TECHNICAL NOTE

Noise Power Spectrum in PROPELLER MR Imaging

Yuki ICHINOSEKI^{1*}, Tatsuo NAGASAKA², Kota MIYAMOTO¹, Hajime TAMURA¹,
Issei MORI¹, and Yoshio MACHIDA¹

¹*Health Sciences, Tohoku University Graduate School of Medicine*

2-1 Seiryō-machi, Aoba-ku, Sendai, Miyagi 980-8575, Japan

²*Department of Radiology, Tohoku University Hospital*

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The noise power spectrum (NPS), an index for noise evaluation, represents the frequency characteristics of image noise. We measured the NPS in PROPELLER (Periodically Rotated Overlapping Parallel Lines with Enhanced Reconstruction) magnetic resonance (MR) imaging, a nonuniform data sampling technique, as an initial study for practical MR image evaluation using the NPS. The 2-dimensional (2D) NPS reflected the k-space sampling density and showed agreement with the shape of the k-space trajectory as expected theoretically. Additionally, the 2D NPS allowed visualization of a part of the image reconstruction process, such as filtering and motion correction.

Keywords: *image quality, MRI, noise power spectrum, PROPELLER*

Introduction

The signal-to-noise ratio (SNR) is an important index for evaluating image quality in a magnetic resonance (MR) imaging system. Several methods have been reported to measure SNR.^{1,2} Typically, the signal is estimated as the mean of the pixel values within a defined region of interest (ROI) in an image, and the noise, as their standard deviation (SD).² The pixel intensity in a magnitude MR image in the presence of statistical image noise is well known to be governed by a Rician distribution.³ When the SNR is greater than 3, the distribution approximates the Gaussian distribution that is the same as the distribution of the original real and imaginary MR images obtained by the Fourier transform of the acquired data. Therefore, the SD of the magnitude MR image is useful as a simple and rational index of noise in MR images. However, the SD only expresses the overall characteristic of the image noise.

The noise power spectrum (NPS) represents the frequency characteristics of image noise not considered in evaluation of the SD. The NPS is a popular index for noise evaluation in the medical imag-

ing field. In X-ray computed tomography (X-CT), for example, the NPS is used to analyze the dependence of noise characteristics on the image reconstruction function.^{4,5} The 2-dimensional (2D) NPS is defined as the square of the absolute value of the 2D Fourier transform of the noise component of an image, which is averaged over an ensemble of images. In addition to the 2D NPS, a one-dimensional (1D) NPS is also generated.

We herein describe the NPS of an MR image. MRI system collects data in a spatial frequency domain (designated as k-space), which is the 2D Fourier transform of an image domain. Therefore, the NPS of an MR image is fundamentally identical to the noise characteristics of k-space data. According to a previous study,⁶ the NPS of an MR image shows a flat property when the k-space data are uniformly sampled along a Cartesian coordinate system, and the shapes of the image reconstruction filters can also be derived using the NPS. These properties can be understood easily by considering the definition of the NPS and the principle of MR image reconstruction. Therefore, little importance has been assigned to the NPS measurement of an MR image. However, nonuniform coverage of k-space data is also used in MR imaging. Conventional half-Fourier imaging⁷ is a simple example, and among several complicated imaging techniques re-

*Corresponding author, Phone: +81-22-717-7940, Fax: +81-22-717-7944, E-mail: ichinoseki@med.tohoku.ac.jp

cently put to practical use is the periodically rotated overlapping parallel lines with enhanced reconstruction (PROPELLER) method.⁸ In such cases, the NPS is presumed in principle to have no flat property. Therefore, understanding the NPS property of the data acquired by these methods may be valuable in evaluating MR images.

In this article, we revisit the NPS of an MR image using the PROPELLER method as an example of recent imaging techniques, examine the feasibility and inherent problems of NPS measurement using common magnitude MR images, and confirm the consistency of experimentally obtained results with those based on theory and past knowledge.

Materials and Methods

The PROPELLER MRI method used for this study is based on the fast spin echo (FSE) sequence. Several multiple k-space lines are collected in each repetition time (TR). The lines constitute a strip domain designated as a blade, and the k-space data are then sampled with a rotating blade centered on the origin of the k-space. Because the circular central region around the k-space origin is covered by all blades in an overlapping fashion, the motion artifact can be reduced by correcting the acquired data by the estimation of rotation and translation of the object between blades.⁸

The MultiVane method in MR systems from Philips Medical Systems (Best, the Netherlands), which corresponds to the PROPELLER method,

has its own parameters. We employed 2 parameters for this study, the MultiVane shot per blade (SPB) and MultiVane percentage (MV%). SPB represents the number of shots that form one blade. The SPB parameter is used to control the blade width (BW), with the number of lines in one blade represented by $BW = ETL \times SPB$, in which ETL is the echo train length in the FSE sequence. The MV% parameter is used to control the number of blades (NB), represented by $NB = \text{the integer part of } (MX \times MV\% / BW)$, in which MX is a reference Cartesian matrix size.

Data acquisition

Phantom imaging was performed using a 1.5-tesla MR imager (Imager A, Intera Achieva Nova Dual, Release 2.6; Philips Medical Systems) with a whole-body coil. A cylindrical MR imaging phantom with inner diameter of 200 mm (conforming to the guidelines of the American Association of Physicists in Medicine [AAPM] and National Electrical Manufacturers Association [NEMA], type 90-401; Nikko Fines Industries Co. Ltd., Tokyo, Japan) was positioned at the magnet center. An FSE-based MultiVane sequence was used with parameters: TR/echo time (TE), 1000/100 ms; ETL, 15; image matrix, 256×256 ; field of view (FOV), 256×256 mm; slice thickness (ST), 5 mm; and number of excitations (NEX), one.

As presented in Fig. 1, we performed imaging in 2 series, fixing the SPB at 3 with an MV% of 100 (NB = 5), 160 (NB = 9), and 200 (NB = 11) and

Fig. 1. The k-space trajectories of a MultiVane (MV) sequence. (a-c) Shots per blade (SPB) fixed at 3 with the percentage of MV (MV%) varied as (a) 100%, (b) 160%, and (c) 200%. (d-f) MV% fixed at 100% with SPB varied as (d) 3, (e) 2, and (f) one. (a) and (d) are identical.

fixing the MV% at 100% with an SPB of one (NB = 17), 2 (NB = 8), and 3 (NB = 5). We obtained 20 phantom images in each imaging condition.

On the console of Imager A, motion correction procedures can be switched on and off. Although we performed the imaging experiments described above without motion correction, we performed the same experiments with motion correction to assess the effect of motion correction procedures on the NPS. In this case, we performed the additional experiments after placing a test tube beside the phantom to make the imaging object asymmetric.

We turned off other image processing options, such as filtering, that could be cancelled on the console, but some built-in filtering process/es may have been resistant to deactivation. To examine the effect of such built-in filtering on the NPS, we conducted an additional experiment using another 3T MR imager (Imager B, Vantage Titan, software version V2.00; Toshiba Medical Systems Corp., Otawara, Japan) on which image reconstruction can be performed without filtering. The following parameters were used: TR/TE, 1000/165 ms; ETL, 43; image matrix, 240×240 ; FOV, 256×256 mm; ST, 5 mm; and NEX, one. Images were obtained with no filtering and with linear filtering L2H1 (filter with an effect of [a little strong Low Pass + slight High Pass]).

Generation of 2D NPS

We obtained a 2D NPS through the following procedures (shown schematically in Fig. 2) using Mathematica software (version 7.0; Wolfram Research Inc., Champaign, IL, USA). We produced a difference image from 2 phantom images acquired under identical conditions; selected the central region (128×128) of the difference image using a rectangular window function to consider only the noise characteristics of the phantom region without background, then removed the average of the region for offset correction; and squared the absolute value of 2D discrete Fourier transform of the selected region to generate a pre-averaging 2D NPS. We performed these steps for 10 pairs of phantom images. Finally, we averaged 10 pre-averaging 2D NPSs to produce a 2D NPS. We herein designate the value of each pixel of a 2D NPS as an NPS value.

We performed the following examination using a 2D NPS obtained in this manner.

Appearance of 2D NPS

We visually investigated the shapes or contours of pre-averaging 2D NPS and 2D NPS in each con-

Fig. 2. Flowchart showing 2-dimensional (2D) noise power spectrum (NPS) calculation. We produced a difference image from 2 phantom images acquired under identical conditions, squared the absolute value of 2D discrete Fourier transform of the difference image to obtain a pre-averaging 2D NPS, and then averaged 10 pre-averaging 2D NPSs to generate a 2D NPS.

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Dependence of NPS values on the number of overlapped blades

As Fig. 3a shows, 3 groups of 10 ROIs were placed on a 2D NPS obtained with MV% of 100% and SPB of 3, with each group corresponding to the number of overlapped blades (NOB) = one, 2, or 3, and a single ROI for NOB = 5, to assess the dependence of the NPS values on the NOB. The mean NPS value in each ROI was normalized by that of NOB = one.

Fig. 3. Measurement of noise power spectrum (NPS) values. **(a)** Dependence on number of overlapped blades (NOB). We set regions of interest (ROIs) in each region of NOB = one (yellow), 2 (blue), 3 (red), and 5 (purple) in the 2-dimensional (2D) NPS, measured the NPS values, and determined the averages of the NPS values at 10 ROIs, except in the region with NOB = 5. We omitted the edge part with a decline in NPS value and the central area with an increase in NPS value from measurements. **(b)** Calculation of one-dimensional (1D) NPS. We measured NPS values on the circumference centering on the origin of 2D NPS and determined the average NPS value at the same frequency. For description, these figures were depicted using k-space trajectories instead of 2D NPS.

Calculation and evaluation of 1D NPS

Figure 3b shows that the NPS values were measured for the circumference centering on the origin of the 2D NPS. The averaged value was calculated for every radius, normalized by the value of NOB = one, and then expressed on a graph. The graph presents the spatial frequency dependence of the NPS values, which is designated as a 1D NPS. Finally, we evaluated the obtained 1D NPS.

Results

Appearance of 2D NPS

Figure 4 shows the 2D NPSs we obtained. The shape or contour of 2D NPS showed agreement with that of the k-space trajectory in all conditions. However, we observed NPS values slightly higher than expected from the k-space trajectory near the center of 2D NPS (Fig. 4d, arrow). In addition, the NPS values of 2D NPS dropped in all directions in the region of high frequency.

Figure 5 shows the effects of motion correction procedures on NPS. With motion correction, the pre-averaging 2D NPS of the symmetric cylindrical phantom had an unexpected shape (Fig. 5, upper row), whereas that of an asymmetrical imaging object had a k-space trajectory shape (Fig. 5, lower row).

Dependence of NPS values on the number of overlapped blades

As Fig. 6 depicts, the NPS value of the region of

NOB = 2 was 50.4%, of NOB = 3, 33.0%, and of NOB = 5, 20.4%, which is proportional to one/NOB.

Calculation and evaluation of 1D NPS

Figure 7 shows calculated 1D NPSs. NPS values were low in the region of low frequency and exhibited a peak at a higher frequency. In addition, the high values observed in the region of low frequency of the 2D NPS are also discernible.

As the MV% increased, the NPS values decreased; when the MV% was 100%, the peak of the NPS was found at a lower frequency than for MV% of 160 and 200% (Fig. 7a). The dependence on SPB was, however, only slight (Fig. 7b). Still, a tendency of the NPS peak to shift to the higher frequency side can be seen as the SPB is decreased.

Figure 8 shows NPS measured using Imager B. In the case of Filter ON, the NPS values of the image post-processed by a linear filter decreased in the region of high frequency compared with those with Filter OFF.

Discussion

Appearance of 2D NPS

In the non-Cartesian sampling case, the sampling density is corrected in the gridding process in image reconstruction using appropriate pre-gridding weights.⁸ In the high-density sampling region in k-space, noise components are averaged through this process. Therefore, the contribution of noise and

Fig. 4. Two-dimensional (2D) noise power spectrum (NPS) in PROPELLER (Periodically Rotated Overlapping Parallel Lines with Enhanced Reconstruction) magnetic resonance imaging (MRI). See Fig. 1 for each k-space trajectory of (a-f). The 2-dimensional NPSs were in agreement with k-space trajectories in all conditions. The points of slightly higher NPS values were observed in the region of low frequency at the central part (see arrow on (d)), and the NPS values decreased at the peripheries (high frequency edges) in all directions.

Fig. 5. Error in motion correction. (a) Shots per blade (SPB) 3 and (b) SPB one (both MultiVane percentage [MV%] 100%). When motion correction was switched ON, a pre-averaging 2-dimensional (2D) noise power spectrum (NPS) had an unexpected shape. An error in motion correction might arise because of the use of a symmetrical cylindrical phantom. The error in motion correction was improved by making an imaging object asymmetrical in both (a) and (b).

Fig. 6. Dependence on the number of overlapped blades (NOB). The noise power spectrum (NPS) values became about 1/2 with NOB = 2, 1/3 with NOB = 3, and 1/5 with NOB = 5.

the NPS values decrease. Consequently, 2D NPS reflects the sampling density and shows agreement with the k-space trajectory shape (Fig. 4). NPS values are detailed below.

Dependence of NPS values on the number of overlapped blades

Generally, a random noise component is evaluated by the SD of signal intensity. It is proportional to the square root of the reciprocal of the number of averages.⁹ PROPELLER reconstruction is conducted using the arithmetic mean of acquired data for the NOB in overlapping parts. A 2D NPS is obtained as a square of the Fourier transform of a noise component. Therefore, an NPS value is proportional to the reciprocal of the NOB, as presented in the following equation (Fig. 6).

$$\text{NPS values} \propto \left(\frac{1}{\sqrt{\text{number of average}}} \right)^2$$

$$= \frac{1}{\text{number of overlapped blades}}$$

The result obtained in this study shows good agreement with the value expected theoretically.

Evaluation of 1D NPS

The NPS values decreased in the lower frequency region with a greater NOB because NPS values decreased in proportion to 1/NOB. The NPS value decreased at a higher MV% because the NOB increased (Fig. 7a). Although the BW changes as the SPB is varied, NPS values differed little (Fig. 7b), presumably because a fixed percentage made the data sampling rate constant. The NPS peak shifts to a slightly higher frequency at a smaller SPB because the overlap of blades spreads to a higher frequency at a smaller SPB.

Fig. 7. One-dimensional noise power spectrum (NPS) in PROPELLER (Periodically Rotated Overlapping Parallel Lines with Enhanced Reconstruction) magnetic resonance imaging (MRI). The horizontal axis represents the spatial frequency. The vertical axis represents NPS values normalized with average NPS value in the region of NOB = one. (a) One-dimensional NPS with percentage of MultiVane (MV%) varied. (b) One-dimensional NPS with shots per blade (SPB) varied.

Results for Imager B show that NPS values drop at a high frequency range (Fig. 8) because of reconstruction filters. Although filtering was turned off on the console in the experiment on Imager A, the distribution of NPS values near the edges fell. A built-in low-pass filter,¹⁰ which users cannot deactivate, should have been used. The results demonstrate that 2D NPS can reveal the characteristics of actual filtering.

The NPS values in the case of MV% of 100% fell at a lower frequency than in the case of either 160 or 200% (Fig. 7a) because the uncollected regions in which data were not sampled were also used to calculate the 1D NPS. Although the influence of the uncollected region can limit this method, the 1D NPS, which is the conventional expression of NPS, can be useful for elucidating the basic properties of the NPS of MR images.

NPS values in the region of low frequency

We performed offset correction² to remove DC components, which are apt to be contaminated by measurement error. This correction nullified the

Fig. 8. Two-(2D) and one-(1D) dimensional noise power spectra (NPS) measured in Imager B. (a) Two-dimensional NPS with filtering OFF; (b) 2D NPS with linear filtering L2H1 (filter with an effect of [a little strong Low Pass + slight High Pass]) ON; and (c) 1D NPS of (a) and (b). Although we observed no decline in NPS values at the peripheries of (a), we did observe a decline at the upper and lower sides of (b) (arrows). Consequently, NPS confirmed the effect of a reconstruction filter.

NPS value at zero frequency. Even so, we observed high NPS values near the zero frequency in some cases (Fig. 4d). Such high values resulting from the residuals of difference images should be attributable to the instability of the MR system.¹¹ Care must be taken in handling low frequency components when estimating random noise by NPS.

Motion correction

Figure 5 shows the effect of motion correction procedures on pre-averaging 2D NPS. Results suggest that the symmetric circular phantom was misjudged to be in motion, although it was standing still.¹²

For SPB of 3, no such error was noticed until production of the pre-averaging 2D NPS because the original MR image demonstrated no artifact (Fig. 5a). For SPB of one, blurring was apparent in the original MR image. We observed a remarkable error in motion correction in the 2D NPS (Fig. 5b) that might be attributable to the narrow blade width. Such a phenomenon was reported in a clinical study of the parietal region of a child. This observation indicates to us that NPS can visualize a part of the image reconstruction procedure.

Practical use of the NPS in the evaluation of MR noise requires further investigation. For example, investigations under clinical conditions are necessary to propose a practical method of measuring the NPS, and the NPS of an image obtained using a multi-channel system or parallel imaging technique should be examined to evaluate the current state of

MR image quality. Moreover, the NPS of background noise in a magnitude image will be used to evaluate the noise of an image with low SNR, such as a diffusion-weighted image.

Conclusion

We examined the NPS of an MR image using the PROPELLER method. The shape of the 2D NPS showed agreement with that of the k-space trajectory. The NPS value was proportional to the reciprocal of the NOB, as expected theoretically. Additionally, we inferred the utility of the 2D NPS for visualizing a part of the image reconstruction process, such as filtering and motion correction, as well as for expressing the noise frequency characteristics.

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