On the Influence of Waterdrops in the Air upon the Atmospheric Radio-refractive Index

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Abstract

Atmospheric radio-refractive index is determined by the well-known formula based on DEBYE's idea. This formula is used unaltered even in rainfall. However, judged from the fact that the electromagnetic scattering effect of waterdrops in rainfalls can be usually detected as radar-echoes, the existence of waterdrops must influence the radio-refractive index in the atmosphere. This problem is approached and discussed assuming Rayleigh scattering for a single drop and approximate mutual reactions among the scattered waves. It is concluded from the result obtained using the data on hydrometeors, that the influence upon the refractive index or the equivalent radio-refractive index could hardly be disregarded in certain cases.

1. Introduction

The radio-refractive index of the atmosphere is determined as a function of atmospheric pressure, ambient temperature and water-vapor pressure by a well-known formula based on DEBYE's idea. This formula is not related to the existence of such waterdrops in the air as raindrops and fogs. However, the following questions arise: Does the existence of waterdrops not affect the properties of the atmosphere as an electromagnetic medium?: Is the usual formula on radio-refractive index of atmosphere still valid in a rainfall? The scattering effect of waterdrops in a rainfall upon electromagnetic waves is a well-established fact usually detected as radar-echoes. Consequently the existence of raindrops must have some influence on the medium properties of the atmosphere, which the author is going to consider in the present paper.

The problem is to obtain the equivalent dielectric constant or equivalent radio-refractive index of a medium in which small spherical particles are uniformly and sparsely spaced, just like the atmosphere with its precipitations. A theory on such a problem has been already proposed by K.W. WAGNER[1]. His theory was,

however, limited to a static field on the basis of the Laplace equation. Therefore it seems unsafe to apply his theory to the radiometeorological case without any examination. So the author solved the problem by a different approach, that is, the consideration of the variation of plane electromagnetic waves, incident upon such a medium as stated above.

2. Scattering of plane electromagnetic waves by uniformly and sparsely distributed small spherical particles and equivalent medium properties

Uniform and sparse distribution of the spherical particles with small sizes is considered enough to allow Rayleigh scattering to each particle. Let a plane wave with its electric vector linearly polarized in the x -direction and with its direction of propagation to the positive z -axis, be incident upon a small spherical particle. Then the scattering will be as follows:

$$(1) \quad \begin{cases} E_\theta = -\frac{1}{4\pi\epsilon} \frac{k^2}{r} \sin\theta \cdot p \cdot e^{-i\omega t + ikr}, \\ H_\phi = -\frac{\omega}{4\pi} \frac{k}{r} \sin\theta \cdot p \cdot e^{-i\omega t + ikr}, \\ p = 4\pi\epsilon \frac{\epsilon' - \epsilon}{\epsilon' + 2\epsilon} a^3 E_0. \end{cases}$$

Here ϵ' is the dielectric constant of the particle with radius a , ϵ is the surrounding dielectric constant, and (r, θ, ϕ) is the polar coordinates with the origin at the center of the particle and with the polar axis coinciding with the x -axis, other symbols being the same as used by STRATTON[2].

Fig. 1. Scattering by slice part with small particles.

Consider a slice with a thickness of ΔZ as illustrated in Fig. 1, where small particles of an equal radius a are distributed in a manner defined above. A plane wave incident upon the slice is scattered by the particles. The total field at point P is a resultant of the primary and the secondary waves, of which the latter is composed of waves scattered by the particles. Electric field of each scattered wave given by equation (1) is expressed in

Cartesian coordinate system as follows:

$$\begin{cases} x\text{-component: } -E_\theta \sin\theta, \\ y\text{-component: } E_\theta \cos\theta \cos\phi, \\ z\text{-component: } E_\theta \cos\theta \sin\phi. \end{cases}$$

By superimposing all the scattered waves at P it can easily be shown that the

y - and z -components are both cancelled out altogether and that the x -component is the only one to survive and to be considered.

The x -component of the waves scattered by a small region $\Delta\xi \Delta\eta \Delta z$ including $Q(\xi, \eta, 0)$ in the slice is, assuming N to be the number of the particles per unit volume,

$$\frac{k^2}{4\pi\epsilon} N \Delta p \xi \Delta\eta \Delta z \frac{\sin^2\theta}{r} e^{-i\omega t + ikr},$$

where the readers are referred to Fig. 1 as for the coordinates. The secondary wave at P is therefore expressed by

$$(2) \quad E_s = \frac{k^2}{4\pi\epsilon} p N \Delta z e^{-i\omega t} \iint_{-\infty}^{\infty} \frac{\sin^2\theta}{r} e^{ikr} \cdot d\xi d\eta.$$

In carrying out the integral contained in equation (2) it is assumed that the point P is detached enough from the origin 0. If we follow the method in FRESNEL'S diffraction theory, equation (2) becomes

$$(3) \quad E_s = i \frac{k^2 \lambda}{4\pi\epsilon} N p \Delta z e^{-i\omega t + ikr} = i s \Delta z E_0 e^{-i\omega t + ikr},$$

with

$$(4) \quad s = \frac{4\pi^2 N \alpha^3}{\lambda} \frac{\epsilon' - \epsilon}{\epsilon' + 2\epsilon} = \frac{3\pi N v}{\lambda} \frac{\epsilon' - \epsilon}{\epsilon' + 2\epsilon},$$

where v is the volume of one particle. It should be noted that the secondary wave is found in equation (3) to be the same linearly polarized wave in x -direction as the primary incident one, as a result of superposition. Consequently the total electric field at P has only the x -component given by

$$(5) \quad E = (1 + i s \Delta z) E_0 e^{-i\omega t + ikr}.$$

Next, the case will be considered where small particles are distributed in the whole space instead of in the slice. equation (5) indicates that the existence of a slice with the thickness Δz , in which small particles are suspended, changes the electric field $E_0 e^{-i\omega t + ikr}$ into $(E + \Delta E) e^{-i\omega t + ikr}$ and that its variation is expressed by

$$(6) \quad \Delta E = i s E \Delta z.$$

Assuming that a certain differential equation holds for equation (6) within the limit of Δz tending to the infinitesimal, we get by integrating the differential equation

$$(7) \quad E = E_0 e^{isz},$$

where E_0 is the value at $z=0$. The total electric field in this case then results in

$$(8) \quad E e^{-i\omega t + ikr} = E_0 e^{-i\omega t + i(k+s)z}.$$

Equation (8) reveals that propagation constant k without the presence of small particles is altered into $(k+s)$ with their presence. Let α indicate the ratio of $(k+s)$ to k , and we get from equation (4),

$$(9) \quad \alpha \equiv 1 + \frac{s}{k} = 1 + 2\pi N \alpha^3 \frac{\epsilon' - \epsilon}{\epsilon' + 2\epsilon} = 1 + \frac{3Nv}{2} \frac{\epsilon' - \epsilon}{\epsilon' + 2\epsilon}.$$

As for α , it should be remembered that the refractive index is increased

equivalently to α -times by the existence of small particles. This fact has hitherto been unnoticed or even neglected in radiometeorology.

At this stage of discussion some considerations will be given to the above process. From a radiometeorological standpoint only a plane wave would be rightly treated as an incident one.

The fact that the total electric field given by equation (8) has the same amplitude but not the same phase or propagation constant as the primary field is entirely due to the method of carrying out the integration of equation (2), i.e., more weight is given to the phase factor than to the amplitude in the integrand. It should be noted, however, that this procedure has little influence upon the phase or the propagation constant of the resultant field from which the equivalent refractive index is derived in the present case.

The treatment is based on perpendicular incidence upon a slice instead of on oblique incidence in general. Judged from the method of integration of equation (2) oblique incidence is easily found not to lead to a different result, because only the absolute value of propagation constant plays an important role in that case.

Mutual reactions among scattered waves are not exerted in a slice with thickness Δz which is of somewhat differential nature, but are approximately taken into consideration in the integration of slices as shown by equations (6) and (7).

In case the particles vary in size and in number per unit volume, it is obvious from the preceding calculations that average values, such as $\langle N \rangle$, $\langle \alpha^3 \rangle$ or $\langle v \rangle$, should be used instead.

The result indicated by equation (9) is interesting in that it is the same as in WAGNER's theory for sparsely distributed particles concerning the static field.

3. Hydrometer and equivalent refractive index

We have merely to consider α for equivalent refractive index, as the latter is α -times of the refractive index without the presence of particles.

Let w indicate the water-content in the air, and equation (9) becomes in liquid precipitation

$$(10) \quad \alpha = 1 + \frac{3}{2} w \frac{\epsilon' - \epsilon}{\epsilon' + 2\epsilon}.$$

Inserting the dielectric constant of water into equation (10) we get

$$(11) \quad \alpha = 1 + \frac{3}{2} w.$$

Published data in the measurements of waterdrops in the air are too scanty for the use of the present discussion, so that the details of the actual state remain still almost unknown, especially for the vertical profiles. Some actual measurements [3] of waterdrops in the air show that the values of w are of the order of 10^{-6} or less in most cases. Theoretical considerations [3], based on the process of adiabatical airlift, give the same results. Therefore the fraction of the equivalent

refractive index or α is usually of the order of 10^{-6} or less. The fractional value thus estimated rather roughly seems small compared with the significant excess of refractive index causing such an anomalous phenomenon as radio-duct. However, it would be too important to be disregarded in the consideration of discontinuities

Fig. 2. Relation between α and visibility.

at the top of clouds accompanied by rainfall, and also of horizontal variations of refractive index in case of rainfall which occurs between a transmitter and a receiver.

The relation between liquid-water content of fogs or rains and visibility, obtained by RADFORD[3], is also valid for the present case, giving the relation between α and visibility as shown in Fig. 2.

As for such crystallized hydrometeors as snow, few data are available for us at present. The effective volume per unit volume of air, or the bulk density, varies with the nature

of snow. In addition, owing to the shapes of snow and its sizes which are abnormally large at times, it is questionable whether Rayleigh scattering holds for the present case. In any case, however, the effect on refractive index probably could hardly be ignored because of the effective bulk density, although the dielectric constant is smaller for snow than for liquid hydrometeors.

4. Conclusions

In this paper a certain electromagnetic property of a medium, in which small spherical particles are uniformly and sparsely spaced, is presented mainly in its radiometeorological aspects through an approximate procedure. The obtained result coincides with WAGNER's limited to a static field. The ratio α of the equivalent refractive index with the presence of the particles to the refractive index without their presence is defined as a function of the dielectric constants of the particles and the surrounding medium, the number of the particles per unit volume and the volume of one particle, as expressed in equation (9).

In application of the result to the atmosphere with hydrometeors, it is seen that the fractional values of α are in most cases of the order of 10^{-6} or less in liquid hydrometeors, which should not be ignored in certain cases, and that on the other hand the details are still so unknown in solid or crystallized hydrometeors that further studies are needed.

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