Collection of Fog Particles with Fine Fibre and Infra-red Absorption of Fog Particles

by

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Abstract

By using a fibre of artificial resin (radius 2μ), the authors investigated the size distribution curve of fog particles and its variation. Some theoretical consideration was also given in the region of 1.2μ~1.5μ in wave length and the infra-red absorption coefficient is estimated to be 1.4%.

1. The method of capturing fog particles

Many on measuring fog particles have found that the smaller the dimension of the detector of fog particles, the finer particles have been captured.

Unless we use a detector of an adequate dimension, our measuring can not offer good information of fog distribution. So in order to catch fine particles included in the real distribution of fog particles, we suspended a fine fibre of artificial resin (radius 2μ) on a small frame, and set it on the mechanical stage of microscope. Supporting the frame vertically to the direction of fog flow, we counted the number and measured the size of the fog particles which settled on the fibre in a unit time, and at the same time measured their evaporating time.

Glass fibre [1] or spider's thread [2] have been generally used as the fibre detector of fog particles, but they are not always hydrophobic and uniform. On the other hand the artificial resin we used consists of Methyl Methacrylate 99.7% and Benzol Peroxide 0.3% is very hydrophobic. (Water absorption ratio; Methyl Methacrylate 0.3~0.5, Polyamide 1.5, Polyvinyl 0.6~1.3, Viniel Archol 100%). If we combine these materials under 85°C, a viscous liquid is obtained, and we can make a fine fibre from it.

2. The purpose of the observation

The southeast coast of Hokkaido is frequently covered with dense fog penetrating from the sea in early summer, and this dense fog brings about shortage of
sunshine and a cold wet weather, so that it is very important to fight these fogs from an agricultural point of view.

By the reason mentioned above there are wooded shelter-belts along the coast, the effect of which we wanted to estimate.

The numbers in the map (Fig. 1) represent the observation points which were selected.

Point (11) was located in front of the forest and point (12) was behind it. Point (21) and point (22) lay by so broad a road that scarcely any effect of these forests and others was perceived for the reference of point (11) and point (12). At these points we observed only those fogs which were regarded to cover these points homogeneously.

The fogs on the 23rd (1645—0850), 24th (1940—2150) and 28th (0730—0850) June 1951, were considered to satisfy this condition.

The mean particle numbers captured on the fibre in ten minutes are shown in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>In case of fog shelter forest</th>
<th>In case of no fog shelter forest</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>near seacoast</td>
<td>far apart seacoast</td>
</tr>
<tr>
<td></td>
<td>(11)</td>
<td>(12)</td>
</tr>
<tr>
<td>23rd</td>
<td>22</td>
<td>16</td>
</tr>
<tr>
<td>24th</td>
<td>24</td>
<td>10</td>
</tr>
<tr>
<td>28th</td>
<td>265</td>
<td>39</td>
</tr>
</tbody>
</table>

Analysing the variance of the data, we obtained Table 2.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Degree of freedom</th>
<th>Sum of square</th>
<th>Unbiased estimate</th>
<th>$F_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>1</td>
<td>103345</td>
<td>103345</td>
<td>2.97</td>
</tr>
<tr>
<td>$S_2$</td>
<td>1</td>
<td>2643</td>
<td>2643</td>
<td></td>
</tr>
<tr>
<td>$S_1x_2$</td>
<td>1</td>
<td>35616</td>
<td>35616</td>
<td></td>
</tr>
<tr>
<td>$S_0(12)$</td>
<td>8</td>
<td>284064</td>
<td>35508</td>
<td></td>
</tr>
</tbody>
</table>
So that we may decide that the effect of the forest in keeping off the fog is significant.

3. Mechanism of fog capturing process of the fibre

The above analyses of variance are calculated under the assumption that the frequency distribution of the numbers of fog particles settling down on the fibre in a constant time is normal distribution. But the assumption is not precisely true to fact.

According to BATEMAN [3], if the probability of particles \( (n+1) \) settling down on the fibre in \( (t+dt) \) time be \( P_{(n+1)}(t+dt) \), the probability should be the sum of the probability \( (1-dt)P_{(n+1)}(t) \), no particle settles on the fibre in \( dt \) time, and the probability \( dtP_{n}(t) \), \( n \) particles settle on the fibre in \( t \) time and one particle settles on it in \( dt \) time, namely

\[
P_{(n+1)}(t+dt) = (1-dt)P_{(n+1)}(t) + dt P_{n}(t).
\]

If we transform the above formula into

\[
dP_{(n+1)}/dt = P_{(n)}(t) - P_{(n+1)}(t)
\]

\[
P_{(n)} = e^{-\varphi t}(\varphi t)^n / n!
\]

this formula shows the Poisson Distribution.

But the frequency distribution curve of particles settling down on the fibre in 30 second shows Polya-Eggenberger Distribution (Fig. 2), namely,

\[
P_{(n)} = (2.625 \times 3.625 \ldots \times (n-1)2.625) \times 4.360^{-0.782(n+1)} / n!.
\]

Testing the curve-fitness,

\[
\chi^2 = 20.1 < \chi^2_{n=21}(0.5) = 21.337
\]

\[
P_r > 0.50
\]

so that the distribution curve may be sufficiently regarded as Polya-Eggenberger Distribution [4] [5] [6] [7].

From such facts, the mechanism of the fog capturing of the fibre could be regarded as Compound Poisson Process.

If \( \varphi \) be the population mean of \( n \), the number of particles observed, and the distribution of \( \varphi \) in general be \( T \)-type Distribution,
\[ F(x) = \int_0^\infty \frac{t^{a+1} \rho^b e^{-\rho t} dt}{\Gamma(a+1)} \]

(a, b are constant),

\[ P(x) = \sum_0^\infty P_n F(x) = \frac{b^{a+1} \Gamma(n+a-1)}{[(1+b)^{n+a+1}n! \Gamma(a-1)]} \]

(P_n is the Poisson Distribution whose population mean is \( n \).) Comparing the above empirical formula with the above \( P(x) \), \( a \) and \( b \) are decided one by one as follows.

\[ a = 0.782 \]
\[ b = 3.360 \]

The calculated curves of \( F(x) \) are illustrated in Fig. 3.

If the particles are homogeneous and distributed in space at random, the capturing mechanism of the fibre should be Poisson Process, but the fact is the Compound Poisson Process.

What is it that caused such a difference? It may be that

(i) each fog has its own particle size distribution,

(ii) the number of fog particles is not always homogeneous in each fog.

4. The particle size distribution

The particle size distribution was measured for the fogs of the 23rd, 24th and 28th. The histograms for fogs’ particle size distribution whose size step is 5 \( \mu \) are shown in following Fig. 4.

We fitted the histograms with the \( \chi^2 \)-distributions (degree of freedom 2P). [8]

\[ f(r) dr = \begin{cases} \left( \frac{r}{a} \right)^{n-1} e^{-\frac{r}{a}} dr / a \Gamma(n) & r \geq 0 \\ 0 & r < 0 \end{cases} \]

\( r \) is particle size of fog, \( a, P \) are parameters), \( \hat{a} \) and \( \hat{P} \), the estimate of parameters \( a \) the \( P \) are separately calculated from the following relation.

\[ \log \hat{P} - \Psi(\hat{P}) = \log (A/G) = g(\hat{P}) \]

\[ a = A/P \]
\( A : \) arithmetic mean \quad \( G : \) geometric mean

\( \Psi : \) digamma function \quad \( \Psi' : \) differentiate by argument

\[
g(x) = \log x - \psi(x) = 1/2x + 1/12x^3 - 1/120x^5 + \cdots (-1)^n B_{2n+2} / (2n+2) x^{2n+2}.
\]

And from the following relation, \( \text{Var} \ P \) and \( \text{Var} \ \hat{a} \), the estimate variance may be calculated,

\[
\text{Var} \ \hat{a} = \Psi' a^2 / N (P \psi - 1).
\]

\[
\text{Var} \ \hat{P} = P / N (P \psi - 1).
\]

So that the significant difference of two estimates, \( \hat{P}_1, \hat{P}_2 \) or \( a_1, a_2 \) should be represented as follows,

\[
| D_p | > \sqrt{F_{\alpha}^2} \left( \frac{\text{Var} \ \hat{P}_1 - \text{Var} \ \hat{P}_2}{\text{Var} \ \hat{a}_1 - \text{Var} \ \hat{a}_2} \right)
\]

Fig. 4. Frequency distribution of droplets radius.
The theoretically deduced mode of the curve may be expressed from the formula
\[ \frac{\partial f(r)}{\partial r} = a, \]
\[ a, \hat{\beta}, \text{Var } a, \text{Var } \hat{\beta} \] and the mode \( \tau_{\text{max}} \) are represented in the following table.

From these data, we can say that in the fogs on the 23rd and 24th there was a tendency for the mode to travel to the smaller-sized part behind the forest, but the case was inverse in the fog of the 28th.

Table 3.

<table>
<thead>
<tr>
<th></th>
<th>23rd</th>
<th></th>
<th>24th</th>
<th></th>
<th>28th</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \hat{a} )</td>
<td>( \hat{\beta} )</td>
<td>Var ( \hat{a} )</td>
<td>Var ( \hat{\beta} )</td>
<td>( \gamma_{\text{max}} (\mu) )</td>
</tr>
<tr>
<td>23rd</td>
<td>0.960</td>
<td>3.510</td>
<td>0.072</td>
<td>0.875</td>
<td>8.04</td>
</tr>
</tbody>
</table>

5. Evaporation time of fog particles

Figs. 5 and 6 show the evaporation of fog particles on the fibre.

![Fig. 5. Evaporation of fog particles on the fibre.](image-url)
\( T_a = a^m(1/2 + D/2a + 15a/4D) \)
\[ a = \frac{u_0}{2D} \]

- **D**: diffusion coefficient of water vapour
- **C_s**: saturated water vapour tension at the water surface
- **C_\infty**: water vapour tension at a place remote from water droplet

\[ k = \beta \sqrt{RT} \sqrt{2\pi M/(1-\beta^2)} \]
- **\beta**: condensation coefficient
- **R**: gas constant
- **M**: molecular weight of water vapour
- **u_0**: falling velocity of droplet

\( m = 2 \).

Excepting \( a^m \), we represent the others by constant \( K \), and transform the above formula logarithmically,

\[ \log T_a = m \log a + \log K \]

Assuming that the standard error of scale in microscope reading is 2.1 \( \mu \), and that of the measurement of evaporating time is 0.5 second [10], we calculated \( m \) and \( \log K \) using the least square method which W.E. Deming [11] had improved. (Table 4 and Fig. 7a.)

<table>
<thead>
<tr>
<th></th>
<th>1_1</th>
<th>1_2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2_1</td>
<td>2_2</td>
</tr>
<tr>
<td>( m )</td>
<td>2.03 \pm 0.66</td>
<td>0.08 \pm 0.31</td>
</tr>
<tr>
<td>\log K</td>
<td>-1.47 \pm 0.35</td>
<td>0.26 \pm 0.48</td>
</tr>
</tbody>
</table>

Fig. 7. The relation between evaporation and radius of droplets.
If we assume the evaporation time $T_a$ is proportional to the cross section of droplet, $m$ is estimated at 2. (Table 5 and Fig. 7b)

Provided that the observed values are distributed normally around $K$ value, the significance of difference between values of $K$ is (in the case of the shelter forest) shown by

$$t = 6.670 > t(0.001) = 4.518 \quad (n=12),$$

then we may say the difference was significant.

<table>
<thead>
<tr>
<th></th>
<th>$1_1$</th>
<th>$1_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2_1$</td>
<td>$2_1$</td>
<td>$2_2$</td>
</tr>
<tr>
<td>$K$</td>
<td>$-1.36$</td>
<td>$-2.24$</td>
</tr>
<tr>
<td>Wind velocity m/sec.</td>
<td>$1.33$</td>
<td>$0.43$</td>
</tr>
</tbody>
</table>

Table 5.

So that it may be true that behind the forest the fog particles evaporate more easily.

Where there was no shelter forest, the difference was not significant.

As to the way of measuring fog particles, we noticed that some fogs were accompanied by so small particles that we could not measure their size, but only the time of evaporation. Such particles, we temporarily called "trace", were always found in the points (21) and (22), near the seashore, but could not be found at the points (11) and (12), remote from the seashore. The life time of these particles may be so short that they can not reach any place remote from the sea.

The number of "trace" particles was often very much more than ordinary fog particle, for instance there were 2,800 "trace" particles when 265 ordinary particles were observed.

If we can use the same evaporating law in the case of "trace" particles, their size may be estimated from the evaporation and size relation.

Then almost all particles were estimated smaller than $1\mu$.

These facts suggested that the size distribution of fog obtained hitherto by many people had neglected the small-sized particles.

6. Infra-red spectrometry of fog

For the estimation of thermal vanishing effects of forest, it is very important to know how much radiant energy the fog absorbs.

We measured the infra-red absorption coefficient of fog by means of infra-red spectrometry, a replica grating 15,000 lines per inch was used in conjunction with the so-called chopped beam of a tungsten lamp.

![Fig. 8. Illustration of optical system.](image)
PbS photo-conductive cell detective and amplifier system, which consists of the LC circuit and three stages of high gain pentode 6SJ7, are employed. The optical system and amplifier system are illustrated in Figs. 8 and 9.

![Amplifier Circuit](image)

Fig. 9. Illustration of amplifier system.

If the radiant energy \( I_0' \) is reduced to \( I' \) by absorption and scattering of fog particles, the relation of \( I' \) and \( I_0' \) may be represented as follows:

\[
I' = I_0' e^{-x},
\]

and \( I_0 \) is reduced to \( I \) by fog particles scattering only, the relation \( I \) and \( I' \) may be also represented as follows

\[
I = I_0 e^{-x}.\]

\( J \) and \( J' \) are extinction coefficient
\( x \): optical path

If we represent the scattering coefficient and the absorption coefficient by \( S \) and \( a \) respectively, then

\[
J = S,
\]

\[
J' = S' + a'.
\]

And when \( 2 \pi r/\lambda > 30 \) (\( r \): radius of the fog particle; \( \lambda \): wave length of the radiant beam), we can regard as

\[
S = S'. \quad [13]
\]

So that

\[
a' = J' - J = \log (I/I_0) - \log (I'/I_0').
\]

Here \( \log (I/I_0) \) is equal to the absorption coefficient of water vapour, and in our case it may be regarded that the space should be saturated by water vapour, whose amount is the function of the temperature in situ only.

Before observation we calibrate the relation between the absorption coefficient and water vapour content as Fig. 10 represents, then the thermometer reading may
give the value by \( \log(I/I_0) \). From the value of \( \log(I'/I'_0) \) measured by spectrometry and of \( \log(I/I_0) \), we can estimate \( a' \), absorption coefficient of the fog particle.

The following curve is absorption curve of fog. There is no absorption at \( \lambda : 1.25 \mu \), but \( \lambda : 1.375 \mu \) is the stretch absorption band, \( \nu_1 + \nu_2 \).

We may calculate as \( a' = 0.144 \) at wave length \( \lambda : 1.375 \mu \), a meter optical path.

From the above observation we may conclude as follows.

(1) The number of fog particles decreases behind the forest.

(2) The fog capturing process of fibre is the Compound Poisson Process.

(3) The size distribution of fog particles fits well \( \chi^2 \)-type distribution. The mode of the curve behind the forest moves to the larger-particle part.

(4) We suggest the existence of the so-called "trace" by the evaporation curve of the fog particle.

(5) The infra-red (1.375\( \mu \)) absorption of fog particles were 14%.

In conclusion the authors wish to express their sincerest thanks to Prof. M. Eguchi and Mr. Y. Yatabe.

References


