The Splitting of Plume Centerline Height in Elevated-Source Diffusion

by

Shin'ichi Sakuraba, Minoru Moriguchi and Isao Yamazi

Meteorological Research Institute, Tokyo

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Abstract

In the tracer experiment of elevated-source diffusion, sometimes there are found two peak values in the vertical concentration profile, which means two centerline-heights of concentration. One corresponds to the tracer release height and the other, the lower-level one, seems to correspond to the lower neutral or near-neutral layer. The vertical spread of concentration in the lower layer is larger than that in the upper layer. In this circumstance the surface concentration near the source is mostly determined by the diffusion from the lower-level centerline height of plume, namely the extraordinarily high surface concentration is expected to occur near the source, which cannot be explained in the light of customary knowledge of elevated-source diffusion.

1. Introduction

In August 1971 the field experiment of atmospheric diffusion was made at the Nyugawa area, Ehime Prefecture, Shikoku District. The Nyugawa area is located at the coastal area facing the Seto Inland Sea and the terrain is flat from the coast to about 7 km inland.

Fig. 1 shows the network of observation, where S is the tracer release site. The vertical temperature profile was measured at four sites, H 's in the figure. The vertical concentration profile was measured at three sites, K 's in the figure.

The tracer release height was 100 m or 200 m. When the tracer release height was 200 m, two peak concentrations were frequently observed in the vertical concentration profile. The level of upper peak concentration is about 200 m and coincides with the tracer release height, while the level of lower peak concentration is about 100 m. In this case the surface concentration near the source is dominated by the latter. In other words, so far as the surface concentration is concerned, this is equivalent to the change of tracer release height from 200 m to 100 m, though the

Fig. 1. Network of observation,
black circle: sampling site of surface concentration.

apparent source intensity decreases. On the other hand, when the tracer release height was 100 m, such distribution was found to be rare.

2. The analysis of centerline height and vertical spread of concentration

The diffusion equation for a continuous point source is

$$\frac{uC}{Q} = \frac{1}{2\pi\sigma_y\sigma_z} \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \left[\exp\left\{-\frac{(z-\bar{z})^2}{2\sigma_z^2}\right\} + \exp\left\{-\frac{(z+\bar{z})^2}{2\sigma_z^2}\right\} \right] \quad (1)$$

Here $\bar{z}$ is the centerline height of concentration. $\bar{z}$ is usually interpreted as the effective height of emission or the tracer release height and assumed constant independent of downwind distance. However, the centerline height of concentration is, in general, not constant, as reported in our previous paper (SAKURABA and others, 1972). The other notations are customary ones.

The vertical distribution of concentration is, from Eq. (1),

$$C(x, y, z)/C_0 = \left[\exp\left\{-\frac{(\eta-1)^2}{2\xi^2}\right\} + \exp\left\{-\frac{(\eta+1)^2}{2\xi^2}\right\} \right] / 2 \exp\left(-\frac{1}{2\xi^2}\right) \quad (2)$$

$$C_0 = C(x, y, 0)$$

$$\xi = \sigma_z/\bar{z} \qquad \eta = z/\bar{z}$$

When the surface concentration C_0 is zero or very small, the second term in brackets of Eq. (1), which shows the reflected part of concentration from the surface, is

negligible and the vertical distribution of concentration becomes

$$C \propto \exp \left\{ -\frac{(z-\bar{z})^2}{2\sigma_z^2} \right\} \quad (3)$$

From the measurement of vertical distribution of concentration downwind of the source, both the centerline height of concentration $\bar{z}$ and the vertical spread of concentration σ_z can be determined, by Eq. (2) or (3).

3. Experiment 1, 10:00–10:30 8th August 1971

The tracer was released at 200 m level and the release period was 30 minutes from 10:00 to 10:30. The pilot balloon observation was made near the tracer release site and the result is shown in Figs. 2 and 3.

The vertical distribution of temperature was measured at four sites: H_1 , H_2 , H_3 and H_4 . Their distances from the tracer release site are as follows:

H_1 : shoreline near the tracer release site

H_2 : 1.8 km H_3 : 3.5 km H_4 : 7.0 km

The vertical temperature profiles at 11:15–11:33 are shown in Fig. 4. Here the dotted line is the dry adiabat. At H_1 , H_2 and H_3 it is drawn through 28°C at the surface, while at H_4 through 29°C.

In these temperature profiles the stable lapse layer aloft is first to be noticed, which seems to have much affected the concentration profile to be stated subsequently. At H_1 , it is between 250 m and 350 m and the lapse rate is 0.2°C/100 m, while at H_3 it is between 150 m and 250 m, the lapse rate being 0.3°C/100 m. At H_2 the cor-

Fig. 2. Wind-direction profiles.

Fig. 3. Wind-speed profiles.

Fig. 4. Vertical temperature profiles at four sites.

Fig. 5. Vertical concentration profile at K_1 .
Ordinate: concentration in relative unit.

Fig. 6. Vertical concentration profiles at K_2 and K_3 .

responding layer seems to lie between 200 m and 300 m.

The inland modification of sea breeze due to the heating from the surface is not yet conspicuous at H_2 , but the potential temperature is the same at the surface and about 200 m level, so the neutral layer is expected to grow rapidly downwind of H_2 . In fact, at H_3 , the temperature profile below 150 m level coincides with the dry adiabat through 28°C at the surface, except for the shallow layer close to the ground. At H_4 the neutral layer extends up to 300 m level.

The vertical distribution of concentration was measured at three sites: K_1 , K_2 and K_3 . Their distances from the tracer release site are as follows:

$$K_1: 0.9 \text{ km} \quad K_2: 2.0 \text{ km} \quad K_3: 3.6 \text{ km}$$

K_2 and H_2 are about the same in distance from the source. The same is the case with K_3 and H_3 .

The vertical concentration profile measured at K_1 is shown in Fig. 5 and those at K_2 and K_3 are in Fig. 6. Fig. 5 shows two peak concentrations, one at about 110 m level and the other at 200 m level. In Fig. 6 the lower part of concentration is much affected by the reflected concentration from the surface. However it is evident that each profile has two peak concentrations.

When the vertical distribution of concentration is represented by one pair of $(\bar{z}, \sigma_z)$, there occurs only one peak value in the vertical distribution, as is evident from Eq. (2) or (3). So, to express the concentration profile as in Fig. 5 or 6, two pairs of $(\bar{z}, \sigma_z)$ are needed.

Assuming that the concentration profile in Fig. 5 is expressible by the sum of two different Gaussian curves, the lower part of the concentration profile is analyzed in Fig. 7 and the upper part in Fig. 8. In both figures the ordinate is the ratio of concentration at any altitude z to that at $\bar{z}$ and the abscissa is $(z-\bar{z})/\sigma_z$. The full curve is the Gaussian curve.

$$\begin{aligned} \bar{z}_1 &= 120 \text{ m} & \sigma_{z1} &= 55 \text{ m (C)} \\ \bar{z}_2 &= 200 \text{ m} & \sigma_{z2} &= 27 \text{ m (D)} \end{aligned}$$

Here $\bar{z}_1$ and σ_{z1} are the centerline height and the vertical spread of concentration respectively representing the lower part of the concentration profile, while $\bar{z}_2$ and σ_{z2} are those of the upper part.

Plotting $\sigma_{z1}=55 \text{ m}$ at $x=0.9 \text{ km}$ on Pasquill's σ_z-x chart (TURNER, 1967), the Pasquill stability of σ_{z1} is found to be C, where x is the downwind distance. In like manner the Pasquill stability of σ_{z2} is D. This shows that the vertical diffusion rate around 100 m level is larger than that of 200 m level.

200 m coincides with the tracer release height and its meaning is self-evident. The mechanism of formation of another lower centerline height $\bar{z}_1=120 \text{ m}$ is not necessarily evident at this stage. It is important to notice here that, as is evident from the visual inspection of Fig. 5, the concentration with the centerline height at 200 m has no effect on the surface concentration. The surface concentration is entirely determined by the concentration with the centerline height at 120 m. This means that, so far as the surface concentration is concerned, the centerline height of concentration is not 200 m but 120 m. Furthermore the Pasquill stability of σ_z is not D but C.

The lower part of concentration profile at K_2 is analyzed in Fig. 9 and the upper

Fig.7. Analysis of the lower part of concentration at K_1 . Fig.8. Analysis of the upper part of concentration at K_1 .

Fig. 9. Same as Fig. 7 but for K_2 .

part in Fig. 10. In Fig. 9 the ordinate is $z/\bar{z}$ and the abscissa is C/C_0 , C_0 being the surface concentration. The parameter ξ is $\sigma_z/\bar{z}$.

$$\begin{array}{lll} \bar{z}=100\text{m} & \xi^2=0.6 & \sigma_{z1}=77\text{m (C-D)} \\ \bar{z}=240\text{m} & & \sigma_{z2}=45\text{m (D)} \end{array}$$

Here C-D means that the Pasquill stability is near the midpoint between C and D.

$\bar{z}=240\text{ m}$ corresponds to the tracer release height, though the height difference between the two is 40 m. Such minor rise or descent of plume centerline is not rare even over a flat terrain. It is to be noticed that $\bar{z}_2$ lies in the stable 200 m–300 m layer observed at H_2 .

$\bar{z}_1=100\text{ m}$ is the lower-level centerline height and the Pasquill stability of σ_z below about 100 m level is C-D, which suggests that the layer from the surface to about 100 m level is neutral or near-neutral. It is well known from the diffusion experiment that the Pasquill stability of σ_z is on the average C-D in the neutral layer when the downwind distance exceeds 1 km. As already remarked, the neutral layer is expected to grow rapidly downwind of H_2 , so it is probable that the diffusion below 100 m level has become that of near-neutral layer at K_2 .

The lower part of concentration at K_3 is analyzed in Fig. 11 and the upper part in Fig. 12.

Fig.10. Same as Fig.8 but for K_2 .

$$\begin{array}{lll} \bar{z}_1 = 90\text{m} & \xi^2 = 0.85 & \sigma_{z1} = 83\text{m} \text{ (C-D)} \\ \bar{z}_2 = 220\text{m} & & \sigma_{z2} = 60\text{m} \text{ (D-E)} \end{array}$$

$\bar{z}_1=90\text{ m}$ is located in the neutral layer extending to 150 m level, as shown in the temperature profile at H_3 , though the coincidence between observation and calculation is not good in Fig. 11. The Pasquill stability C-D of σ_{z1} is acceptable as the vertical diffusion rate in the neutral layer. $\bar{z}_2=220\text{ m}$ is in the stable 150 m – 250 m layer and corresponds to the tracer release height. The Pasquill stability D-E of σ_{z2} is reasonable as the vertical diffusion rate in the stable lapse layer.

The splitting of centerline height of concentration into two seems to have some connection with the thermal stratification of the layer concerned, namely the stable layer aloft and the underlying relatively unstable layer, though the role of wind-direction shear shown in Fig. 2 cannot be ignored. In case of K_2 and K_3 , the neutral or near-neutral layer extends up to 100 m or 150 m level, above which lies the stable lapse layer.

4. Experiment 2, 10:40–11:10 8th August 1971

40 minutes later the second experiment of this day was made. This time the tracer release height was 100 m .

The vertical distribution of concentration at K_1 is shown in Fig. 13 and those at K_2 and K_3 in Fig. 14. Except K_1 , each profile has one peak value of concentration, normal profile, which is quite different from the case of Experiment 1.

Aug. 8. 1971. (NYUGAWA)
 10:00-10:30 $h=200\text{m}$ $K_3(X=3.6\text{Km})$
 $Z/\bar{z}$ $\bar{z}=90\text{m}$ $\sigma_z=83\text{m}$ (C-D) $\xi^2=0.85$

Fig.11. Same as Fig.7 but for K_3 .

Aug. 8. 1971. (NYUGAWA)
 10:00-10:30 $h=200\text{m}$ $K_3(X=3.6\text{Km})$
 $C(z)/C(\bar{z})$ $\bar{z}=220\text{m}$ $\sigma_z=60\text{m}$ (D-E) $o > \bar{z}$

Fig.12. Same as Fig.8 but for K_3 .

Fig. 13. Vertical concentration profile at K_1 . Fig. 14. Vertical concentration profiles at K_2 and K_3 .

Fig. 15. Analysis of the concentration profile at K_1 .

Though the profile at K_1 has two peak values of concentration, it is difficult to analyze the lower part of concentration. So, assuming that the concentration profile consists of a single Gaussian curve, the analysis is shown in Fig. 15.

$$\bar{z}=60\text{m} \quad \xi^2=1 \quad \sigma_z=60\text{m} \text{ (C)}$$

The Pasquill stability of σ_z is C and equal to that of the lower part of concentration at K_1 in Experiment 1. $\bar{z}=60\text{m}$ corresponds to the tracer release height.

The concentration profile at K_2 is analyzed in Fig. 16.

$$\bar{z}=130\text{m} \quad \xi^2=0.4 \quad \sigma_z=82\text{m} \text{ (C-D)}$$

Similarly the Pasquill stability of σ_z is the same as that of the lower part of concentration at K_2 in Experiment 1. The concentration at K_2 is very low and correspondingly the accuracy of analysis is not good compared with that of K_1 or K_2 , which is reflected in Fig. 17.

$$\bar{z}=70\text{m} \quad \xi^2=3 \quad \sigma_z=121\text{m} \text{ (C-D)}$$

It must be emphasized here that, when the tracer release height is 100 m, the

Fig. 16. Same as Fig. 15 but for K_2 .

splitting of centerline height of concentration does not occur, in contrast with a conspicuous splitting when the tracer release height is 200 m. The atmospheric layer is considered almost the same in both cases.

5. The concentration in case of two centerline heights of concentration

Normally there is only one centerline height of concentration, $\bar{z}$ in the present notation, corresponding to the tracer release height or the effective height of emission. However, in some conditions, there occur two centerline heights, one corresponding to the tracer release height in the stable layer and the other to the underlying relatively unstable layer, as stated in Section 3. In this case the low-level centerline dominates the surface concentration near the source.

Let the centerline height of concentration in the upper layer be $\bar{z}_2$ and that in the lower layer be $\bar{z}_1$, then the diffusion equation is

$$\frac{2\pi\sigma_y}{\exp(-y^2/2\sigma_y^2)} \frac{uC}{Q} = \frac{\alpha_1}{\sigma_{z1}} \left[\exp\left\{-\frac{(z-\bar{z}_1)^2}{2\sigma_{z1}^2}\right\} + \exp\left\{-\frac{(z+\bar{z}_1)^2}{2\sigma_{z1}^2}\right\} \right] + \frac{\alpha_2}{\sigma_{z2}} \left[\exp\left\{-\frac{(z-\bar{z}_2)^2}{2\sigma_{z2}^2}\right\} + \exp\left\{-\frac{(z+\bar{z}_2)^2}{2\sigma_{z2}^2}\right\} \right] \quad (4)$$

$$\alpha_1 + \alpha_2 = 1 \quad (5)$$

Eq. (5) comes from the continuity condition

$$\int_0^\infty \int_{-\infty}^\infty \frac{uC}{Q} dy dz = 1 \quad (6)$$

Fig. 17. Same as Fig. 15 but for K_s .

σ_{z1} and σ_{z2} correspond to $\bar{z}_1$ and $\bar{z}_2$ respectively, and the horizontal spread of concentration σ_y is assumed to be common to $\bar{z}_1$ and $\bar{z}_2$. The surface axial concentration C_a is

$$\frac{uC_a}{Q} = \frac{1}{\pi\sigma_y} \left[\frac{\alpha_1}{\sigma_{z1}} \exp\left(-\frac{\bar{z}_1^2}{2\sigma_{z1}^2}\right) + \frac{\alpha_2}{\sigma_{z2}} \exp\left(-\frac{\bar{z}_2^2}{2\sigma_{z2}^2}\right) \right] \quad (7)$$

or

$$\frac{u C_{a1}}{\alpha_1 Q} = \frac{1}{\pi \sigma_y \sigma_{z1}} \exp(-\bar{z}_1^2/2\sigma_{z1}^2) \quad (8)$$

$$\frac{u C_{a2}}{\alpha_2 Q} = \frac{1}{\pi \sigma_y \sigma_{z2}} \exp(-\bar{z}_2^2/2\sigma_{z2}^2) \quad (9)$$

$$C_a = C_{a1} + C_{a2} \quad (10)$$

To assess quantitatively the surface concentration given by Eq. (8) or (9), the following values corresponding to the analysis in Section 3 will be adopted:

$$\begin{array}{ll} \bar{z}_1=100\text{m} & \bar{z}_2=200\text{m} \\ \sigma_{z1} (C-D) & \sigma_{z2} (D) \end{array}$$

Here $\sigma_{z1}(C-D)$ means that the Pasquill stability of σ_{z1} is $C-D$, and so on. The magnitude of σ_z varies with the downwind distance x as follows:

x	1	2	3	4	km
$\sigma_{z1} (C-D)$	44	76	104	130	m
$\sigma_{z2} (D)$	32	50	64	77	m
σ_{z2}/σ_{z1}	0.73	0.66	0.62	0.59	

To determine α_1 and α_2 , let $C(\bar{z}_1)$ be the concentration at $\bar{z}_1$ and $C(\bar{z}_2)$ that at $\bar{z}_2$, then

$$\begin{aligned} & \frac{C(\bar{z}_1)}{C(\bar{z}_2)} \\ &= \frac{\frac{\alpha_1}{\sigma_{z1}} \left[1 + \exp\left(-\frac{4\bar{z}_1^2}{2\sigma_{z1}^2}\right) \right] + \frac{\alpha_2}{\sigma_{z2}} \left[\exp\left\{-\frac{(\bar{z}_1-\bar{z}_2)^2}{2\sigma_{z2}^2}\right\} + \exp\left\{-\frac{(\bar{z}_1+\bar{z}_2)^2}{2\sigma_{z2}^2}\right\} \right]}{\frac{\alpha_1}{\sigma_{z1}} \left[\exp\left\{-\frac{(\bar{z}_2-\bar{z}_1)^2}{2\sigma_{z1}^2}\right\} + \exp\left\{-\frac{(\bar{z}_2+\bar{z}_1)^2}{2\sigma_{z1}^2}\right\} \right] + \frac{\alpha_2}{\sigma_{z2}} \left[1 + \exp\left(-\frac{4\bar{z}_2^2}{2\sigma_{z2}^2}\right) \right]} \end{aligned} \quad (11)$$

At $x=1$ km or so where the magnitude of σ_z is rather small, Eq. (11) is approximately

$$\frac{C(\bar{z}_1)}{C(\bar{z}_2)} \approx \frac{\alpha_1}{\alpha_2} \frac{\sigma_{z2}}{\sigma_{z1}} \quad (12)$$

In case of $C(\bar{z}_1) \approx C(\bar{z}_2)$, as is the case in Fig. 5,

$$\alpha_1=0.58 \quad \alpha_2=0.42$$

by $\sigma_{z2}/\sigma_{z1}=0.73$ and the condition (5).

The righthand side of Eq. (8) is calculated in Fig. 18 and that of Eq. (9) in Fig. 19. Exactly speaking, σ_y of Fig. 18 is different from that of Fig. 19, which, however, has no practical effect on the subsequent discussion.

From Fig. 18 and 19

x	1	2	3	4	km
$\frac{u C_{a1}}{\alpha_1 Q}$	6.6	11.0	8.2	6.0	$\times 10^{-6} \text{m}^{-2}$
$\frac{u C_{a1}}{Q}$	3.8	6.4	4.8	3.5	"
$\frac{u C_{a2}}{\alpha_2 Q}$	0	0	0.2	0.6	"
$\frac{u C_{a2}}{Q}$	0	0	0.1	0.3	"

Evidently the surface concentration is determined by the low-level centerline height and the high-level centerline height has practically no effect down to 4 km or so. This means the lowering of centerline height from 200 m to 100 m, so far as the surface concentration is concerned, though the source intensity is apparently reduced.

Fig. 18. Normalized surface axial concentration versus downwind distance curves in case of C-D Pasquill stability. Numeral shows the centerline height of concentration in meters.

For reference the surface concentration when only one centerline height is at 200 m and the Pasquill stability of σ_z is C-D is given below.

x	1	2	3	4	km
$\frac{u C_a}{Q}$	0	0.8	2.1	2.5	$\times 10^{-6} \text{m}^{-2}$

6. Concluding remarks

It is usual that there is one centerline height of concentration corresponding to one release height of tracer. However, the diffusion experiment shows that, on some occasions, there appear two centerline heights. It is not necessarily made clear here under what meteorological condition one centerline height is split into two. A tentative conclusion at the present stage is that it is associated with the thermal stratification such that the upper stable layer lies on the lower relatively unstable layer and the tracer is released in the upper stable layer. When the tracer is released in the lower layer, there occurs only one centerline height.

Fig.19. Same as Fig.18 but for D Pasquill stability.

Anyway, when the splitting of centerline height occurs, the surface concentration is dominated by the lower-level centerline height and is extraordinarily high near the source in the light of tracer release height. The higher Pasquill stability of σ_z in the lower layer is also responsible for the high surface concentration.

The present conclusion comes from the diffusion experiment when the tracer release height is 200 m. There is no evidence whatever that, when the effective stack height or tracer release height exceeds 300 m, the same phenomenon occurs.

However, the present result of analysis is serious from the standpoint of practical air pollution control and suggests that, when the effective stack height is 200 m to 300 m, there may occur an extraordinarily high surface concentration near the source not expected from the effective stack height.

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高煙源拡散実験において見出された2つの濃度の中心線高度

桜庭信一, 森口 実, 山路 勲

濃度の中心線高度を $\bar{z}$, 濃度の鉛直幅を σ_z とすると, 点源に対する濃度の鉛直分布は, ふつう一組の $(\bar{z}, \sigma_z)$ で表現できる。このとき濃度の鉛直分布に極大値は一つしかない。すなわち 煙源付近では極大値は $\bar{z}$ の高度に, 煙源から遠いところで地表からの反射濃度が卓越してくると極大値は地表に起る。ところが濃度の鉛直分布に2つの極大値が起る例が, いくつかトレーサーによる拡散実験で見出された。典型的なのが壬生川地域(愛媛県の臨海地域)における拡散実験結果である。このような濃度分布を表現するには $(\bar{z}, \sigma_z)$ の2組の値を要する。すなわち $\bar{z}$ は2つあり, 対応して異なる σ_z を持つ。

8月8日の壬生川拡散実験では第一回はトレーサー散布高度が200mで濃度鉛直分布は0.9, 2.0, 3.6kmの3点で測定したが, 解析結果は

$$\begin{array}{ll} \bar{z}_2 \approx 200\text{m} & \text{対応する } \sigma_z \text{ の Pasquill 安定度 } D \text{ または } D-E \\ \bar{z}_1 \approx 100\text{m} & \sigma_z \text{ の Pasquill 安定度 } C \text{ または } C-D \end{array}$$

$\bar{z}_2 \approx 200\text{m}$ がトレーサー散布高度に対応することは自明だが, それ以外に $\bar{z}_1 \approx 100\text{m}$ の高度に新しい中心線高度が出現したことになる。この結果は極めて重要であって, 地表濃度は煙源付近(少なくとも風下距離4km位まで)では $\bar{z} \approx 100\text{m}$ に中心線高度をもつ濃度によって決まり, $\bar{z}_2 \approx 200\text{m}$ は関係しない。すなわち地表濃度の立場からは $\bar{z}$ は200mでなく100mであるということになる。

約40分遅れて第2回の実験がなされ, トレーサー散布高度は100mに変更された。この時の鉛直分布は濃度の極大値が1つのふつうの型で

$$\bar{z} \approx 100\text{m} \quad \sigma_z \text{ の Pasquill 安定度 } C \text{ または } C-D$$

濃度の中心線が2つに分裂する現象が何によるかは今の段階では必ずしも明瞭ではないが, トレーサー散布高度が安定なてい減層中にあり, この下に地表付近まで中立層か, これに近い層があるとき, 起るのではないかと考えている。このような温度成層は海風が日中, 内陸で変質するときふつうに見られる。

ほとんど同じ大気成層内でトレーサー散布高度200mのときに中心線高度が2つ出現し, 100mのとき1つというのは注目すべきである。いずれにせよ有効煙突高200m程度のとき煙源付近に異常な高濃度の出現の可能性を示している。有高煙突高が300m以上のとき同じ現象が起るかどうかはもち論わからない。