Magnitude of Deep-focus Earthquakes in and near Japan

by

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Abstract

The writer tried to obtain the same M as those given by the Pasadena seismological laboratory for the deep-focus earthquakes in and near Japan by using the seismological data obtained at Japanese stations.

And the writer succeeded in finding a method to get M with a difference of ± 0.3 unit of magnitude as compared with the Pasadena determination.

It is shown that the calculated $\log A/T - \Delta$ curves for deep earthquakes of various depths on the basis of Norman Ricker's wavelet theory fit well with the observed values.

It is shown that the magnitude of deep earthquakes can be given by the formula,

$$0.63M = \log A_0/T_0 + \alpha(H),$$

so far as the magnitude does not exceed 7.

If M exceeds 7, we must use the other formula,

$$\{0.63 + 0.08(\log A_0/T_0 + \alpha(H) - 4.4)\}M = \log A_0/T_0 + \alpha(H).$$

And $\alpha(H)$ is given by the formula

$$\alpha(H) = 2.5 \log t_0 - 2.8,$$

where t_0 is the travel time of the S phase at the epicenter.

It is shown that the station corrections for deep earthquakes are quite different from those for shallow earthquakes.

Some examples are shown to suggest that there is the azimuthal factor among the observed $\log A/T$, having some intimate connections with the mechanism of the occurrence of the earthquakes.

Some theoretical considerations are made on the formula

$$\log A_0/T_0 = 0.63M - \alpha(H).$$

1. Introduction

The magnitudes were given for many deep-focus earthquakes which occurred in and near Japan by the Pasadena seismological laboratory (See B. GUTENBERG and C. F. RICHTER, 1954).

B. GUTENBERG and C. F. RICHTER (1956) say, "Until 1954, the writers generally reported magnitudes for large deep shocks, and for large shallow teleseisms as determined from body waves, effectively in terms of M_s , first determining m_B and then correcting to M_s by applying equation

$$M_s - m_B = a(M_s - b),$$

or an earlier approximation to it. The correction was usually applied only to shocks of magnitude 7 or over."

Further, they say, "For a number of years reductions were carried out with $a=1/4$, $b=7$, converting values of m_B into the corresponding M_s ."

Where M_s is the magnitude determined from the amplitudes of surface waves for shallow teleseisms and m_B is found from the amplitude/period ratio of body waves for teleseisms, shallow and deep-focus. As shown above, we must remember that the magnitude of a deep-focus earthquake is given effectively in terms of M_s .

Prof. H. HONDA (1951), Dr. T. HIRONO and Mr. Y. IWAI (1952), Dr. K. WADATI and Dr. T. HIRONO (1956) and Mr. K. KATSUMATA (1958) tried to obtain the same M as those given by the Pasadena seismological laboratory as close as possible for the deep-focus earthquakes in and near Japan by using the seismological data obtained at Japanese stations.

They succeeded in finding methods to get M with a difference of ± 0.3 unit of magnitude as compared with the Pasadena determination.

The writer, also, studied the same subject taking it into consideration that the Pasadena magnitude was given effectively in terms of M_s . The writer considered the relation between the common logarithm of the amplitude/period ratio at the epicenter and the corresponding magnitude, and obtained an equation

$$\log A_0/T_0 = 0.63M - \alpha(H).$$

Where A_0 and T_0 are respectively the maximum amplitude in microns and the period in seconds of the wave with the maximum amplitude at the epicenter and $\alpha(H)$ is a constant for a given focal depth H . And the writer succeeded in getting M with the same accuracy as his predecessors.

2. Station correction

As is well known, the common logarithm of the amplitude/period ratios obtained at many stations in case of a deep earthquake scatter about its mean value at each epicentral distance more intensely than in case of a shallow earthquake.

The reason of the scattering might be found in the difference among the geological structures around stations, that is the matter connected with the station correction, as well as in the azimuthal difference in the intensity of the wave emission at the hypocenter. The latter will be discussed in a later article.

After several trials, the writer determined the station correction to be added to $\log A/T$ observed at each station for deep earthquakes, and could lessen the scattering of the observed values to one half of the original one by applying these corrections. A/T in μ/sec is the mean value of the two horizontal components.

The station corrections are shown in Table 1.

Table 1. Station correction *C*.

Station	<i>C</i>	Station	<i>C</i>	Station	<i>C</i>
Aikawa	+0.07	Kumagaya	-0.07	Osaka	-0.08
Akita	-0.38	Kumamoto	-0.16	Oshima	-0.01
Aomori	-0.68	Kyoto	+0.18	Saga	-0.59
Chichibu	-0.05	Maebashi	+0.13	Sakata	-0.45
Choshi	-0.25	Matsumoto	+0.10	Sapporo	-0.19
Fukui	+0.04	Matsushiro	+0.70	Sendai	-0.31
Fukushima	-0.48	Matsuyama	+0.19	Shionomisaki	+0.28
Fukuoka	+0.07	Mishima	-0.18	Shimizu	+0.41
Fukuoka (B)	0	Mito	-0.51	Shirakawa	-0.31
Funatsu	-0.15	Miyako	-0.35	Shizuoka	+0.31
Gifu	+0.10	Miyazaki	+0.10	Sumoto	+0.51
Hachinohe	-0.08	Mizusawa	-0.60	Takada	-0.25
Hamada	+0.55	Mori	-0.47	Tokyo	-0.06
Hamamatsu	+0.10	Morioka	-0.38	Tomie	+0.82
Hikone	-0.16	Murotomisaki	+0.56	Tomisaki	-0.09
Hiroshima	+0.52	Nagano	0	Toyama	+0.21
Inawashiro	-0.40	Nagoya	-0.08	Toyooka	+0.24
Ishinomaki	-0.19	Niigata	-0.45	Tsuruga	+0.25
Kagoshima	-0.32	Numazu	-0.23	Ustunomiya	-0.55
Kakioka	-0.69	Nagasaki	+0.49	Uwajima	+0.01
Kameyama	+0.14	Oita	-0.35	Wajima	-0.40
Kanazawa	-0.09	Oiwake	+0.02	Wakayama	+0.29
Kobe	+0.26	Omaezaki	-0.24	Yokohama	-0.65
Kochi	+0.46	Onahama	-0.83	Yagi	+0.20
Kofu	-0.16	Okayama	+0.13		

The station corrections for many stations in Japan for shallow earthquakes have already been determined by various authors. Recently, Mr. M. ICHIKAWA (1958) determined them newly and compared them with those given by various authors.

The writer compared his station corrections for deep earthquakes with ICHIKAWA's for shallow earthquakes. The results are shown in Fig. 1, where the numerals on each station show the value obtained by subtracting the latter from the former. Therefore, the station which has a negative or a positive value is shaken by a deep earthquake respectively more intensely or less than in case of a shallow earthquake.

As will be seen from the figure, the stations in the east part of the main island and Kyushu have mainly negative

Fig. 1. Geographical distribution of (station correction for deep earthquakes) - (station correction for shallow earthquakes).

values, and in the west part of the main island and Shikoku they mainly have positive values. The former regions correspond to the zones of abnormal seismic intensity well known in the case of a deep earthquake.

This fact seems to show that the station correction for deep earthquakes reflects deeper geological structures around each station than in case of shallow earthquakes.

3. $\log A/T-\Delta$ curves for deep-focus earthquakes

To obtain the common logarithm of the amplitude/period ratio at the epicenter of an earthquake from the data obtained at various epicentral distances, we must determine the standard $\log A/T-\Delta$ curve for each depth of the focus.

K. WADATI and T. HIRONO (1956) devised a method to obtain an attenuation coefficient as a function of depth, and showed in a figure the standard $\log A/T-\Delta$ curve for an earthquake of $H=240$ km as an example. The curve given by them varies with M .

It seems to the writer that there are yet some ambiguities about the velocity and the attenuation coefficient of the medium at each depth in the upper part of the Earth. Moreover, the laws governing the attenuation of the wave travelling through the Earth are not well known.

N. RICKER (1949) summarized his wavelet theory on the propagation of shock waves in an absorbing medium as follows:

- (1) It is the center of the wavelet which travels with the classical velocity of sound in the medium, this velocity being given by the square root of the elasticity over the density.
- (2) The breadth of the wavelet, measured in second, say, is proportional to the square root of the propagation time of its center.
- (3) The amplitude of the displacement-type wavelet decays as the wavelet advances through the Earth in such a manner that the amplitude is inversely

Fig. 2. Standard $\log A/T-\Delta$ curve and observed values of the earthquake on February 7, 1938, $H=100$ km.

Fig. 3. Standard $\log A/T-\Delta$ curve and observed values of the earthquake on April 15, 1935, $H=230$ km.

proportional to the $4/2$ power of the travel time of its center, etc.

By his theory, it is expected that the amplitude/period ratio decreases inversely proportional to the $5/2$ power of the travel time of the wave.

The writer calculated $\log A/T-\Delta$ curve for deep earthquakes of various depths on the basis of N. RICKER's theory, and compared them with the observed data corrected with the station factor. In general, the curves rather well fitted with the observed values. Some examples are shown in Figs. 2, 3 and 4.

We used the travel time of the S phase given by Dr. K. SAGISAKA and Mr. M. TAKEHANA (1935) as the basis of the calculation, supposing that the wave with the maximum amplitude of a deep earthquake might be found in a group of S waves, when observed at an epicentral distance less than about 1,000 km.

Some examples of the travel time of the S phase and the standard $\log A/T-\Delta$ value calculated on the assumption that A/T decreases inversely proportional to the $5/2$ power of the travel time of the S phase are given in Table 2. In the table, $\log A/T$ at the epicenter is taken as zero.

Fig. 4. Standard $\log A/T-\Delta$ curve and observed values of the earthquake on June, 30, 1931, $H=350$ km.

Table 2. Travel time of S phase and the standard $\log A/T$.

Δ km	$H=100$ km		$H=200$ km		$H=300$ km		$H=400$ km		$H=500$ km	
	t_s sec.	$-\log A/T$	t_s sec.	$-\log A/T$	t_s sec.	$-\log A/T$	t_s sec.	$-\log A/T$	t_s sec.	$-\log A/T$
0	25.6	0	47.9	0	69.4	0	89.7	0	108.3	0
20	26.3	0.03	48.0	0	69.6	0.01	89.9	0.01	108.4	0
40	27.7	0.09	49.0	0.03	70.1	0.02	90.2	0.01	108.7	0
60	29.7	0.16	50.2	0.05	70.8	0.03	90.8	0.02	109.2	0.01
80	32.3	0.25	51.7	0.09	71.7	0.04	91.6	0.03	109.8	0.01
100	35.2	0.35	53.5	0.12	72.9	0.06	92.5	0.04	110.5	0.02
140	42.2	0.54	57.7	0.20	75.9	0.10	94.8	0.06	112.3	0.04
180	49.7	0.72	63.1	0.30	79.8	0.16	97.8	0.10	114.9	0.06
220	57.4	0.88	69.5	0.41	84.5	0.22	101.4	0.14	117.7	0.09
260	65.3	1.02	76.3	0.51	89.9	0.29	105.5	0.18	121.4	0.12
300	73.2	1.14	83.6	0.61	95.7	0.35	110.3	0.23	125.4	0.16
340	81.4	1.26	91.0	0.70	101.9	0.42	115.5	0.28	129.8	0.19
380	89.9	1.37	98.5	0.78	108.3	0.49	121.0	0.33	134.5	0.23
420	98.6	1.47	106.0	0.86	115.2	0.55	126.8	0.38	139.4	0.27
460	107.4	1.56	114.0	0.94	122.2	0.62	132.8	0.43	144.5	0.31
500	116.4	1.65	122.0	1.02	129.3	0.68	139.0	0.48	149.8	0.35
580	135.2	1.81	138.2	1.15	143.9	0.80	152.0	0.58	161.0	0.43
660	154.1	1.95	154.3	1.27	159.0	0.90	165.7	0.67	173.1	0.51
740	172.8	2.08	171.0	1.38	174.3	1.00	179.8	0.76	185.7	0.58
820	191.2	2.18	188.0	1.49	190.0	1.10	194.3	0.84	199.1	0.66
900	209.6	2.28	205.2	1.60	206.1	1.19	209.1	0.92	212.7	0.73
980	227.5	2.42	222.6	1.67	222.5	1.27	223.9	1.00	226.9	0.80
1060	245.1	2.45	239.8	1.75	238.8	1.35	239.1	1.07	241.1	0.87

4. Relation between $\log A_0/T_0$ and M

We obtained $\log A_0/T_0$, that is $\log A/T$ at the epicenter, for many deep earthquakes of which M were given already, using the standard $\log A/T-d$ curves.

Fig. 5. $\log A_0/T_0$ with respect to M for the earthquakes, $H=250-450$ km. Full line shows the values calculated by $\log A_0/T_0 = 0.63M - 1.88$.

We compared $\log A_0/T_0$ with the corresponding M in the case of deep earthquakes of $H=250-450$ km, only because in this range of the depths we had comparatively many examples. The result is shown in Fig. 5.

If we assume a linear relation between $\log A_0/T_0$ and M , we get a formula

$$\log A_0/T_0 = 0.63M - 1.88,$$

by the method of least squares.

A similar equation will be given for shallow earthquakes as follows.

B. GUTENBERG and C. F. RICHTER (1956) obtained the following two relations,

$$m_B = 0.63M_S + 2.5,$$

and

$$\log A_0/T_0 = m_B - 2.3.$$

Combining these formulae, we obtain

$$\log A_0/T_0 = 0.63M + 0.2,$$

for the Pasadena group of seismological stations.

In general, it is expected that the relation between $\log A_0/T_0$ and M will be given by a formula, that is

$$\log A_0/T_0 = 0.63M - \alpha(H),$$

where, $\alpha(H)$ is a constant for a given focal depth H , and will be determined in the next article.

5. The values of $\alpha(H)$

Fig. 6. $\alpha(H)$ with respect to H . Full line shows the values calculated by $\alpha(H) = 2.5 \log t_0 - 2.8$.

The values of $\alpha(H)$ are obtained by a formula

$$\alpha(H) = 0.63M - \log A_0/T_0,$$

using the known $\log A_0/T_0$ and M of deep earthquakes of various focal depths. They are shown in Fig. 6.

By N. RICKER's wavelet theory, it is expected that $\alpha(H)$ is proportional to $5/2 \log t_0$, where t_0 is the travel time of the S phase at the

epicenter.

In fact, as will be seen from the figure, the values of $\alpha(H)$ obtained above are well represented as a function of H by a formula

$$\alpha(H) = 2.5 \log t_0 - 2.8.$$

The values of $\alpha(H)$ calculated by the equation are shown in Table 3.

Table 3. Values of $\alpha(H)$.

H km	100	120	160	200	240	280	320
$\alpha(H)$	0.72	0.90	1.17	1.40	1.57	1.72	1.87

H km	360	400	450	500			
$\alpha(H)$	1.97	2.07	2.20	2.28			

6. Comparison between M' and M_G

We evaluated the magnitudes of deep earthquakes by a formula

$$0.63M = \log A_0/T_0 + \alpha(H), \dots\dots\dots (\alpha)$$

and designated them as M' .

We compared M' with M_G , that is the M given by the Pasadena seismological laboratory, and found that M' coincided well with M_G so far as the magnitude does not exceed 7.

If M' exceeds 7, we must use the other formula

$$\{0.63 + 0.08(\log A_0/T_0 + \alpha(H) - 4.4)\}M = \log A_0/T_0 + \alpha(H) \dots\dots\dots (\beta)$$

to reduce M' to M_G .

It seems to the writer that the reason of this change of the formula at $M=7$ may be found in the correction which was applied only to shocks of magnitude 7 or over by the Pasadena seismological laboratory.

M' obtained by the formulae (α) or (β) according as M' is smaller or greater than 7 and the corresponding M_G are shown in Fig. 7.

As will be seen from the figure, the difference between M' and M_G is generally within the limits ± 0.3 unit of magnitude.

Fig. 7. Comparison between M' and M_G .

The date of occurrence, the location of the epicenter, the focal depth, $\log A_0/T_0$, M' and M_G of all earthquakes used in this investigation are shown in Table 4.

Table 4.

Date (J. S. T.) m d			Epicenter			$\log A_0/T_0$	M'	M_G
			$\lambda(^{\circ}\text{E})$	$\varphi(^{\circ}\text{N})$	$H\text{km}$			
1930	V	24	139.9	34.1	100~120	3.46	6.8	$6^{3/4}$
	VII	23	147	$44^{1/3}$	140	3.50	7.1	7.1
1931	I	9	140.6	39.8	130	2.67	5.8	6
	I	21	145.6	43.4	100	3.38	6.5	$6^{1/4}$
	II	20	135.7	44.5	350	3.20	7.5	7.4
	III	1	143.7	46.0	250	2.45	6.5	$6^{1/2}$
	IV	21	134.2	38.5	350	1.92	6.1	6
	VI	2	137.5	36.0	240	2.51	6.5	$6^{1/2}$
	VI	30	136.8	33.9	350	2.13	6.5	$6^{1/2}$
1932	II	19	140.3	32.9	150	2.43	5.6	$5^{1/2}$
	IV	28	136.7	34.0	320	1.80	5.8	$5^{3/4}$
	V	5	135.3	34.6	360	1.86	6.1	$6^{1/2}$
	VII	25	135.9	35.2	360	2.14	6.5	$6^{3/4}$
	VII	27	139.0	31.2	300	1.90	5.9	$5^{3/4}$
	X	14	138.8	31.6	300	1.72	5.6	$5^{1/2}$
	X	26	144.6	46.2	370	2.42	7.0	$6^{1/2}$
	XI	13	136.7	43.8	300	3.0	7.2	7.0
	XII	5	137.0	33.7	350	1.65	5.7	$5^{3/4}$
1933	II	9	138.8	31.7	250	2.12	5.9	6
	V	24	145.6	46.7	450	1.66	6.1	6
	V	29	138.0	32.4	400	1.08	5.0	$5^{1/4}$
	IX	20	136.6	34.1	380	1.30	5.3	$5^{1/2}$
1935	IV	15	137.1	36.2	230	2.25	6.0	$6^{1/4}$
	V	31	133.8	38.5	450	2.10	6.8	$6^{1/2}$
	X	15	135.4	37.7	280	1.95	5.8	$5^{3/4}$
1936	VI	26	137.9	32.5	320	2.13	6.4	$6^{1/4}$
	X	20	$135^{1/4}$	37.0	350	1.60	5.6	$5^{3/4}$
	X	26	136.3	34.5	340	1.95	6.1	$6^{1/4}$
	XII	1	129.0	$30^{1/2}$	230	2.65	6.6	$6^{1/2}$
1937	XI	22	135.7	35.1	370	1.35	5.3	$5^{3/4}$
1938	II	7	139.1	36.1	100	3.34	6.4	$6^{1/2}$
	IV	23	130.1	28.2	165	2.75	6.3	$6^{1/4}$
	X	18	141.3	44.0	260	2.38	6.4	$6^{1/2}$
1939	IV	21	140.0	47.6	500	2.58	7.3	7.0
1940	VI	27	138.8	30.6	400	1.40	5.5	$5^{1/2}$
	VII	4	144.5	44.3	200	2.35	6.0	$5^{3/4}$
	VII	10	130.6	44.8	560	2.50	7.3	7.3
	XI	7	138.5	30.3	480	1.95	6.7	$6^{3/4}$
	XI	27	139.5	36.5	120	2.48	5.4	$5^{1/4}$
1942	III	6	141.7	43.0	300	2.76	7.1	6.9
	IV	20	137.3	33.9	350	2.50	7.0	$6^{1/2}$
1943	XI	17	138.0	33.0	320	2.40	6.8	7.0
1950	II	28	143.8	46.0	320	3.90	7.8	7.8
1951	IV	17	138.0	31.2	470	1.90	6.6	$6^{3/4}$
1952	V	28	135.8	35.1	370	2.33	6.9	$6^{3/4}\sim 7$
1954	V	15	137.4	36.0	230~240	2.97	7.1	7

7. Theoretical considerations on the formula $\log A_0/T_0 = 0.63M - \alpha(H)$

In article 4, it is shown that the formula

$$\log A_0/T_0 = 0.63M - \alpha(H)$$

is an elementary equation connecting $\log A_0/T_0$ with M .

Now, A_0/T_0 is the ratio of the maximum ground amplitude of body waves to the corresponding period at the epicenter, and M is effectively given in terms of M_s , that is the magnitude determined from the amplitudes of surface waves for shallow teleseisms.

To connect the two quantities A_0/T_0 and M_s , we must consider theoretically A/T of body waves at the epicenter and the amplitudes of surface waves at a distance produced by a disturbance in the Earth.

Prof. T. SAKAI (1934) treated theoretically the propagation of waves over the plane surface of an elastic solid produced by an internal source.

As a distortional point source, he assumed a primary disturbance which can be written in the form

$$D_R^{(0)} = \frac{2A \cos \theta}{c^2 R_0^3} \frac{1}{\tau^2 + 1}, \quad D_\theta^{(0)} = \frac{A \sin \theta}{c^2 R_0^3} \frac{1}{\tau^2 + 1},$$

where

$$\tau = (t - R_0/v_2)/c,$$

$D_R^{(0)}$ = radial component of the displacement at the focus,

$D_\theta^{(0)}$ = co-latitudinal component of the displacement at the focus,

R_0 = radius of a small sphere taken at the focus,

v_2 = velocity of the distortional wave,

θ = co-latitude, the polar axis being directed vertically upward from the focus,

A = constant.

c may be taken as a measure of the period of the shock wave or the duration of the aperiodic disturbance.

The horizontal and the vertical components of the displacement of the distortional wave near the epicenter can be given respectively as follows;

$$D_r^{(2)} = \frac{4A\theta_r}{v_2^2 dc^4} \frac{3\tau^2 - 1}{(\tau^2 + 1)^3}, \quad D_z^{(2)} = -\frac{4A\theta_z}{v_2^2 dc^4} \frac{3\tau^2 - 1}{(\tau^2 + 1)^3},$$

where

$$\tau = (t - R/v_2)/c,$$

θ_r, θ_z = certain functions of θ ,

d = depth of the source,

R = hypocentral distance.

If we represent the displacement at the focus by $D^{(0)}$, we have a relation

$$D_r^{(2)} \propto D^{(0)}/c^2.$$

The components of the displacement of the Rayleigh wave at a large epicentral distance are given by

$$D_r^{(3)} = B_3 c_3 \frac{\cos\left(\frac{7}{2} \tan^{-1} \frac{1}{\tau}\right)}{(\tau^2 + 1)^{7/4}}, \quad D_z^{(3)} = -B_3 \frac{\sin\left(\frac{7}{2} \tan^{-1} \frac{1}{\tau}\right)}{(\tau^2 + 1)^{7/4}},$$

where

$$\tau = (t - r/v_3) / \left\{ c \left(1 + \frac{d\delta_0'}{cv_2} \right) \right\},$$

$$B_3 = \frac{-15A\pi\sqrt{v_3}\delta_0' M_z(-i\delta_0')}{2^{5/2}c^{9/2}v_2^3 E'(-i\delta_0')(1 + d\delta_0'/cv_2)^{7/2} \sqrt{r}},$$

$$\delta_0' = \left\{ \left(\frac{v_2}{v_3} \right)^2 - 1 \right\}^{1/2},$$

$c_3, M_z(-i\delta_0'), E'(-i\delta_0')$ = certain functions of v_1, v_2 and v_3 ,

v_1 = velocity of the dilatational wave,

v_2 = velocity of the distortional wave,

v_3 = velocity of the Rayleigh wave,

r = epicentral distance.

If we assume $d/cv_2 \gg 1$, that is, the depth of the focus is greater than the breadth of the shock wave, we have a relation

$$D_r^{(3)} \propto D^{(0)}c.$$

After B. GUTENBERG (1945), M_S is given by

$$M_S = \log A_S - \log B + C + D,$$

where

A_S = horizontal component of the maximum ground movement in microns during surface waves having periods of about 20 seconds,

B = the same quantity for a shock of magnitude zero,

C = constant for each station, ground, instrument,

D = depth, azimuth, absorption.

Therefore, we may take

$$M_S \propto \log D_r^{(3)},$$

and we have the following relations:

$$\log A_0/T_0 \propto \log D_r^{(2)}/c \propto \log D^{(0)}/c^3 \propto \log D_r^{(3)}/c^4 \propto (M_S - 4 \log c).$$

B. GUTENBERG and C. F. RICHTER (1956) examined the relation between the period of the wave of maximum amplitude and the corresponding M , and obtained a result that a rough representation for $M_L < 7$ is

$$\log T_0 = -1.1 + 0.1M_L,$$

where M_L is the magnitude determined from records of local earthquakes in California according to the original definition by RICHTER.

If we use M_S instead of M_L , the formula is transformed approximately to

$$\log T_0 = -1.03 + 0.093M_S.$$

Therefore, we may assume

$$\log c \propto 0.093M_s$$

and have lastly

$$\log A_0/T_0 \propto 0.63M_s.$$

8. Azimuthal factor for $\log A/T$

From our experiences, we notice that the $\log A/T$ observed at each station fairly deviates from the standard value at each epicentral distance even after applying the station correction.

To examine whether there is any azimuthal factor among the observed $\log A/T$ or not, the writer calculated the amount of the deviation, that is

$$\Delta \log A/T = (\log A/T)_{\text{obs}} - (\log A/T)_{\text{cal}},$$

and plotted it on a geographical map.

Some examples are shown in Figs. 8–12. As will be seen from these figures, the geographical distributions of $\Delta \log A/T$ are rather systematic, and we can separate the stations with positive $\Delta \log A/T$ from those with negative one by one or two lines.

Moreover, in these figures the nodal lines of initial motions of the P phase taken mainly from the paper given by H. HONDA and A. MASATSUKA (1952) are also shown.

It seems to the writer that one of the nodal lines frequently goes through the midst of the domains with negative $\Delta \log A/T$.

The points of intersection of the lines, P and P' as shown in Figs. 9–11, do not coincide with each other. The polar angle θ between P and P' , taking the origin of the polar coordinates at the focus, was calculated by

$$\cos \theta = \cos \theta_p \cos \theta_{p'} + \sin \theta_p \sin \theta_{p'} \cos \varphi,$$

where

$$\sin \theta_p = \frac{r_0}{r} \frac{v}{v_0} \alpha_p, \quad \sin \theta_{p'} = \frac{r_0}{r} \frac{v}{v_0} \alpha_{p'},$$

α_p = cosine of the emergent angle at P ,

Fig. 8. Geographical distribution of $\Delta \log A/T$ of the earthquakes on November 27, 1940, $H=120$ km. Full lines represent the boundaries between positive and negative values. Dotted lines represent the nodal lines of initial motions of the P phase.

Fig. 9. Geographical distribution of $\Delta \log A/T$ of the earthquake on April 15, 1935, $H=230$ km.

Fig. 10. Geographical distribution of $\Delta \log A/T$ of the earthquake on June 30, 1931, $H=350$ km.

Fig. 11. Geographical distribution of $\Delta \log A/T$ of the earthquake on July 25, 1932, $H=360$ km.

Fig. 12. Geographical distribution of $\Delta \log A/T$ of the earthquake on May 15, 1954, $H=230-240$ km. Full line represents the boundary between positive and negative values. Dotted line represents the nodal line of initial motions of the P phase.

α_p , =cosine of the emergent angle at P' ,

r_0 =radius of the Earth,

r =radial distance of the focus from the center of the Earth,

v_0 =wave velocity near the surface,

v =wave velocity at the focus,

φ =angle between two lines EP and EP' , connecting P and P' to the epicenter E respectively.

The polar angles take values between 30° — 40° in cases shown in the figures.

From these examples and some others not shown here, the writer is inclined to accept the reality of the azimuthal factor for $\log A/T$.

Sometimes, the observed $\log A/T$ — Δ curves were much deformed from the standard curve by the azimuthal factor, so that we must use sufficient data obtained by many earthquakes to determine the standard $\log A/T$ — Δ curve for an earthquake of any given depth.

Although the existence of the azimuthal factor for $\log A/T$ is not yet thoroughly proved, some examples such as shown above seem to suggest that the azimuthal factor might have some intimate connections with the mechanism of the occurrence of the earthquakes.

H. KAWASUMI (1933, 1934) gave the displacements of P wave and S wave due to a doublet force with moment respectively as follows;

$$\begin{aligned}
 u_1 &= A \frac{d}{dr} \left\{ \frac{H^{(2)}_{2+1/2}(hr)}{\sqrt{r}} \right\} \sin^2 \theta \sin 2\varphi e^{i\phi t} \\
 &\approx -\sqrt{\frac{2h}{\pi}} A \frac{e^{i(\phi t - hr)}}{r} \sin^2 \theta \sin 2\varphi, \\
 v_1 &= A \frac{H^{(2)}_{2+1/2}(hr)}{r^{3/2}} 2 \sin \theta \cos \theta \sin 2\varphi e^{i\phi t} \approx O\left(\frac{1}{r^2}\right), \\
 w_1 &= A \frac{H^{(2)}_{2+1/2}(hr)}{r^{3/2}} 2 \sin \theta \cos 2\varphi e^{i\phi t} \approx O\left(\frac{1}{r^2}\right), \\
 u_2 &= B \frac{H^{(2)}_{2+1/2}(kr)}{r^{3/2}} \frac{3}{2} \sin^2 \theta \sin 2\varphi e^{i\phi t} \approx O\left(\frac{1}{r^2}\right), \\
 v_2 &= B \frac{1}{r} \frac{d}{dr} \left\{ \sqrt{r} H^{(2)}_{2+1/2}(kr) \right\} \frac{1}{2} \sin \theta \cos \theta \sin 2\varphi e^{i\phi t} \\
 &\approx \sqrt{\frac{2k}{\pi}} B \frac{e^{i(\phi t - kr)}}{r} \frac{1}{2} \sin \theta \cos \theta \sin 2\varphi, \\
 w_2 &= -B \left\{ \frac{k H^{(2)}_{2+1/2}(kr)}{\sqrt{r}} \sin^2 \varphi + \frac{1}{r^{3/2}} H^{(2)}_{2+1/2}(kr) \cos 2\varphi \right\} \sin \theta e^{i\phi t} \\
 &\approx \sqrt{\frac{2k}{\pi}} B \frac{e^{i(\phi t - kr)}}{r} \sin \theta \sin^2 \varphi,
 \end{aligned}$$

where, u_1 , v_1 , w_1 are displacements of P wave and u_2 , v_2 , w_2 are displacements of S wave in the direction of r , θ and φ respectively.

The nodal planes of P wave are $2\varphi=0$ and π . There is a nodal plane $\varphi=0$ or

π for S wave, and this is coincident with one of the nodal planes of P wave and corresponds to the fault plane.

Moreover, the amplitude of S wave, that is $\sqrt{v_2^2 + w_2^2}$, is large in the direction perpendicular to the fault plane.

In the above examples, the nodal line of P wave which passes through the negative $\Delta \log A/T$ domain might be considered as a locus of the intersection of the fault plane with the Earth's surface. But we cannot understand why P and P' do not coincide with each other.

9. Summary

The writer tried to obtain the same M as those given by the Pasadena seismological laboratory for the deep-focus earthquakes in and near Japan by using the seismological data obtained at Japanese stations. The results are summarized in the following.

(1) It is shown that the calculated $\log A/T - \Delta$ curves for deep earthquakes of various depths on the basis of NORMAN RICKER's wavelet theory fit well with the observed values.

(2) It is shown that the magnitude of deep earthquakes can be evaluated by the formula

$$0.63M = \log A_0/T_0 + \alpha(H)$$

so far as the magnitude does not exceed 7.

If M exceeds 7, we must use the other formula

$$\{0.63 + 0.08(\log A_0/T_0 + \alpha(H) - 4.4)\} M = \log A_0/T_0 + \alpha(H),$$

where A_0/T_0 is the amplitude/period ratio at the epicenter. And $\alpha(H)$ is given by the formula

$$\alpha(H) = 2.5 \log t_0 - 2.8,$$

where t_0 is the travel time of the S phase at the epicenter.

(3) It is shown that the station corrections for deep earthquakes are quite different from those for shallow earthquakes. The former reflect deeper geological structures around each station than do the latter.

(4) Some examples are shown to suggest that these is the azimuthal factor among the observed $\log A/T$, and it is shown that the factor may be explained to some extent by assuming a doublet force with moment as the disturbance at the hypocenter.

(5) Some theoretical considerations are made on the relation between $\log A_0/T_0$ and $\log A_s$. Where A_s is the amplitudes of surface waves, and M_s is defined to be proportional to $\log A_s$.

Taking the relation between the period of the wave of maximum amplitude and the corresponding M into consideration, the formula

$$\log A_0/T_0 = 0.63M - \alpha(H)$$

is proved theoretically to some extent, where M is given effectively in terms of M_s .

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日本附近の深発地震の規模について

井 上 宇 胤

日本の地震観測の資料から深発地震の規模を求めるという試みは、既に和達、本多、広野、岩井、勝又によつてなされている。

然し、地球上層の地震波の速度分布、波動の減衰係数の分布などが十分分っていないばかりでなく、波動減衰の機構も明確でない。また、地震動に対する地盤補正も、深発地震では異常震域を伴うので当然浅発地震の場合とは異なる筈である。

なお、発震機構によつて実体波の振幅の方位差も考えられるので、蛇足ようであるが、この問題を取扱つてみた。

主な結果は次のようなものである。

- (1) NORMAN RICKER の連波説によつて計算した $\log A/T-D$ 曲線は観測値と比較的よく一致する。
- (2) $M \leq 7$ の場合は

$$0.63M = \log A_0/T_0 + \alpha(H)$$

によつて、

$M > 7$ の場合は

$$\{0.63 + 0.08(\log A_0/T_0 + \alpha(H) - 4.4)\} M = \log A_0/T_0 + \alpha(H)$$

によつて深発地震の M が求められる。

但し、 $\alpha(H)$ は深さによつて異なる常数であつて、 $\alpha(H) = 2.5 \log t_0 - 2.8$ によつて与えられる。

此処に、 A_0/T_0 は震央における A/T の値であり、 t_0 は震央における S 相の走時である。

- (3) 深発地震に対する地盤補正は浅発地震に対するものと著しく異なっている。

(4) 観測される $\log A/T$ は震源における方位に関係があり、断層面に直角の方向に大きくなっているようである。

- (5) M を求めるときの基本となる

$$\log A_0/T_0 \propto 0.63M$$

という関係は、坂井の半無限弾性体の内部に震源のある場合の波動理論によつて説明される。

然し、最も大事な地球上層の地震波の速度分布および波動の減衰係数の分布は将来の問題として残されている。