The Numerical Prediction of the Typhoon Movement by the Relay Computer

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Abstract

Using the relay computer, numerical predictions of the typhoon movement are performed by the barotropic model successively with many time steps of small time interval. In the problems of the typhoon prediction, the large truncation errors especially near the typhoon center seriously affect the forecast of typhoon movement. So, in this paper, to reduce such truncation errors, a certain smoothing procedure is taken, and the barotropic forecast is applied to the smoothing map at 700 mb and 500 mb level.

1. Introduction

We have had no opportunity to use the high-speed electronic computer for numerical prediction in Japan. Therefore, the numerical computations in all studies concerning numerical prediction, have been performed by hand up to the present. As this hand computation is a very laborious task, it is difficult to calculate 12 hr or 24 hr numerical forecast successively by many time steps of small time intervals as 1 hr or 2 hr. And we are compelled to compare the computed instantaneous height changes with the observed 12 hr or 24 hr ones, to verify the accuracy of numerical prediction. So, it is one of the earnest desires for us Japanese researchers to compute numerical prediction successively by many time steps of small time intervals.

Fortunately, the relay computer (E.T. Mark II) in the Electrotechnical Laboratory has been made available since last summer. And it was used for the numerical prediction of typhoon movements by the Forecast Section of Japan Meteorological Agency in Tokyo under the simple baroclinic model. This model was proposed by T. MURAKAMI, Y. MASUDA and A. ARAKAWA [1] and named the parabolic model in which the vertical- p -velocity is assumed to be parabolic. However, this machine has only 200 memories and the time needed for each computation is considerably long. So, in order to be in time for the routine forecast, the 24 hr prediction was computed by only two time steps of 12 hr time intervals. Then, in spite of using this relay computer, the numerical computation for the routine forecast is not essentially different from the one performed by hand. Therefore, the problem of numerical computation by as small time interval as possible, remains still to be solved. So, in this report, the numerical prediction of the typhoon movement is

calculated by the relay computer with successively many time steps of as small time interval as possible.

The typhoon taken as example in this paper is not so suitable for the parabolic model as described in a later section, so we take up here as the main subject the prediction of the typhoon movement with successively many time steps of as small time interval as possible under the barotropic model. In the case of typhoon prediction, as the typhoon profile is very sharp, truncation errors which arise from the description of the space derivatives by a considerably large grid size such as 300 km, come into question. So, in this paper, in order to reduce such truncation errors as much as possible, a certain smoothing procedure, proposed by MURAKAMI et al. (1956), is taken. After smoothing the initial maps to be used for the prediction, the barotropic forecast is applied at 700 mb and 500 mb level respectively and successive forecast of the typhoon movement is performed.

2. The method of calculation by the relay computer

Our forecasting problems are computed by the same relay computer (E.T. Mark II) that was used in the attempt of the routine numerical forecasting of typhoon movements at Forecast Section of Japan Meteorological Agency in the summer of this year (1956), and we also use the control tapes* prepared for that computation. This machine has a relay memory for only 200 words. Because of the limited capacity and moderate computation speed of this machine our computations are made over the grid of 12×13 points, spaced about 300 kilometers apart along the grid axes, though it is desirable to calculate over a larger area diminishing the effects of the insufficient boundary condition.

The procedure of computation is the same as the method of solution A given by CHARNEY and PHILLIPS [2]. That is, the set of equations leading from time t to time $t + \Delta t$ is

$$(1) \quad \left(\frac{\partial \xi}{\partial t} \right)^t = J(gf^{-1}m^2\nabla^2 Z^t, Z^t),$$

$$(2) \quad \nabla^2 \left(\frac{\partial Z}{\partial t} \right)^t = \left(\frac{\partial \xi}{\partial t} \right)^t,$$

$$(3) \quad Z^{t+\Delta t} = Z^{t-\Delta t} + 2\Delta t \left(\frac{\partial Z}{\partial t} \right)^t,$$

where the change of Coriolis parameter f with latitude is neglected. Here $J(\alpha, \beta) \equiv \frac{\partial(\alpha, \beta)}{\partial(x, y)}$ is the Jacobian of α and β , $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ the Laplacian operator in the plane, ξ the vertical component of vorticity, g the gravitational acceleration, Z the height at a constant pressure level, m the magnification factor for the map, and Z^t etc. denotes the value of Z at the time t . In our computations f and m are regarded as constants.

The Jacobian term in equation (1) is written in the finite difference form as follows:

* The programming and control tapes were prepared by M. FUJINAKA (Electrotechnical Laboratory), K. GAMBO, S. MATSUMOTO and H. ITOO (Meteorological Research Institute).

$$(4) \quad \frac{m^2 g}{f} J(\overline{F^2 Z}, Z) \simeq \frac{m^2}{4d^4} \frac{g}{f} \mathbf{J}(\overline{Z} - 4Z, Z) = \frac{m^2}{4d^4} \frac{g}{f} \mathbf{J}(\overline{Z}, Z),$$

where the bar denotes the sum of Z in the four neighbouring lattice-points, $\mathbf{J}$ the Jacobian with finite difference, and d is the mesh-size. For the sake of simplicity for programming, the Jacobian is directly calculated from the heights stored in the relay memory by the following scheme: (Fig. 1)

$$(5) \quad \begin{aligned} \mathbf{J}(\overline{Z}_0, Z_0) = & \{ (Z_5 + Z_{11} + Z_{12}) \\ & - (Z_7 + Z_8 + Z_9) \} (Z_1 - Z_3) \\ & - \{ (Z_5 + Z_6 + Z_7) \\ & - (Z_9 + Z_{10} + Z_{11}) \} (Z_4 - Z_2). \end{aligned}$$

Fig. 1. Schematic distribution of the grid points using for the calculation of the Jacobian.

In the case where the calculated points are neighbouring to the boundary, at the outside points, for example, the points corresponding to No. 6, 8, 10 or 12 in Fig. 1, the values of height are not given. In that case the values at these points are estimated by linear extrapolation using the neighbouring two points along the same grid axes, and we calculated the Jacobian at the neighbouring points to the boundary using these extrapolated values. Therefore, Jacobians are computed over the grid of 11×10 points.

On the boundary we assume

$$Z(i, j, t) \equiv Z(i, j, 0) \quad \text{for } 1, 2, \dots, 10 \text{ hours,}$$

and for the time since then

$$Z(i, j, t) \equiv Z(i, j, 0) + \frac{1}{2} \{ \delta Z(i-1, j, 10) + \delta Z(i-2, j, 10) \} \times n \Delta t$$

or

$$Z(i, j, t) \equiv Z(i, j, 0) + \frac{1}{2} \{ \delta Z(i, j-1, 10) + \delta Z(i, j-2, 10) \} \times n \Delta t,$$

respectively on the boundary perpendicular to i -axis or j -axis, where δZ is the height change per unit hour, n the number of time step and Δt the time interval. The above-mentioned conventional method of the Jacobian computations at points neighbouring to the boundary and insufficient boundary conditions are used in each time step. Thus, since the errors near the boundary come to extend to the inner part of the grid, these assumptions may affect the forecast considerably. But it is impossible to avoid such a defect, as the capacity of this machine is limited.

The computation speed of this machine is very slow in comparison with that of the electronic computer. It takes about 0.25 seconds for transfer, addition and subtraction, and takes about 2 seconds for multiplication and division. The speed of the tape-reader is 4 words per second and the printer takes about 2 seconds for

a word. Since the routine forecast must be calculated within a limited time, saving the computation time is needed. For this reason the Liebmann process is performed in the following manner: the calculation for each iteration starts from the grid

Fig. 2. Schematic illustration for the sequence of Liebmann process. The symbol 6 represents the initial typhoon centre from which the all iterations are start, the bar represents the final grid point of the third approximation and the double bar, that of the fourth approximation.

point with the symbol 6, corresponding to the initial location of the typhoon center and is made successively point to point in the order shown by arrows in Fig. 2. That is to say, the first and second approximations are computed throughout all points, and the third approximations are calculated from the starting point to the point marked by the bar, and the fourth approximations from the starting point to the point marked by the double bar. A few iterations at the outer part of the grid will not lead to remarkably incorrect results, since, generally speaking, height changes are relatively small near the boundary in case of a typhoon. Using the above-mentioned procedure, about 10 minutes are saved for one Liebmann process. The computation time for the barotropic model is about 50 minutes per one time step in our programming.

As the first attempt to calculate step by step with small time interval, we use the following several time intervals: for the first 12 hr a time interval is 1 hr, for the next 12 hr it is 2 hr, and for the suc-

cessive 12 hr, 4 hr. The height values $Z^{t+\Delta t}$ for $t=1, 2, 3, \dots$ are computed by the equation (3), that is, by the formula of centered difference, but those for $t=0$ are obtained from the equation

$$Z^1 = Z^0 + \Delta t \cdot \left(\frac{\partial Z}{\partial t} \right)^0,$$

where notations are the same as before.

3. Results of the prediction

It is a matter of experience that one of the most difficult parts of the prediction of typhoon movement is the prediction of its recurvature. So, in this report, the numerical forecast is focussed on the prediction of the recurvature of typhoon Harriet, which attacked the Japan Islands on 27 September 1956 after recurving at a point to the south of Okinawa. As the initial maps for the forecast, we select the maps of 700 mb and 500 mb level on 0000 LMT 25 September 1956. As an example, an initial 700 mb level map is shown in Fig. 3.

In this figure, 700 mb height contour at 100 feet intervals, the typhoon track and the location of the grid points, the interval of which is 300 km, are shown.

The small circles on the typhoon track are the positions of the typhoon center at 12 hr intervals. Therefore, the speed of typhoon movement can be roughly estimated, and that of about 18 km/hr before the recurving point and about 28 km/hr after it are obtained.

The contours immediate to the typhoon center are nearly circular, but at the outer region of the typhoon area, they are deformed considerably from the circular and show somewhat eccentric patterns. From these patterns we may presume that the basic current near the typhoon region is considerably far from uniform. We may expect from the result obtained by MURAKAMI, et al. that this kind of non-uniform basic current may produce a large truncation error in the course of numerical computation. So, some procedure must be taken to reduce these errors as much as possible.

At the upper region of this prediction area, a considerably marked trough appears. This trough moves eastward

Fig. 4. The initial smoothed 700mb height contour at 100 feet intervals on 0000 LMT 25 September 1956.

Fig. 3. The initial 700mb height contour at 100 feet intervals on 0000 LMT 25 September 1956 for the typhoon Harriet and locations of the grid points used for the computation. The full line with the arrow in front represents the typhoon track, the symbol 9 the position of the initial typhoon center and circles on the track the position of the typhoon center for 12 hr interval.

with an almost uniform speed in this forecast period, and seems to affect the typhoon movement in some degree.

In order to reduce truncation errors near the typhoon center, MURAKAMI et al. proposed a tentative smoothing procedure. That is, let Z_0 be the height of a certain isobaric level at the grid point where the smoothing procedure is taken and Z_1, Z_2, Z_3 and Z_4 be the heights at the four grid points neighbouring to this point, (see Fig. 1), then the smoothing height $\tilde{Z}_0$ is defined by the following formula:

$$\tilde{Z}_0 = \frac{1}{6} (Z_1 + Z_2 + Z_3 + Z_4 + 2Z_0).$$

In this report, we also adopt the same procedure. Assuming that the barotropic vorticity equation is also applied to this smoothing field automatically, the

barotropic forecasts of this smoothing field are performed. As an example, we present the initial 700 mb smoothing field in Fig. 4. Compared with Fig. 3, it is apparent that the contour patterns in the outer region remain almost the same as those in Fig. 3. In the typhoon area, however, the height contours change markedly and the sharpness of the typhoon profile diminishes greatly.

The predicted 700 mb height contours of the first 12 hr forecast calculated by successive 1 hr time steps and the observed ones which are smoothed by the same smoothing procedure on 1200 LMT 25 September are shown in Fig. 5. In this

Fig. 5. The predicted 700 mb height contour of 12 hr forecast (a) and the observed one on 1200 LMT 25 September 1956 (b) at 100 feet intervals. The symbols 9 represents the predicted and the observed typhoon center after 12 hr and the symbols 6, the initial typhoon center.

figure, the symbols 9 represent the new positions of the predicted and observed typhoon centers, and the symbols 6, the initial typhoon center where the barotropic forecast is started. It is difficult to determine the predicted typhoon center objectively, using the predicted heights on each grid point only. At first, adopting the quadratic equation to the height contour near the typhoon center, and determining the coefficients of this equation by the least-square method using the predicted heights on each grid point, we tried to determine the typhoon center objectively. However, in spite of taking the smoothing procedure, we failed to determine the typhoon center satisfactorily because of the sharpness of typhoon profile near the typhoon center. So, in this paper, the typhoon center is estimated subjectively after drawing the predicted contour patterns.

The typhoon Harriet moved in the north-northwestward direction in this forecast period and, having developed in some degree, was ready to turn to the north-northeastward at the end of this period. From Fig. 5, it is clear that the 12 hr predicted typhoon center moves in the same direction as that of the observed one from the initial position, but shifts somewhat southward from the observed

center. The mean speed of displacement of the predicted typhoon center is about 13 km/hr in this period, and is only about 70 percent of the observed one. However, as the predicted heights near the typhoon center become deeper in some degree than the initial height, the development of typhoon is predicted fairly well.

It is of interest to compare the predicted 12 hr height change with the observed one in order to verify the accuracy of the prediction. Fig. 6 presents the predicted

Fig. 6. The predicted 12hr height change at 25 feet/12hr intervals (a) and the observed one at 50 feet/12 hr intervals (b). The symbols 5 represent the initial typhoon center. Full lines show the positive height change and dashed lines, the negative one.

12 hr height change at 25 feet/12 hr intervals and the observed one at 50 feet/12 hr intervals. The negative center of the predicted 12 hr height change is situated just north of the initial typhoon center and shows 125 feet/12 hr. On the other hand, that of the observed one shifts a little to the east from the predicted one, and reaches 200 feet/12 hr. The positions of the center of the predicted and observed positive height change agree with each other. The maximum values of the negative and the positive center of the predicted height change are both smaller than those of the observed one. This fact means that the mean speed of displacement of the predicted typhoon center is only about 70 percent of the observed one.

At the northern part of this figure, the discrepancy between the predicted and the observed 12 hr height change is considerably large. That is to say, the negative height change caused by the eastward movement of the trough is not predicted very well. This discrepancy is partly caused by the fact that the boundary condition assumed in the course of the calculation is not appropriate, and partly by neglecting the baroclinic effect.

The next 12 hr 700 mb prediction is performed by successive time steps of 2 hr time intervals. The predicted 24 hr contour pattern from the initial map and the corresponding observed one on 0000 LMT 26 September 1956 are shown in Fig. 7.

Fig. 7. The predicted 700 mb height contour of 24 hr forecast (a) and the observed one on 0000 LMT 26 September 1956 (b). The illustrations are the same as in Fig. 5.

It is apparent from this figure that the discrepancy between the positions of the predicted and the observed typhoon center become increasingly larger. However, the position of the predicted typhoon center is situated north-northeastward of the initial typhoon center in the same manner as the observed one, and shifts to the northeast from the position of the preceding 12 hr predicted typhoon center. That is to say, the recurvature of the typhoon movement is predicted in a fairly good sense. On the other hand, the trough near the upper boundary deepens during this forecast period and seems to exert considerable influence on the typhoon movement. However, because of insufficient assumptions concerning the boundary condition, the prediction of this trough is very wrong. This badness of the prediction of the development of this trough may be one of the large causes of the defective prediction of the typhoon movement itself.

The predicted and the observed 24 hr height change at 50 feet/24 hr intervals are shown in Fig. 8. The patterns of the height change in the typhoon region are similar to those in Fig. 6. The maximum value of the negative center of the predicted height change is smaller than that of the observed one. On the contrary, however, the maximum value of the positive center is larger than that of the observed one. The discrepancy between the predicted and the observed height change near the upper boundary becomes large for the reason mentioned above.

The successive 12 hr predicted height contour computed at 4 hr time interval and the corresponding observed one on 1200 LMT 26 September 1956 are presented in Fig. 9. At a glance of this 36 hr after predicted height pattern, it is apparent that considerable irregularity appears in the outer region of the typhoon, especially in the rear of it. This irregularity may come from computational errors accumulating in the course of computation. The position of the predicted typhoon center is situated about 300 km south of that of the observed one. Though the actual

Fig. 8. The predicted 24 hr height change (a) and the observed one (b) at 50 feet/24 hr intervals. The illustrations are the same as in Fig. 6.

Fig. 9. The predicted 700 mb height contour of 36 hr forecast (a) and the observed one on 1200 LMT 26 September 1956. The illustrations are the same as in Fig. 5.

speed of the typhoon gradually increases in this forecast period, this acceleration of the typhoon movement is not predicted sufficiently. This defective prediction of the typhoon movement seems mainly to come from the imperfect prediction of the retrogression of the ridge in front of the typhoon. In fact, in the area of this frontal ridge, the disagreement between the predicted and the observed height pattern becomes increasingly large in this forecast period.

Fig. 10. The predicted 36 hr height change (a) and the observed one (b) at 100 feet/36 hr intervals. The illustrations are the same as in Fig. 6.

The predicted and observed 36 hr height change from the initial map is shown in Fig. 10 at 100 feet/36 hr intervals. As the typhoon deepened to the minimum height during this period, the maximum value of the observed negative height change reaches 700 feet/36 hr, while that of the predicted one only 300 feet/36 hr. This discrepancy between the observed and the predicted negative height change mainly comes from the inaccurate prediction of the typhoon deepening as well as of the typhoon movement. Another disagreement between the predicted and the observed height change appears near the northern boundary for the same reason as mentioned in the case of 12 hr and 24 hr forecast.

The barotropic forecast on the 500 mb level is performed in the same manner as described in the case of 700 mb level. However, in this case the prediction is extended only up to 24 hr after the initial forecast time. As an example, the predicted 24 hr after contour pattern on 500 mb level and the corresponding observed one are presented in Fig. 11. In this case also, the position of the predicted typhoon center is situated to the north-northeast of the initial one and in the same direction as the observed typhoon center 24 hr after. That is to say, the recurvature of the typhoon movement is also predicted fairly well in the case of 500 mb barotropic forecast. However, the discrepancy between the position of the predicted and the observed typhoon center is larger than in the case of 700 mb level forecast. That is to say, the predicted speed of the typhoon movement on the 500 mb level is smaller than that on the 700 mb level. So, we can deduce from this fact that the barotropic level is near the 700 mb level rather than the 500 mb level in this example. From this we may expect that simple baroclinic forecast with the parabolic model, in which the vertical- p -velocity is assumed to be parabolic and the barotropic level is assumed to be 500 mb level, is not suitable for the prediction of 700 mb level. In fact, the results of the 700 mb baroclinic prediction calculated by this simple parabolic model —though it is not shown in this report—

Fig. 11. The predicted 500 mb height contour 24 hr forecast (a) the observed one on 0000 LMT 26 September 1956 (b). The illustrations are the same as in Fig. 5.

is considerably worse than that of the 700 mb barotropic one.

As the summary of this section, the predicted typhoon tracks on the 700 mb and 500 mb level calculated by the barotropic forecast and the observed ones are shown in Fig. 12 and Fig. 13 respectively. In these figures, the full line with an

Fig. 12. The predicted (full line) and the observed (dashed line) typhoon track. The Symbol 6 represents the initial typhoon center, the white small circles, the positions of 12 hr intervals and the back small circle, the positions of 4 hr intervals.

Fig. 13. The predicted (full line) and the observed (dashed line) typhoon track. The illustrations are the same as in Fig. 12.

arrowhead shows the predicted typhoon track and the dashed line with an arrowhead, the observed one. The small white circles on the tracks are the positions of the typhoon center at 12 hr intervals, the small black circles, that of the predicted one at 4 hr intervals and the symbols $\odot$ the initial typhoon center at which the prediction starts.

The typhoon tends to recurve from north-northwest to north-northeast at about 10 hr after 0000 LMT 25 September. At 700 mb level, the predicted typhoon center also tends to recurve in the same manner at the same time as the observed one. And the predicted typhoon track draws almost the same curve as that of the observed one. However, the calculated speed of the typhoon movement is always smaller than the actual one, being only about 70 percent. So the discrepancy between the predicted and the observed typhoon center becomes larger with time. On the other hand, at 500 mb level, though the recurving time of the typhoon movement is predicted with considerable accuracy, its predicted speed is smaller than at 700 mb level. So, the disagreement between the predicted and the observed typhoon center is larger than that of the 700 mb level. This fact suggests that, generally speaking, steering current at 700 mb level is stronger than that at 500 mb level. So we may conclude that, in the case of typhoon Harriet, barotropic level lies near the 700 mb level rather than the 500 mb level in the middle latitudes.

4. Conclusions

Difficulties in numerical predictions of typhoon movement are partly due to the large truncation error near the typhoon center. So, in this paper, the forecasts are applied to the smoothed map with the grid of 300 km mesh-size. As shown in the preceding section, the agreement between the observed and the calculated height changes are fairly good. This shows that the truncation error is sufficiently reduced by the smoothing procedure.

The recurvature of the typhoon track is successfully forecasted. In the subjective prediction the forecast of recurvature has been difficult. That problem seems to be solved by computing the location of the typhoon center step by step with small time intervals using the procedure of numerical prediction.

The calculated velocity of the typhoon is about 70 percent of the observed one. MAGATA [3] shows that the travelling speed is always underestimated, especially in the case of the disturbances with short wave length, when the computation is done with a relatively large grid size, such as 300 km, for instance. Though we apply the smoothing procedure to the contour pattern, the components of short wave length will considerably remain as yet. On the other hand, we use the geostrophic approximations in the problem of typhoons as in large-scale disturbances in middle latitudes. Since the contour of a typhoon has a very large curvature, the actual wind in a typhoon area is not geostrophic. So, the geostrophic assumption considered in this report is a too large assumption for typhoon prediction. Therefore, the underestimation of the predicted typhoon speed is caused partly by the fact that the smoothed contour patterns contain disturbances of relatively short wave length as yet, and partly by the assumption of geostrophic wind near the typhoon region.

Since typhoons have nearly circular symmetry, the solutions of the balance

equation are nearly equal to the gradient wind. At the same time the stream line patterns obtained from the balance equation will be expected not to contain disturbances of relatively short wave length. Thus, in our next studies we are trying to solve a balance equation and forecast a typhoon track by using stream line patterns.

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