and x . The linear part of this curve may be represented by

$$\delta' = 0.718 x^{\frac{4}{5}} \quad (\text{cm}) \quad (3.7)$$

Another of BRADLEY's observations concerns the grass-tarmac transition. The relation between δ and x in this case is shown in Fig. 3. The linear part of the curve is represented by

$$\delta = 0.347 x^{\frac{4}{5}} \quad (\text{cm}) \quad (3.8)$$

The PANOFSKY-TOWNSEND interface height δ' is also e times the above-mentioned δ .

Since the velocity profile at each fetch downwind of the transition is not observed simultaneously, each velocity profile in region 1 corresponding to each observation run is usually different, so that z_{01} , z_{02} , v_{*1} and v_{*2} in each run are also different as shown in Table 1 to be given later. But it is interesting to note that there is, nevertheless, a definite relation between δ and x .

4. Variation of surface shearing stress downwind of rough-smooth transition

If the shearing stresses on the surfaces of region 1 and region 2 are represented by τ_{01} and τ_{02} , respectively, since

$$\tau_{01} = \rho v_{*1}^2, \quad \tau_{02} = \rho v_{*2}^2$$

Fig. 4. Variation of surface shearing stress downwind of rough-smooth (spikes-tarmac) transition.

then

$$\frac{\tau_{02}}{\tau_{01}} = \left(\frac{v_{*2}}{v_{*1}} \right)^2 = R^2 = \left(\frac{\log \delta - \log z_{01}}{\log \delta - \log z_{02}} \right)^2 \quad (4.1)$$

As seen from the above relation, τ_{02}/τ_{01} is the function of z_{01} , z_{02} and δ or x . When z_{01} , z_{02} and δ at each x are given τ_{02}/τ_{01} at each x may be estimated by the relation (4.1).

BRADLEY gave $z_{01}=0.25$ cm, $z_{02}=0.002$ cm for the spikes-tarmac transition. When we calculate τ_{02}/τ_{01} at each fetch x using these values of z_{01} and z_{02} and further δ obtained from the empirical formula (3.4), we obtain the curve shown by the solid line in Fig. 4. On the other hand, τ_{02}/τ_{01} at each fetch x estimated by using v_{*1} and v_{*2} obtained from the observed velocity profiles is shown by the broken line in the figure. There is considerable difference between the two curves. It seems that this is due to differences in z_{01} and z_{02} between the two cases. Values of z_{01} estimated by the observed velocity profiles range from 0.14 cm to 0.008 cm, and those of z_{02} range from 0.0015 cm to 0.00023 cm (Table 1, shown later). These values differ considerably from those chosen by BRADLEY, who estimates them by the extreme velocity profiles above both surfaces (rough and smooth), which are considered to be typical. The value of z_{02}/z_{01} determined by the observed velocity profiles is 0.029 on the average and that chosen by BRADLEY is 0.008.

Fig. 5. Variation of surface shearing stress downwind of rough-smooth (grass-tarmac) transition.

Another of BRADLEY's observations, which concerns the grass-tarmac transition, is shown in Fig. 5. In this case BRADLEY choses $z_{01}=0.5$ cm and $z_{02}=0.002$ cm, but on the other hand $z_{01}=0.5$ cm and $z_{02}=0.0015$ cm are estimated by the set of velocity profiles observed. Since the values of z_{01} and z_{02} of the two cases are not widely different, the two curves are comparatively close to each other. But eventually it will be found from these results that τ_{02}/τ_{01} is sensitive to the change of roughness parameters.

5. Comparison of the observed velocity profiles with those predicted by the PANOFSKY and TOWNSEND theory

According to BRADLEY, the velocity profiles predicted by the PANOFSKY and TOWNSEND theory coincide well with the observed ones for shorter fetches, but turn out to have smaller values than the observed ones for longer fetches, that is, the observed velocity profiles seem to be accelerated (cf. BRADLEY's paper, Figs. 9 and 10). He did not discuss the reason for this acceleration, whose mechanism seems difficult to explain.

According to the PANOFSKY and TOWNSEND theory, as described above, the velocity profile in region 2 may be represented by

$$\frac{u}{v_{*2}} = \frac{1}{\kappa} \left[(1-S) \ln \frac{z}{z_{02}} + S \frac{z}{\delta'} \right]$$

This equation is written as

$$\frac{u}{v_{*2}} = \frac{1}{\kappa} \ln \frac{z}{z_{02}} + \frac{1}{\kappa} \left(\frac{1}{R} - 1 \right) \frac{z}{\delta'} \quad (5.1)$$

by the substitution of R for v_{*2}/v_{*1} . In this equation, the first term of the right-hand side represents the velocity profile of the ELLIOTT theory.

Since BRADLEY has adopted the velocity $u_{112.5}$ at $z=112.5$ cm in region 1 as the reference velocity to represent the velocity profiles downwind of the spikes-tarmac transition, we also use the following equation to be derived from equations (2.1) and (5.1), to make comparison easy,

$$\frac{u}{u_{112.5}} = \frac{R \ln \frac{z}{z_{02}} + (1-R) \frac{z}{\delta'}}{\ln \frac{112.5}{z_{01}}} \quad (5.2)$$

First, we estimate z_{01} , v_{*1} , z_{02} , v_{*2} , z_{02}/z_{01} , τ_{02}/τ_{01} , δ and δ' at each x by the use of the set of velocity profiles observed by BRADLEY. The results are shown in Table 1. Since BRADLEY observed velocity profiles at $x=0.12, 2.1, 4.0, 6.1, 8.3, 10.2, 12.2$ m by moving the anemometer mast in turn, the velocity profiles at each fetch were generally different, because the natural wind cannot remain steady for long; therefore z_{01} , z_{02} , v_{*1} and v_{*2} are also different in each run. However, the values of z_{02}/z_{01} take an almost

constant value (0.0297 on the average) for fetches more than 4.1 m, except for shorter fetches.

Table 1. Roughness parameters, friction velocities, ratio of shearing stresses and interface heights estimated by the use of the set of velocity profiles observed by Bradley (spikes-tarmac transition).

x m	z_{01} cm	v_{*1} cm/s	z_{02} cm	v_{*2} cm/s	z_{02}/z_{01}	τ_{02}/τ_{01}	δ cm	δ' cm
0.12	0.18	42.0	0.047	31.6	0.261	0.565	10.9	29.6
2.10	0.14	38.2	0.0015	19.8	0.011	0.269	19.1	52.0
4.00	0.12	42.3	0.0035	26.0	0.0292	0.378	32.0	87.0
6.10	0.075	42.0	0.0024	27.2	0.0320	0.419	42.7	116.1
8.30	0.027	35.6	0.0008	24.3	0.0296	0.467	56.2	152.9
10.20	0.038	40.5	0.0011	27.6	0.0290	0.465	70.8	192.5
12.20	0.008	28.1	0.00023	20.3	0.0288	0.518	83.2	226.2

Although BRADLEY predicted the velocity profiles by using $z_{01}=0.25$ cm and $z_{02}=0.002$ cm, which were estimated by the extreme velocity profiles in each region (1 and 2), we considered that the wind with a logarithmic profile characterized by z_{01} and v_{*1} as shown in each run in Table 1 was modified by the spikes-tarmac transition. That is, we predicted the velocity profiles from equation (5.2) by using z_{02} estimated by multiplying z_{01} shown in the table by 0.0297 and δ' at each corresponding x . These results are shown by solid lines in Fig. 6–Fig. 10. Velocities observed at each height (BRADLEY) are also shown by $\bigcirc$ in the figures. On the other hand, the velocity profiles predicted from equation (5.2) by using z_{01} and z_{02} chosen by BRADLEY are shown by broken lines.

It will be seen from the above figures that there is no significant difference between the velocity profiles predicted by the author and those predicted by BRADLEY for shorter fetches, that is, for 4.0 and 6.1 m, and both velocity profiles also agree well with the observed ones, but the velocity profiles represented by the broken line indicate lower velocity for fetches more than 6.1 m, while the velocity profiles shown by the solid line fit the observed ones. That is, the observed velocity profile seems to show acceleration over the one which was predicted by BRADLEY by using the PANOFSKY-TOWNSEND theory. On the contrary, the velocity profiles predicted by the author well fit the observed ones at each fetch and acceleration can not be found. This is due to difference in the choice of z_{01} and z_{02} between the cases. BRADLEY has determined the values of z_{01} and z_{02} by the use of the extreme velocity profiles in each of regions 1 and 2, but it seems that these values do not always show the actual situation.

The case of grass-tarmac transition will be described in the following as another example. The values of z_{01} , v_{*1} , z_{02} , v_{*2} , z_{02}/z_{01} , τ_{02}/τ_{01} , δ and δ' at each fetch over the tarmac surface in this case are shown in Table 2.

Fig. 6. Velocity profile downwind of rough-smooth (spikes-tarmac) transition (fetch over tarmac surface $x=4.0$ m, $\delta=32$ cm).

Fig. 7. Velocity profile downwind of rough-smooth (spikes-tarmac) transition ($x=6.1$ m, $\delta=42.7$ cm).

Fig. 8. Velocity profile downwind of rough-smooth (spikes-tarmac) transition ($x = 8.3$ m, $\delta = 56.2$ cm).

Fig. 9. Velocity profile downwind of rough-smooth (spikes-tarmac) transition ($x = 10.2$ m, $\delta = 70.8$ cm).

Fig. 10. Velocity profile downwind of rough-smooth (spikes-tarmac) transition ($x = 12.2$ m, $\delta = 83.2$ cm).

Fig. 11. Velocity profile downwind of rough-smooth (grass-tarmac) transition ($x = 5.5$ m, $\delta = 46.4$ cm).

Fig. 12. Velocity profile downwind of rough-smooth (grass-tarmac) transition ($x = 13.9 \text{ m}$, $\delta = 106 \text{ cm}$).

Fig. 13. Velocity profile downwind of rough-smooth (grass-tarmac) transition ($x = 27.0 \text{ m}$, $\delta = 199.5 \text{ cm}$).

Table 2. Roughness parameters, friction velocities, ratio of shearing stresses and interface heights estimated by the use of the set of velocity profiles observed by Bradley (grass-tarmac transition),

x m	z_{01} cm	v_{*1} cm/s	z_{02} cm	v_{*2} cm/s	z_{02}/z_{01}	τ_{02}/τ_{01}	δ cm	δ' cm
0.3	0.72	17.5	0.13	11.0	0.181	0.391	12.6	34.2
5.5	0.50	13.0	0.0015	5.7	0.003	0.192	46.4	126.1
13.9	0.50	18.8	0.0015	9.0	0.003	0.230	106.0	288.0
27.0	0.50	23.8	0.0015	12.1	0.003	0.258	199.5	543.0

In this case, the values of z_{02}/z_{01} at each x estimated by the observed velocity profiles are 0.003 except for $x=0.3$ m. On the other hand BRADLEY has determined $z_{01}=0.5$ cm and $z_{02}=0.002$ cm, hence $z_{02}/z_{01}=0.008$. Comparisons between the velocity profiles predicted by the author and those predicted by the use of BRADLEY's z_{01} and z_{02} at each fetch are shown in Fig. 11–Fig. 13, where the velocity u_{297} at $z=297$ cm is chosen as the reference velocity.

It will be seen from these figures that there is no significant difference between our velocity profiles and those predicted by using BRADLEY's values of z_{01} and z_{02} at fetches of 5.5 and 13.9 m, but the latter give slightly lower velocity at $x=27.0$ m. Also in this case, the tendency of the observed velocity profile to show higher velocity can not generally be found (although it seems that the velocity profile predicted by using BRADLEY's z_{01} and z_{02} gives slightly lower velocity at $x=27.0$ cm).

6. Conclusion

The results mentioned above may be summarized as follows:

- 1) The PANOFSKY-TOWNSEND interface height δ' is e times the ELLIOTT interface height δ .
- 2) The interface height δ (or δ') is the function of both roughness parameters (z_{01} , z_{02}) and friction velocities (v_{*1} , v_{*2}). But it may be represented empirically by the function of x except for the region close to the transition.
- 3) The ratio of the shearing stresses on both surfaces (rough and smooth) τ_{02}/τ_{01} is sensitive to the change of roughness.
- 4) The velocity profiles observed agree fairly well with those predicted by the PANOFSKY and TOWNSEND theory, and the acceleration reported by BRADLEY can not be found.

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地表面における内部境界層についての 2, 3 の考察

根 本 茂

BRADLEY (1968) によれば、粗い地表面から滑らかな地表面に急に変化する際、その境目からある程度離れた風下側の各地点における風速の鉛直分布は PANOFSKY-TOWNSEND 理論によって推定された分布とあまりよく一致しない。理論的に推定された風速の鉛直分布は実測された分布よりも風速の小さい方へずれる、即ち実測された分布は加速されたような形になる。

しかし、その理由がよくわからないので、BRADLEY の観測値を用いてこの問題を再検討してみた。

その結果、Roughness parameter を適切に選べば実測された風速の鉛直分布は PANOFSKY-TOWNSEND 理論によって推定された分布とよく一致し、BRADLEY のような加速は認められなかった。

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