Detection of Weak Magnetic Fields Induced by Electrical Currents with MRI: Theoretical and Practical Limits of Sensitivity

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Detection of weak magnetic fields induced by neuronal electrical activities with magnetic resonance imaging (MRI) is a potentially effective method for functional imaging of the brain. In this study, we compared the theoretical and practical limits of sensitivity for detecting weak magnetic fields with a columnar phantom. The theoretical limit of sensitivity was estimated from signal and noise intensities in magnetic resonance images. The theoretical limit of sensitivity was approximately $10^{-8}$T. The practical limit was 10 times the theoretical limit. The dependence of the theoretical limit of sensitivity on acquisition parameters, such as the repetition time (TR), echo time (TE), number of pixels, and spectral width, was quantitatively evaluated. The results indicated the existence of an optimal value in $T_E/T_2^*$.

Keywords: MRI, gradient echo, theoretical limit, sensitivity, electrical currents

Introduction

Imaging of temporal and spatial distributions of neuronal electrical activities in the brain is important for the study of brain functions. Some attempts have been made to use magnetic resonance imaging (MRI) for detecting weak magnetic fields induced by neuronal electrical activities in order to develop a new method of mapping brain functions.1-5 While this method has the potential to provide high temporal and spatial resolutions, the detection of a magnetic field from neurons requires extremely high sensitivity. Scott et al. investigated the theoretical limit of sensitivity of magnetic resonance current-density imaging.6 However, their discussion is based on the detection of externally applied bipolar currents, which is not necessarily applicable to the detection of magnetic fields induced by neuronal electrical activities.

In this paper, we performed an analysis of the theoretical limit of sensitivity using gradient echo (GRE) according to the theory of the signal-to-noise ratio in MRI and compared the theoretical limit with the practical limit by means of a phantom experiment. In addition, we investigated the dependence of the theoretical limit of sensitivity on acquisition parameters.

Theory

Signal intensity

The magnetization $M_0$ (nuclear magnetic moments per unit volume) induced in a sample by the main static magnetic field $B_0$ is calculated with the following equation:

$$M_0 = N_S g^2 h^2 I(I + 1) B_0 B_1 M_0 L^2 th \times \left(1 - \exp\left(-\frac{T_R}{T_1}\right)\right) \exp\left(-\frac{T_E}{T_2^*}\right) \sin \theta$$

where $N_S$ is the $^1$H density of the sample, $\gamma$ is the gyromagnetic ratio of $^1$H ($\gamma = 2.67 \times 10^8$ rad/T·s), $h$ is Planck’s constant ($1.05 \times 10^{-34}$ J·s), $I$ is the spin quantum number ($^1$H: $I = 1/2$), $k_B$ is the Boltzmann constant (1.38 $\times 10^{-23}$ J/K), and $T_S$ is the absolute temperature of the sample.7 When a magnetic resonance (MR) image is obtained with a field of view of $L \times L$ and a slice thickness of $th$, the signal intensity $S$ per voxel is

$$S = \gamma B_0 B_1 M_0 L^2 th \times \frac{1 - \exp\left(-\frac{T_R}{T_1}\right)\exp\left(-\frac{T_E}{T_2^*}\right)}{1 - \cos \theta \exp\left(-\frac{T_R}{T_1}\right)} \sin \theta$$

where $T_R$ is the repetition time, $T_1$, and $T_2^*$ are the relaxation times of the sample, and $\theta$ is the flip angle. $B_1$ is the intensity of magnetic field that the unit current flowing in the receiver coil produces at
the voxel.8

Noise

In MR images, noise is attributable partly to conductors in the receiver coil and partly to the sample. The effective resistance $R_C$ of a one-turn coil is

$$R_C = \frac{\rho l}{pr} = \frac{l}{p}(\mu_0 \omega_0 \rho/2)^{1/2}$$ (3)

where $\rho$ is the resistivity of the coil, $l$ its length, $p$ its circumference, $\mu_0$ its permeability, and $\delta$ the radio frequency skin depth.9 The effective resistance $R_S$ of a columnar sample is

$$R_S = \frac{1}{8} \pi \sigma \omega_0^2 B_1^2 r_S^4 h$$ (4)

where $\omega_0$ is the magnetic resonance frequency ($\omega_0 = \gamma B_0$), $\sigma$ the conductivity of the sample, $r_S$ its radius, and $h$ its height.10 In an MR image consisting of $n \times n$ voxels, the Johnson noise $N$ due to the coil and sample per voxel is

$$N = n \sqrt{4k_B \Delta f (T_C R_C + T_S R_S)}$$ (5)

where $\Delta f$ is the spectral width of the receiver circuit and $T_C$ is the absolute temperature of the coil.9

Theoretical limit of sensitivity

The intensity $\beta$ of a weak magnetic field in a sample is estimated from a change in the MR signal.6 The uncertainty $\sigma_\beta$ in the estimated value of the magnetic field due to the noise in the MR signal is

$$\sigma_\beta = \frac{N}{S \gamma T_E}$$ (6)

The weak magnetic field $\beta$ can be detected when $\beta$ is higher than the uncertainty $\sigma_\beta$. Thus, $\sigma_\beta$ gives the theoretical limit of sensitivity for magnetic fields.

Experiments

To evaluate the practical limit of sensitivity for detecting the magnetic field induced by currents, we obtained GRE phase images of a columnar phantom with a 4.7T MRI system. Figure 1 shows a schematic of the columnar phantom. Copper electrodes were attached to the top and bottom. The phantom was filled with a gel containing 1% agarose and 0.9% NaCl. To correct phase drifts, four columnar references filled with the gel were placed around the phantom. To obtain images, the phantom was located at the center of a one-turn surface coil. Table 1 shows parameters of the phantom, the references, and the surface coil. The radius $r_S$ and height $h$ of the phantom were $6.5 \times 10^{-3}$ m and $5.0 \times 10^{-2}$ m, respectively. Rectangular electrical current pulses (20 mA, 10 mA, 5 mA, ..., 100 $\mu$A) were applied to the phantom with a duration $t_i$ of 2 ms between the echo time $T_E$. The current densities in the phantom were 150 A/m$^2$, 75 A/m$^2$, 38 A/m$^2$, ..., 0.75 A/m$^2$, and the corresponding voltages were 16 V, 8 V, 4 V, ...

![Fig. 1. Columnar phantom fitted with circular copper electrodes on top and bottom.](image)

The phantom was filled with a gel containing 1% agarose and 0.9% NaCl. To correct phase drifts, four columnar references filled with the gel were placed around the phantom. The phantom was located at the center of a one-turn surface coil to obtain images.

### Table 1. Parameters of the phantom, references, and surface coil

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Phantom</th>
<th>Surface Coil</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius $r_S$</td>
<td>$6.5 \times 10^{-3}$ m</td>
<td>$8.8 \times 10^{-2}$ m</td>
<td>$1.5 \times 10^{-3}$ m</td>
</tr>
<tr>
<td>Height $h$</td>
<td>$5.0 \times 10^{-2}$ m</td>
<td>$5.0 \times 10^{-1}$ m</td>
<td>$4.0 \times 10^{-2}$ m</td>
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<tr>
<td>Conductivity $\sigma$</td>
<td>2.0 S/m</td>
<td>$1.26 \times 10^{-6}$ H$\cdot$m$^{-1}$</td>
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</tr>
<tr>
<td>Temperature $T_S$</td>
<td>295 K</td>
<td>$1.72 \times 10^{-4}$ Q$\cdot$m</td>
<td></td>
</tr>
<tr>
<td>T1 relaxation time $T_1$</td>
<td>2.50 s</td>
<td>4.49 $\times 10^{-3}$ T/A</td>
<td></td>
</tr>
<tr>
<td>T2* relaxation time $T_2^*$</td>
<td>103 ms</td>
<td>$295$ K</td>
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</table>
80 mV. Table 2 shows the imaging parameters used in the phantom experiment. The static magnetic field $B_0$ was applied in the $+z$ direction with an amplitude of 4.7T. The imaging parameters were not optimized because the optimized parameters could result in an extremely small noise intensity.

Phase angle images were generated from the measured GRE signals. The phase images were unwrapped to eliminate discontinuities in phase. To compute the phase shift $\Delta \phi$, the phase image without electrical current pulses was subtracted from the phase images with the pulses. The intensity of the magnetic fields generated by the electric currents was calculated from the phase shift with the following equation

$$\Delta B = \frac{\Delta \phi}{\gamma T_i}$$

where $\Delta B$ is the magnetic field generated by the electrical currents.

Using equation (6), we analyzed the dependence of the theoretical limits of the sensitivity $\sigma_B$ on parameters: $T_R/T_1$, $T_E/T_2^*$, number of voxels $n$, and spectral width $\Delta f$.

**Results and Discussion**

Figure 2 shows magnetic field images with electrical currents of 10 mA and 1 mA and a theoretical prediction of the magnetic field. The top of the images is the $+z$ direction, and the right of the images is the $+x$ direction. The solid lines indicate the theoretical predictions of the magnetic field induced by electrical currents in the x-axis of the phantom. The dots indicate the experimentally obtained magnetic fields. The dashed lines indicate the theoretical limit of sensitivity. According to Ampere’s rule, the magnetic field on the x-axis can be calculated with the following equation:

$$\Delta B = \frac{\mu_0}{2} \frac{I_x}{\pi r_x^2} x$$

Table 2. Imaging parameters used in the phantom experiment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Repetition time $T_R$</td>
<td>900 ms</td>
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<tr>
<td>Echo time $T_E$</td>
<td>5 ms</td>
</tr>
<tr>
<td>Flip angle $\theta$</td>
<td>90°</td>
</tr>
<tr>
<td>Field of view $L$</td>
<td>32 mm</td>
</tr>
<tr>
<td>Number of voxels $n$</td>
<td>128</td>
</tr>
<tr>
<td>Spectral width $\Delta f$</td>
<td>80321.3 kHz</td>
</tr>
<tr>
<td>Slice thickness $th$</td>
<td>0.5 mm</td>
</tr>
</tbody>
</table>

![Fig. 2. Magnetic field images and theoretical predictions for electrical current intensities of (a) 10 mA and (b) 1 mA. The top of the images is the $+z$ direction and the right of the images is the $+x$ direction. The solid lines indicate the theoretical predictions of the magnetic field induced by electrical currents on the x-axis of the phantom. The dots indicate the experimentally obtained magnetic fields. The dashed lines indicate the theoretical limit of sensitivity.](image)
where $I_S$ is the intensity of the electrical current applied to the phantom.

Under these conditions, the theoretical limit of sensitivity $\sigma_B$ was $1.5 \times 10^{-8}$T. At 10 mA, the experimentally obtained magnetic field was in good agreement with the theoretical prediction. At 1 mA, the image marginally showed the generated magnetic field, though the noise in the image was comparable in intensity to the magnetic field.

The practical limit of sensitivity was statistically evaluated with Student’s t-test. Figure 3 shows the levels of statistical significance in the difference between the phase images obtained with and without the electrical currents. The pixels with a level of statistical significance below 1% were extracted and superimposed on a magnetic field image. An increase in the intensity of the electrical current resulted in a decrease in the level of significance. The pixels exhibiting statistical significance were found mainly on the left and right edges of

**Fig. 3.** Results of Student’s t-test for electrical current intensities of (a) 10 mA and (b) 1 mA. The color indicates the level of statistical significance in Student’s t-test.

**Fig. 4.** Dependence of the theoretical limits of sensitivity for detecting magnetic fields in parameters (a) $T_R/T_1$, (b) $T_E/T_2^*$, (c) number of pixels $n$ and (d) spectral width $\Delta f$. 

...
the phantom, which was consistent with the theoretical prediction. We defined the practical limit of sensitivity as the level of 1%. The practical limit of sensitivity was about 10 times the theoretical limit of sensitivity. In calculating the theoretical limit of sensitivity, we attributed the noise in the MR images to the Johnson noise from the coil and sample. However, the practical limit was greater because of unconsidered factors such as radiation loss and system noise.

Figure 4 shows dependences of the theoretical limit of sensitivity on the following parameters: (a) $T_R/T_1$, (b) $T_E/T_2^*$, (c) number of pixels $n$ and (d) spectral width $\Delta f$. An increase in $T_R/T_1$ caused a decrease in the theoretical limit of sensitivity, while an increase in the number of pixels $n$ or the spectral width $\Delta f$ caused an increase in the theoretical limit of sensitivity. The dependence on $T_E/T_2^*$ indicates the existence of an optimal value in $T_E/T_2^*$.

Conclusion

In this study, we analyzed the theoretical limit of sensitivity for detecting magnetic fields induced by electrical currents by computing the theoretical value of signal and noise in GRE phase images. We also evaluated the practical limit of sensitivity. In the case of the columnar phantom with injection of pulsed electrical currents, the theoretical and practical limits of sensitivity were approximately $10^{-9}$T and $10^{-7}$T, respectively. The dependence of the theoretical limits of sensitivity on acquisition parameters was quantitatively evaluated.

Acknowledgments

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References

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