Weather Forecast and Entropy in Information Theory

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Abstract

Weather forecast is generally regarded as social information containing a certain kind of uncertainty, and so information theory must be very effective for the verification of forecast technique and forecasted result.

The author tried to introduce the idea of entropy in information theory to the various problems of weather forecast, and could show that the several verification measures hitherto known were systematically identified to the ideas of relative entropy or transmitted amount of information in information theory.

Moreover, as a by-product of such induction process, a simplified method to estimate correlation coefficient could be offered, and an effective procedure to issue weather forecast under several restricting conditions was gained by the principle of maximum total entropies.

Lastly, the hypothetical examples of such procedure and actual situations were schematically shown for the daily temperature forecasting etc.

1. Introduction

The economic utility or the operations research of weather forecasts were recently discussed by THOMPSON and others.

Verification indices of weather forecasts (results) were already studied by many authors, but their quantitative measure of uncertainty in view of operations research is not yet established.

In this paper, the author tried to introduce the entropy ideas in information theory as the quantitative measure of uncertainty in weather forecast, and showed the effective usefulness of such entropy representations in the systematic identifications and specifications of the verification indices obtained already.

Moreover, as a more effective method for issuing weather forecast, the author proposes to adopt the entropy maximum principles in view of the cost needed for weather forecast, and denoted several examples of weather forecasts as an application of such principles schematically.

2. Specification of entropy idea in information theory for the verification of weather forecast

The multi-variate probability distribution is generally denoted by

$$F(x_1, x_2, \dots, x_k; \theta_1, \theta_2, \dots, \theta_h),$$

where, $x_1, x_2, \dots, x_k$ are stochastic variables and $\theta_1, \theta_2, \dots, \theta_h$ are population parameters of distribution respectively.

Average amount of information (i.e., entropy of information)

$$(2.1) \quad H = - \int \log F dF,$$

is exactly the function of $\theta_1, \theta_2, \dots, \theta_h$ (i.e. $H(\theta_1, \theta_2, \dots, \theta_h)$), but the author will write as $H = H(x_1, x_2, \dots, x_k)$ in conformity with the ordinary custom.

The following recurrence relations are satisfied (without proof)

$$\begin{aligned} H(x_1, x_2) &= H(x_1) + H_{x_1}(x_2) \\ H(x_1, x_2, x_3) &= H(x_1, x_2) + H_{x_1 x_2}(x_3) = H(x_1) + H_{x_1}(x_2) + H_{x_1 x_2}(x_3) \\ H(x_1, x_2, \dots, x_k) &= H(x_1, x_2, \dots, x_{k-1}) + H_{x_1 x_2 \dots x_{k-1}}(x_k) \\ &= H(x_1, x_2, \dots, x_{k-2}) + H_{x_1 x_2 \dots x_{k-2}}(x_{k-1}) \\ &\quad + H_{x_1 x_2 \dots x_{k-1}}(x_k) \\ &\dots\dots\dots \\ &= H(x_1) + H_{x_1}(x_2) + H_{x_1 x_2}(x_3) \\ &\quad + H_{x_1 x_2 \dots x_{k-2}}(x_{k-1}) + H_{x_1 x_2 \dots x_{k-1}}(x_k), \end{aligned}$$

where, $H_{x_1}(x_2), H_{x_1 x_2}(x_3), \dots, H_{x_1 x_2 \dots x_{k-1}}(x_k)$ are the conditional entropy of information with fixed variate $x_1; x_1, x_2; \dots; x_1, x_2, \dots, x_{k-1}$ respectively.

In the case of multi-variate normal distribution

$$(2.2) \quad F'(x_1, x_2, \dots, x_k; \theta_1, \theta_2, \dots, \theta_h) = \frac{\sqrt{|A|}}{(2\pi)^{\frac{k}{2}}} e^{-\frac{1}{2} \sum_{i,j=1}^k A_{ij} (x_i - \mu_i) (x_j - \mu_j)},$$

where

$$\begin{aligned} |A| &= \det |A_{ij}|, \quad A^{ij} = \sigma_i \sigma_j \rho_{ij} \quad |A^{ij}| = |A_{ij}|^{-1} \quad (i, j = 1, 2, \dots, k) \\ \mu_i &= \text{expectation of } x_i, \quad \sigma_i^2 = \text{variance of } x_i \\ \rho_{ij} &= \text{correlation coefficient between } x_i \text{ and } x_j. \end{aligned}$$

Entropy function can be expressed as follows

$$\begin{aligned} H(x_i) &= \log_e \sqrt{2\pi e} \sigma_i \quad (i = 1, 2, \dots, k), \\ (2.3) \quad H(x_1, x_2, \dots, x_k) &= \log_e \frac{(2\pi)^{\frac{k}{2}}}{\sqrt{|A|}} + \frac{1}{2} \sum_{i,j=1}^k A_{ij} \sigma_i \sigma_j \rho_{ij}, \\ (2.4) \quad H_{x_1 x_2 \dots x_{k-1}}(x_k) &= \log_e \sqrt{2\pi e} \sigma_k \sqrt{1 - R_{k \cdot 12 \dots k-1}^2}, \end{aligned}$$

($R_{k \cdot 12 \dots k-1}$ = multiple correlation coefficient between x_k and $x_1, x_2, \dots, x_{k-1}$).

(a) Verification of forecast technique

In the statistical forecasting (linear least square prediction procedure), the above stochastic variables $x_1, x_2, \dots, x_k$ are specified as

(b) *Verification of forecasted results*

When we assume $k=2$ in (2.5) and x_1 =actual value (or observed value), x_2 =forecasted value, conditional entropy $H_{x_1}(x_2)$ or conditional relative entropy $I(x_1, x_2)$

$$(2.7) \quad I(x_1, x_2) = H(x_2) - H_{x_1}(x_2) = H(x_1) + H(x_2) - H(x_1, x_2) = I(x_2, x_1)$$

can be adopted as the rigorous verification measure of forecasted results.

In the case of the two variate normal distribution, a formally similar relation can be easily induced as follows:

$$(2.8) \quad I(x_1, x_2) = -\log \sqrt{1 - \rho^2(x_1, x_2)},$$

(where $\rho(x_1, x_2)$ is the correlation coefficient). Generally we can consider $H_{x_1}(x_2)$ and $I(x_1, x_2)$ as the verification measure of weather forecast from the viewpoint of Information theory, and such consideration can essentially correspond to $\sigma\sqrt{1-\rho^2}$ and ρ (correlation coefficient between forecast values and observed (or readed) values.) which have been used to verify the forecasted result. So $H_{x_1}(x_2)$ and $I(x_1, x_2)$ are the more general exclusive concepts containing the verification measure, obtained already.

Moreover, in order to test the mutual independency of forecasted and observed values, we have already qualified verification measure χ^2 for the corresponding contingency tables consisting of categorically forecasted values and corresponding observations, and even such measure χ^2 can be reduced as a special case of I as follows.

Table 1. Schematic contingency table of verification.

forecast observed	A_1	A_2	A_3		A_r	Total
A_1	n_{11}	n_{12}	n_{13}		n_{1r}	$n_{1.}$
A_2	n_{21}	n_{22}	n_{23}		n_{2r}	$n_{2.}$
A_3	n_{31}	n_{32}	n_{33}		n_{3r}	$n_{3.}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
A_r	n_{r1}	n_{r2}	n_{r3}		n_{rr}	$n_{r.}$
Total	$n_{.1}$	$n_{.2}$	$n_{.3}$		$n_{.r}$	$n_{..}$

As already well known, we have the following formula for the test of the contingency table (Table 1):

$$(2.9) \quad \chi^2 = \sum_{ij=1}^r \frac{\left(n_{ij} - \frac{n_{i.}n_{.j}}{n}\right)^2}{(n_{i.}n_{.j}/n)},$$

and likelihood function λ has the well-known asymptotic property

$$(2.10) \quad -2 \log \lambda \rightarrow \chi^2 \quad (n \rightarrow \infty).$$

On the other hand we can prove the following formula without neglecting any terms for the sample value of I :

$$(2.11) \quad I = +\log \lambda/n,$$

$$\begin{aligned}
 (\because \lambda &= [\text{Max}_{p_{ij}} \pi p_{ij}^{n_{ij}}] / [\text{Max}_{p_i q_j} \pi p_i^{n_i} \cdot \pi q_j^{n_j}] \\
 &= \pi (n_{ij}/n)^{n_{ij}} / \pi_i (n_i/n)^{n_i} \cdot \pi_j (n_j/n)^{n_j}, \\
 -\log \lambda &= -\sum_{ij} n_{ij} \log (n_{ij}/n) + \sum_i n_i \cdot \log \frac{n_i}{n} + \sum_j n_j \log \frac{n_j}{n} \\
 &= [H(x_1, x_2) - H(x_1) - H(x_2)] n = -nI(x_1, x_2),
 \end{aligned}$$

and so I of sample contains χ^2 as the special case that the verification measure I of categorically forecasted and observed values essentially reduce χ^2 when n tends to infinity.

Other verification measures which were gained already can be reduced to conditional entropy $H_{x_1}(x_2)$ or conditional relative entropy $I(x_1, x_2)$ by slight deformations (not essential) or taking the special cases of such entropy idea.

After all, the various verification measures can be unified to the entropy idea in Information theory.

3. A simplified procedure of the estimation of correlation coefficient as a by-product

As a by-product of the induction process of the entropy idea in weather forecasting, a more simplified technique estimating correlation can be easily suggested.

As is already shown, conditional relative entropy (amount of transmitted information) I is denoted by

$$\begin{aligned}
 I(x_1, x_2) &\doteq \frac{1}{n} (\sum_{ij} n_{ij} \log n_{ij} + n \log n - \sum_i n_i \cdot \log n_i - \sum_j n_j \log n_j) \\
 &= -\log \sqrt{1 - \rho^2} \quad (\rho = \text{correlation coefficient between } x_1 \text{ and } x_2).
 \end{aligned}$$

Therefore, the author readily prepared the table consisting of n and $n \log_2 n$ ($n = 1 \sim 1000$), and the figure to estimate ρ from I .

Table 2. Contingency Table of factor x_1 and x_2

x_1/x_2	A_i	Total
A_j	$n_{ij} (A)$	$n_j (B)$
Total	$n_i (C)$	$n (D)$

Already S. GOLDMAN (1953) proposed the table consisting of n and $\log_2 n$ ($n = 1 \sim 10000$) as the preparation of entropy calculation, and the author could make out the required Table of $n \log_2 n$ according to his Table. So as to estimate correlation coefficient ρ , firstly $n_{ij} \log_2 n_{ij}$, $n \log_2 n$, $n_i \cdot \log_2 n_i$ and $n_j \log_2 n_j$ must be read by the above prepared table for n_{ij} , n , n_i and n_j respectively, and I can be easily calculated by the procedure,

$$I = \frac{1}{\text{total number } (n)} [(\text{Summation of } n_{ij} \log n_{ij} \text{ in the column } A + n \log n \text{ in the column } D) - (\text{Summation of } n_i \cdot \log n_i \text{ in the column } C + n_j \log n_j \text{ in the column } B)].$$
 Secondly, ρ can be read by Fig. 4 (sign of ρ is intuitively determined in the contingency table). Actually, the estimated value of correlation coefficient is sufficiently close to the exact value of it (calculated by exact definition formula) (difference between such two values is at most within ± 0.03 in the case of two variate normal distributions).

Several examples of such technique given by the author as following Table.

Table 3. Correlation coefficients of forecasted values and actual values in the several numerical predictions.

Factors	Exact value of $\hat{\rho}$	Estimated value of $\hat{\rho}$	Sample size n
24 hr prediction by ϕ	0.60	0.63	120
" " Z(500 mb)	0.78	0.79	120
" " Z(600 mb)	0.85	0.88	120

* Exact values are due to T. Murakami. ϕ =stream function.

For this procedure, the sampling error of estimated values is not yet fully discussed. The problems of such sampling errors and efficiency will be examined some day.

4. Effective issue of weather forecast

In order to formulate the operational effect of weather forecast, the author tried to use the following principles of maximum total entropy (Ideally perfect effective use of weather forecast must be discussed by the consideration of its various economic utilities).

Numerous forecasts for the various meteorological and climatological elements are actually issued at present from the Forecast Section of Japan Meteorological Agency under conditions of a limited staff and observation. Hence the following notation for the probabilistic treatment of this problem can be used.

X_i =Meteorological or Climatological element (or phenomenon) to be forecasted on.

p_{i0} =Empirical occurrence probability of X_i (i.e. predictability without technical cost in forecasting),

p_i =the best confidence probability attainable by the sufficient use of the cost* in the present meteorological knowledge for weather forecasting,

c_i =cost available for the forecasting of X_i ,

$p_i(c_i)$ =confidence probability of forecasted X_i by the use of cost c_i .

Fig. 2. Schematic illustration of the relation between p_i and c_i .

Then, the Schematic representation of such situation can be shown as an example in Fig. 2.

When the total costs of weather forecasting $\sum_i c_i$ are limited, and the possible upper limit of cost A must not be exceeded by $\sum_i c_i$ (i.e., $\sum_i c_i \leq A$), we must make the following U maximum for the purpose of maximizing the total amount of information

$$\sum_i \log \frac{p_i(c_i)}{p_{i0}} \text{ under the restriction } \sum_i c_i = A,$$

* Such cost contains, as a rule, the numerous efforts of the staff and the equipment cost to be used.

$$(4.1) \quad U = \sum_i \log \frac{p_i(c_i)}{p_{i0}} + \lambda (\sum_i c_i - A),$$

where, λ = Lagrange's indeterminate multiplier. One of the curves $p_i(c_i)$ satisfying the above representation is evidently the logistic curve like the following :

$$(4.2) \quad p_i(c_i) = p_{i0} p_{i1} e^{k_i c_i} / [p_{i1} + p_{i0} (e^{k_i c_i} - 1)] \quad (i=1, 2, \dots, N),$$

where k_i = a constant referring to X_i .

Actual meaning of this formulation can be explained as follows. In this formula, k_i is regarded as a constant k for the forecast of the same element.

So, intuitively, the following case may be taken to explain the above functional representation of p_i and c_i :

$$\begin{array}{lll} \text{a)} & p_{i0}=0.1 & p_i=0.6 & p_{i1}=0.9 \\ \text{b)} & p_{i0}=0.3 & p_i=0.8 & p_{i1}=0.9. \end{array}$$

The two costs which are needed to forecast for a) and b) are almost equal, but the merit of information from a) is evidently twice as large as that of information from b), and actually it is rather difficult to issue weather forecast with reliable probability $p_i=0.8$ for the case of $p_{i0}=0.3$, but it is not so difficult to attain the reliable probability $p_i=0.6$ for the case of $p_{i0}=0.1$ in general.

And as the probability p_i increases with the cost c_i , constant little improvement rapidly becomes difficult.

The above schematic formula can be theoretically solved for the case of $k_i=k=\text{const.}$ as follows :

$$(4.3) \quad \begin{cases} \frac{\partial U}{\partial c_i} = \lambda + \frac{1}{p_i} \frac{\partial p_i}{\partial c_i} = 0 & (i=1, 2, \dots, N) \\ \frac{1}{p_i} \frac{\partial p_i}{\partial c_i} = k \left(1 - \frac{p_{i0} e^{k c_i}}{p_{i1} + p_{i0} (e^{k c_i} - 1)} \right). \end{cases}$$

If we put

$$F = N^{-1} \left(kA - \sum_i \log \frac{p_{i1} - p_{i0}}{p_{i0}} \right),$$

then the solution is

$$(4.4) \quad \begin{cases} p_i = p_{i0} \exp[k(e^F + 1)^{-1} c_i] \\ p_i = p_{i0} p_{i1} e^{k c_i} / [p_{i1} + p_{i0} (e^{k c_i} - 1)]. \end{cases}$$

Similarly expanded results for the case of $k_i \neq \text{const.}$ (for example, k_i is a function of c_i) can be shown as a more complicated form. After all, if the actual values of parameters k_i , p_{i0} , p_{i1} ($i=1, 2, \dots, N$) and A are known by the any suitable method, an operational optimum policy of weather forecast will be formally established by the use of the above results.

In spite of the fact that the ideal probability (p_{i1}) can hardly be known on account of various complicated conditions, transmitted information amount $\log(p_i/p_{i0})$ is generally apt to be large when p_{i0} is small (about ≤ 0.2), but not large when p_{i0} is not small (about $\leq 0.5 \sim 0.6$) even if p_i is large (about ≈ 0.8), so long as we are concerned with the information theory.

So, if we are allowed to express the above situation in an extreme form, the daily forecast of clear weather is generally not favourable for the forecaster, but the suitable forecast with the usual accuracy for the comparatively rare events in the atmosphere (gust etc.) is rather profitable to the public and favourable for the forecaster.

5. A hypothetical example of Section 4

In the case that the daily mean values of temperature x_i are forecasted with the confidence probability of 75% (that is the probability that the actual value falls within the estimated value $\pm 2^\circ\text{C}$) over one month ($N=30$ days), the total transmitted information amount will easily be $\sum_{i=1}^{30} \log(0.75/p_{i0})$, where p_{i0} is the probability of the daily data of temperature fallings climatologically within the forecated value of the i th day $\hat{T}_i \pm 2^\circ\text{C}$.

Maximum of p_{i0} will evidently be presented when $\hat{T}_i = \bar{T}_i$ (climatologically normal value of the i th day), so the value of $\sum_{i=1}^{30} \log(0.75/p_{i0})$ is minimum when $\hat{T}_i = \bar{T}_i$ ($i=1, 2, \dots, 30$) (Properly speaking, p_{i0} must be estimated for each forecasted value $\hat{T}_i$ as the probability that T_i falls climatologically within $\hat{T}_i \pm 2^\circ\text{C}$).

When the standard deviation of daily mean value $\sigma_{\bar{T}}$ equals to 2°C , maximum value of p_{i0} equals to 0.68 approximately.

$$\text{Min} \sum_{i=1}^{30} \log(0.75/p_{i0}) \doteq 30 \times \log(0.75/0.68) \doteq 4.2 \text{ bits.}$$

Total transmitted amount of information by the daily forecast over one month is at least greater than 4.2 bits. Similarly, if we assume $p_i=0.80$,

$$\text{Min} \sum_{i=1}^{30} \log(0.80/p_{i0}) \doteq 6.7 \text{ bits.}$$

On the other hand, if the possible maximum confidence probability p_{i1} is assumed as 0.9 and the climatological confidence probability p_{i0} is assumed as 0.68, the term referring to the cost $e^{k_i c_i}$ can be estimated for $p_i=0.68, 0.75, 0.80$ and 0.85 respectively. And the actual standard deviation of daily mean values of temperature has

Table 4. Increasing rate of p_i corresponding the increase of cost c_i

p_i	$\exp[k_i c_i]$
0.68	1.00
0.75	1.65
0.80	2.59
0.85	5.51
0.90	∞

small in the seacoast parts of Japan, but large in the inland parts.

After all, the greater part of standard deviation of daily mean temperature is generally considered as larger than 2°C throughout the year as a rule (exact situation must be compared with actual calculation of such statistics), and so the forecast of daily mean temperature with success probability 75% allowing the error

not yet regularly calculated for all the weather stations in Japan, but Mr. YOSIMUCHI has shown his calculated result for the data of Hiroshima in summer season as in Fig. 3, and Mr. Tōjō has suggested to the author that the standard deviation of daily minimum temperature of Mae-basi in Spring equals to 3°C approximately, and in general, standard deviation $\sigma_{\bar{T}}$ is comparatively small in summer and winter, but is comparatively large in spring and autumn, and moreover, standard deviation of temperature is generally

$\pm 2^\circ\text{C}$ will be available for the public, while, on the other hand, the increase of p_i is quite desirable when σ_T is comparatively small. However, in order to increase actually the probability p_i , we must take into account the various complicated factors and conditions involved in the daily forecast of the daily mean value of temperature,

Fig. 3. An example of standard deviation of daily mean temperature of Hiroshima in Summer (due to Mr. YOSIMUCHI).

so the coefficient k_i in the schematic model mentioned already will actually become the very complicated function c_i , p_i and p_{i1} etc, and the actual determination of such functions for k_i will remain a future problem until suitable data of the necessary cost of forecast are sufficiently accumulated.

Suffice it, therefore, that the following hypothetical examples are considered. We have the following formula to determine the values of c_i ($i=1, 2, \dots, N$) from (4.2) and (4.4):

$$c_i = \frac{e^F + 1}{k} \log_e \frac{p_i}{p_{i0}} = \frac{e^F + 1}{k} \log \left[\frac{p_{i1} e^{k c_i}}{p_{i1} + p_{i0} (e^{k c_i} - 1)} \right] \\ = (e^F + 1) k^{-1} [k c_i + \log p_{i1} - \log \{p_{i1} + p_{i0} (e^{k c_i} - 1)\}].$$

Ex 1. The case of $A=100$, $k=0.6$, $p_{i1}=0.9$, $p_{i0}=0.3$, $N=30$.

$$F = (0.6 \times 100 - 30 \log_e 2) / 30 = 2 - \log_e 2 \doteq 1.3$$

$$c_i \doteq 3.3 \quad p_i \doteq 0.49$$

$$\sum_i \log_2 \frac{p_i}{p_{i0}} = \frac{kA}{e^F + 1} \log_2 e \doteq 20 \text{ bits (Maximum amount of information).}$$

So the forecaster issues the forecasts with probability of success ~ 0.5 by using cost about 3.3 over the 30 times of a certain period.

(For example, the daily forecast of rainfall at a certain rainy station over one month will probably be a case in point).

Ex 2. The case of $A=10$, $k=0.5$, $p_{i0}=0.3$, $p_{i1}=0.9$, $p_{20}=0.2$, $p_{21}=0.9$ and $N=2$.

$$F = \frac{1}{2} \left(10 \times 0.5 - \log \frac{0.9-0.3}{0.3} - \log \frac{0.9-0.2}{0.2} \right) \doteq 2.5 - \log \sqrt{7},$$

$$c_1 \doteq 4 \quad c_2 \doteq 6 \quad p_1 \doteq 0.42 \quad p_2 \doteq 0.35 \quad (p_i \doteq p_{i0} e^{0.072 c_i} \quad i=1, 2)$$

Maximum amount of information $\doteq 1.3$ bits.

So the forecaster issues the forecast with $p_1 \sim 0.42$ by using cost 4 for the predictand having $p_{i0}=0.3$, $p_{i1}=0.9$ and issues the forecasts with $p_2 \sim 0.35$ by using cost 6 for the predictand having $p_{20}=0.2$, $p_{21}=0.9$ respectively.

6. Summary and Conclusion

The author tried to introduce the idea of information theory to the several problems of weather forecast, and the main results can be summarized as follows:

1) The verification techniques of weather forecast gained already, can be identified by the idea of conditional entropy or relative conditional entropy, that is, the various verification measures already obtained can be expressed as the special case or the slight deformation (not essential) case of the idea of entropy.

Table 5. The values of n and $n \log_2 n$.

n	$n \log_2 n$	n	$n \log_2 n$	n	$n \log_2 n$	n	$n \log_2 n$	n	$n \log_2 n$
1	0	51	289.30005	101	672.49335	155	1127.8234	510	4587.2201
2	2.00000	52	296.42860	102	680.60214	160	1171.5344	520	4691.7260
3	4.75497	53	303.58718	103	688.72495	165	1215.4675	530	4796.5318
4	8.00000	54	310.77000	104	696.85928	170	1259.6235	540	4901.5800
5	11.60990	55	317.98140	104	705.01095	175	1303.9915	550	5006.9140
6	15.51006	56	325.21944	106	713.17648	180	1348.5618	560	5112.5144
7	19.65194	57	332.48100	107	721.35120	185	1393.3349	570	5218.3500
8	24.00000	58	339.77038	108	729.54216	190	1438.3019	580	5324.4638
9	28.52991	59	347.08284	109	737.74906	195	1483.4586	590	5430.8084
10	33.22000	60	354.46080	110	745.96500	200	1528.8040	600	5537.4060
11	38.05450	61	361.78307	111	754.19505	205	1574.3241	610	5644.2507
12	43.02048	62	369.16784	112	762.44112	210	1620.0261	620	5751.3184
13	48.10663	63	376.57683	113	770.69729	215	1665.8995	630	5858.6283
14	53.30416	64	384.00000	114	778.96428	220	1711.9344	640	5966.1632
15	58.60455	65	391.46185	115	787.24975	225	1758.1365	650	6073.9185
16	64.00000	66	398.93766	116	795.54424	230	1804.5041	660	6183.3420
17	69.48835	67	406.43540	117	803.85201	235	1851.0221	670	6290.0940
18	75.06018	68	413.95680	118	812.16922	240	1897.6944	680	6398.5280
19	80.71219	69	421.49754	119	820.50262	245	1944.5209	690	6507.1554
20	86.44040	70	429.05940	120	828.84480	250	1991.4900	700	6615.9940
21	92.24061	71	436.64219	121	837.20263	255	2038.6051	710	6725.0419
22	98.10944	72	444.24360	122	845.56585	260	2085.8578	720	6834.2760
23	104.04441	73	451.86635	123	853.94964	265	2133.2606	730	6943.7235
24	110.04144	74	459.50892	124	862.33816	270	2180.7846	740	7053.3692
25	116.09900	75	467.17125	125	870.74250	275	2228.4487	750	7163.2125
26	122.21378	76	474.85180	126	879.15618	280	2276.2516	760	7273.2380
27	128.38446	77	482.55284	127	887.58014	285	2324.1693	770	7383.4684
28	134.60916	78	490.27056	128	896.00000	290	2372.2261	780	7493.8656
29	140.88461	79	498.01047	129	904.46802	295	2420.3983	790	7604.4847
30	147.20970	80	505.76480	130	912.92630	300	2468.6970	800	7715.2480
31	153.58330	81	513.54000	131	921.39767	310	2565.6530	810	7826.2200
32	160.00000	82	521.32976	132	929.87796	320	2663.0752	820	7937.3376
33	166.46817	83	529.13994	133	938.37219	330	2760.9417	830	8048.6594
34	172.97772	84	536.96664	134	946.87482	340	2859.2572	840	8160.1464
35	179.52900	85	544.81005	135	955.38960	350	2957.9900	850	8271.8005
36	186.12108	86	552.67126	136	963.91632	360	3057.1308	860	8383.6326
37	192.75372	87	560.54883	137	972.45066	370	3156.6772	870	8495.6283
38	199.34514	88	568.44128	138	980.99784	380	3256.6114	880	8607.7728
39	206.13541	89	576.35243	139	989.55212	390	3356.9250	890	8720.1043
40	212.88162	90	584.27910	140	998.12160	400	3457.6160	900	8832.5910
41	219.66406	91	592.22163	141	1006.6991	410	3558.6606	910	8945.2363
42	226.48248	92	600.18132	142	1015.2872	420	3660.0648	920	9058.0532
43	233.33477	93	608.15397	143	1023.8857	430	3761.8077	930	9170.9997
44	240.21976	94	616.14556	144	1032.4901	440	3863.8776	940	9284.1356
45	247.13820	95	624.14905	145	1041.1101	450	3966.2820	950	9397.3905
46	254.08974	96	632.16960	146	1049.7356	460	4069.0174	960	9510.8160
47	261.07184	97	640.20485	147	1058.3764	470	4172.0584	970	9624.3885
48	268.08384	98	648.25726	148	1067.0208	480	4275.3984	980	9738.1326
49	275.12716	99	656.32248	149	1075.6816	490	4379.0516	990	9852.0048
50	282.19900	100	664.40000	150	1084.3455	500	4482.9900	1000	9966.0000

Fig. 4. Diagram to estimate $-\log_2 p$, $-p \log_2 p$ and ρ (correlation coefficient) from p . (ρ can be estimated from $-\log_2 p$, $p = \sqrt{1 - \rho^2}$). The right hand and left hand of vertical axis AB denote the value of $-\log_2 p$ and $-p \log_2 p$ respectively.

2) The idea of entropy is intuitively easier to understand than the other idea of verification, i.e. all formulations by the idea of entropies can be calculated in expressing by the number of bits (1 bit shows the amount of information for two equally likely events and N bits generally show the amount of information for the 2^N equally likely events).

3) Numerical calculations of these entropies and the estimation of correlation are simply made by using Table 5 and Fig. 4.

4) Comparatively accurate forecasts for the rare events are generally favourable for the forecasters and necessary for the public uses, and such situation may be quantitatively explained by information theory.

5) The effective use of cost A referring weather forecasts when A is divided into c_i ($i=1, 2, \dots, N$, $\sum_i c_i = A$) for the issuing of the various forecasts, could be generally formulated by using the maximum entropy principle and solved by Lagrange's indeterminate multiplier, but the actual application of such solution remains a future problem.

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天気予報と情報のエントロピー

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情報理論を天気予報に応用する研究の第1歩として、次の諸点を調査した。

1. 天気予報は不確実さを含んだ1つの情報であり、情報の価値は定量化したエントロピーその他の確率的測度を導入することにより、これまでの検証方式を統一的立場でまとめることができる。
2. 分割表から相関係数を簡単に推定する方法を図表化した(1つの副産物)。
3. 天気予報の効果的な出し方を情報理論の立場から定式化し、仮説的な例によつてその意味を説明した。ただ、情報量は情報の経済的価値を直接反映するものでないから、この報告で用いた“最大情報の原理”は1つの尺度であつて、万能の尺度ではない。