Quasi-Static Finite Element Analysis of Magnetostrictive Vibration Energy Harvester

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The paper aims at the behaviour analysis of the Galfenol rods under bending conditions that are employed in a vibration energy harvester by illustrating the spatial variations in stress and magnetic field. Armstrong model is employed to predict the magnetization and magnetostriction as a function of stress and magnetic field and it is incorporated into a static finite element model of vibration-driven generator by which the compressive and tensile stresses contributions to change in magnetic flux densities in the Galfenol rods were calculated and the effectiveness of the inspected vibration-driven generator in voltage generation and energy harvesting is demonstrated.

Key words: magnetostrictive material, vibration energy harvester, finite element method.

1. Introduction

Magnetostrictive materials are being employed in a host of applications ranging from active control and energy harvesting to torque and force sensing. The development of iron-gallium alloys (Galfenol) has improved the possibilities to build devices based on magnetostrictive phenomenon.

Recently, a micro energy harvester has been proposed using Galfenol to produce electricity from vibration energy¹⁾. Contrary to Terfenol-D and piezoelectric materials, Galfenol's high strength and ductility have made it a popular option in applications involving bending²⁾.

Studies on Galfenol behaviour involving bending mode have been performed using Galfenol unimorph sensors and laminated composites having Galfenol attached to other structural materials^{3),4)}. In this paper, we consider the vibration energy harvester^{1),9),10)} in order to investigate the behaviour of Galfenol in bending conditions. A model for simulation of the vibration energy harvester, based on static finite element method, is developed and the multiaxial magneto-elastic behaviour of Galfenol is included in the model using Armstrong model^{5),6)}. Contrary to the models previously developed and based on a strong coupling approach of magnetostrictive problem⁷⁾, the approach employed in this paper is based on a so-called weak coupling approach to consider the bidirectional coupling between magnetic and mechanical problems⁸⁾.

The multiphysics finite element package COMSOL allows the magnetostrictive strain tensor to be implemented directly using the actual properties of the materials involved within the system. Finally, experimental results are presented which show the agreement between the numerical derivations and experimental results.

2. Vibration energy harvester

The energy harvester consists of two parallel square rod of Galfenol ($\text{Fe}_{81.6}\text{Ga}_{18.4}$, 0.5mm by 1mm area and 10 mm length, magnetically easy axis in

longitudinal direction) is shown in Figs. 1 and 2. On each Galfenol rod a coil of 312 turns is wound (0.05 mm diameter wire, 12 Ω). In addition, one yoke is bonded to a fixture and the other one to a mover (Al, 0.64g) which oscillates by external force. Two pieces of Nd-B-Fe permanent magnets (2mm diameter and 2mm length) are used to provide adequate bias flux for the rods and the attached back iron yokes close the magnetic circuit.

The fundamental operation principle of the energy harvester is based on the inverse magnetostrictive effect that the magnetization changes with the stress. When a transverse load is applied to the mover as shown in Fig. 2, one Galfenol rod is compressed and the other one is stretched leading to relative permeability change in both Galfenol rods, which causes the magnetic flux density to drop inside the compressed rod and to rise inside the stretched rod. Therefore, voltages are induced in the coils around Galfenol rods due to time-varying magnetic fields and the vibration energy is harvested.

Fig. 1 Sectional view of the device.

Fig. 2 Principle of energy harvesting (electricity generation by vibrating the device).

3. Finite Element Modeling

3.1 The Armstrong model

The Armstrong model is an energy based model, that its suitability to capture the behavior of Galfenol in both actuation as well as sensing applications has been shown^{(3),(4),(5),(6),(13)}. This model finds the distribution of orientation of various domains to a particular stress and magnetic field, and this distribution determines the macroscopic response of the material. In this work, we will use the anhysteretic modeling technique which is deemed suitable for Galfenol as Fe-Ga alloys exhibit negligible hysteresis.

Consider a magnetic field (H) and stress (σ) applied to a single crystal sample as shown in Fig. 3, so that the direction cosines they form with the crystallographic directions are respectively $(\beta_{1F}, \beta_{2F}, \beta_{3F})$ and $(\beta_{1S}, \beta_{2S}, \beta_{3S})$. The energy corresponding to the magnetization orienting in a direction $(\alpha_1, \alpha_2, \alpha_3)$ is given by⁽¹³⁾:

$$W_a(\sigma, H) = K_1(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_1^2\alpha_3^2) + K_2(\alpha_1^2\alpha_2^2\alpha_3^2) + \left\{ -\frac{3}{2}\lambda_{100}\sigma(\alpha_1^2\beta_{1S}^2 + \alpha_2^2\beta_{2S}^2 + \alpha_3^2\beta_{3S}^2) - 3\lambda_{111}\sigma(\alpha_1\alpha_2\beta_{1S}\beta_{2S} + \alpha_2\alpha_3\beta_{2S}\beta_{3S} + \alpha_1\alpha_3\beta_{1S}\beta_{3S}) \right\} - \mu_0 M_s H(\alpha_1\beta_{1F} + \alpha_2\beta_{2F} + \alpha_3\beta_{3F}) \quad (1)$$

where M_s , μ_0 , K_1 and K_2 are saturation magnetization, vacuum permeability, cubic and uniaxial magnetocrystalline anisotropy constants, respectively. It is reasonable to assume that the orientation of the domains follows a Boltzmann distribution under the condition of non-interaction of domains as well as independence of the present domain orientation on its previous state (this assumption results in an anhysteretic model). The probability of the magnetization to be oriented along a direction (θ, φ) will therefore be:

$$f(\theta, \varphi) = \frac{e^{-W_a(\theta, \varphi)/\omega}}{\int e^{-W_a(\theta, \varphi)/\omega} d\Omega} \quad (2)$$

where ω is the potential distribution parameter⁽⁵⁾, θ and φ are the azimuthal and elevation angles, respectively, and $d\Omega = \sin\theta d\theta d\varphi$. The direction cosines α are related to (θ, φ) as $\alpha_1 = \sin\theta \cos\varphi$, $\alpha_2 = \cos\theta \sin\varphi$ and $\alpha_3 = \cos\theta$. Once the probability $f(\theta, \varphi)$ is defined, the net magnetization in any direction $(\beta_i, \beta_j, \beta_k)$ is calculated using Eq. 3. Magnetic flux density B is then determined by Eq. 4:

$$M(\sigma, H) = M_i \beta_i + M_j \beta_j + M_k \beta_k \quad (3)$$

$$B(\sigma, H) = \mu_0 (M(\sigma, H) + H) \quad (4)$$

where M_i , M_j and M_k correspond to the macroscopic (average) magnetization along [100], [010] and [001] directions respectively. For instance M_i is given by:

$$M_i = \int M_s \sin(\theta) \cos(\varphi) f(\theta, \varphi) d\Omega \quad (5)$$

for: $(\beta_{1F}, \beta_{2F}, \beta_{3F}) = (1, 0, 0)$

The expected value of magnetostriction $\lambda(\sigma, H)$ along the direction $(\beta_i, \beta_j, \beta_k)$ is similarly calculated by taking the ensemble average of λ defined in terms of λ_{100} , λ_{111} and direction of magnetization. A more detailed development of the model can be found in references^{(5),(11)}.

The rod of Fe_{81.6}Ga_{18.4} is annealed under compressive stress (219MPa, 600°C, for 20 minutes), that its corresponding parameters could be found in [2] as $K_1 = -13.4 \text{ kJ m}^{-3}$, $K_2 = 13.6 \text{ kJ m}^{-3}$, $\lambda_{100} = 170 \times 10^{-6}$, $\lambda_{111} = -4.7 \times 10^{-6}$ and $M_s = 1281 \text{ kA m}^{-1}$. The Armstrong model has been fitted into the measured coupling variables in the direction of [001] by choosing the above-mentioned parameter values (Fig. 4, 5 and 6) and it can predict the coupling variables in other directions⁽¹¹⁾.

Fig. 3 Crystallographic frame showing stress (σ), magnetic field (H) and orientation of magnetization (M).

Fig. 4 Comparison between model predicted and experimentally determined $M(\sigma, H)$ curves in the direction of [001] for Fe_{81.6}Ga_{18.4}.

Fig. 5 Relative permeability $\mu_r(\sigma, H)$ versus stress σ and magnetic field H in the same direction of [001].

(a) $\lambda(\sigma, H)$ curves for compressive stress (b) $\lambda(\sigma, H)$ curves for tensile stress (c) $\lambda(\sigma, H)$ in a mesh grid
Fig. 6 Magnetostriction curves in the direction of [001].

3.2 Modeling of the Mechanical Aspect

In the static case, the mechanical problem is formulated by the following formulas⁴⁾:

$$\tilde{\varepsilon} = \nabla^s \tilde{u} \quad (6)$$

$$\nabla \cdot \tilde{\sigma} + \tilde{F}_{body} = 0 \quad (7)$$

$$\tilde{\sigma} = \tilde{C}(\tilde{\varepsilon} - \tilde{\lambda}(\tilde{\sigma}, \tilde{H})) \quad (8)$$

where F_{body} is the body force, u the displacement, ε the mechanical strain and $\lambda(\sigma, H)$ is the magnetostriction's contribution to strain, which varies depending on mechanical stress σ and magnetic field H in the Galfenol rod. The symbol $\sim$ over a quantity denotes a vector or a tensor. For instance, the strain displacement relation in Eq. 6 represents:

$$\{\varepsilon_{11} \ \varepsilon_{22} \ \varepsilon_{33} \ \gamma_{12} \ \gamma_{23} \ \gamma_{31}\}^T = \begin{bmatrix} \partial/\partial x & 0 & 0 & \partial/\partial y & 0 & \partial/\partial z \\ 0 & \partial/\partial y & 0 & \partial/\partial x & \partial/\partial z & 0 \\ 0 & 0 & \partial/\partial z & 0 & \partial/\partial y & \partial/\partial x \end{bmatrix}^T \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad (9)$$

In passive materials, a constitutive equation is used to relate stress to strain according to Hooke's law. In active materials, however, this relation is modified using the coupling variable $\lambda(\sigma, H)$ calculated by the Armstrong model as shown in Eq. 8. When calculating the stress, it is necessary to subtract the contribution of magnetostriction from the corresponding strain. The stiffness matrix C for isotropic materials can be obtained from the Young's modulus (E) and Poisson's ratio (ν):

$$\tilde{C} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-2\nu \end{bmatrix} \quad (10)$$

that $E=70$ GPa and $\nu=0.45$ are considered for Galfenol.

Eqs. 6-8 can be used together with boundary conditions to formulate a weak form that can be solved using the finite element method:

$$\int_{\Omega} \delta \tilde{\varepsilon}^T \tilde{\sigma} dV - \int_{\Omega} \delta \tilde{u}^T \tilde{F}_{body} dV - \int_{\partial\Omega} \delta \tilde{u}^T \tilde{F}_s dS = 0 \quad (11)$$

The boundary conditions for mechanical problem are prescribed displacements ($\tilde{u} = \tilde{0}$ where constrained) and applied traction force $\tilde{\sigma} \cdot \hat{n} = \tilde{F}_s$.

3.3 Modeling of the Magnetic Aspect

For magnetostatic problems involving no electrical currents, the complex set of Maxwell's equations can be reduced to Eqs. 12 and 13:

$$\nabla \cdot \tilde{B} = 0 \quad (12)$$

$$\nabla \times \tilde{H} = 0 \quad (13)$$

Eq. 13 can be expressed in terms of a hypothetical scalar potential, V_m :

$$\tilde{H} = -\nabla V_m \quad (14)$$

The following constitutive relation relates magnetic flux density with the magnetic field:

$$\tilde{B} = \mu_0 \tilde{\mu}_r(\tilde{\sigma}, \tilde{H}) \tilde{H} + \tilde{B}_r \quad (15)$$

Here, B_r is the remnant magnetic flux density, which is non-zero for hard magnetic materials like permanent magnets and zero for all other materials. For active materials, the relative permeability $\mu_r(\sigma, H)$ is a diagonal matrix with each element corresponding to the relative permeability of the material along one of the i, j and k axes. Relative permeability along any axis is computed using the coupling variable $M(\sigma, H)$ (magnetization) calculated by the Armstrong model corresponding to the direction of the axis. The following equation shows the relative permeability of Galfenol for non-zero magnetic field which depends on H and σ :

$$\tilde{\mu}_r(\tilde{\sigma}, \tilde{H} \neq 0) = \frac{\tilde{M}(\tilde{\sigma}, \tilde{H}) + \tilde{H}}{\tilde{H}} \quad (16)$$

Eqs. 12-15 can be used together with boundary conditions to formulate a weak form that can be solved using the finite element method:

$$\int_{\Omega} \delta \tilde{H}^T \tilde{B} dV = 0 \quad (17)$$

The boundary conditions for the magnetostatic problem are magnetic insulation $\tilde{B} \cdot \hat{n} = 0$ at the boundaries of the air domain.

Fig. 7 Diagram of the weak coupling approach.

3.4 Weak Coupling Approach

The coupled problem is treated by an iterative process of successive magnetic and mechanical finite element computations. Fig. 7 illustrates the diagram of the weak coupling approach that has been implemented by COMSOL Multiphysics software. It includes two segregated steps, one step for magnetostatic problem while the mechanical variables are considered constant, and another step for mechanical problem in which the magnetic variables are considered constant. This process is iterated in a major loop until a user imposed convergence criterion is met. For instance, the relative error of magnetic flux density $\Delta B < 0.1\%$ and the relative error of stress $\Delta \sigma < 0.1\%$ have been considered in this work. Arbitrary initial guesses, preferably close to zero are used for H and σ and by considering the magnetic field of the permanent magnets and also the application of external stress, the iterative algorithm converges to the solution.

The highly nonlinear behavior of Galfenol is modeled well by employing the Armstrong energy-based model to obtain magnetization $M(\sigma, H)$ and

magnetostriction $\lambda(\sigma, H)$. Then relative permeability is computed using Eq. 16 and also the equivalent displacement to the magnetostriction vector $\lambda(\sigma, H)$ is computed and it is considered as initial displacement through software implementation. It is worth noting that the mechanical problem is solved by an iterative scheme due to the dependence of $\lambda(\sigma, H)$ on the applied stress σ .

4. Results

Fig. 8 depicts the vibration energy harvester in 3D model and shows a slice cut through the Galfenol rods that demonstrates the stress distribution inside it, caused by transverse load of 2 N. As a matter of fact the bending device is similar to a cantilever as one of the rods is compressed and the other one is stretched. It is worth mentioning that, conventionally, a tensile stress is considered positive whereas a compressive stress is considered negative.

Fig. 9(a) shows the corresponding spatial distribution of magnetostriction $\lambda(\sigma, H)$ of the Galfenol rods in microstrain unit and along the direction [001]. Because of the magnetomechanical coupling and non-uniformity in the magnetic field and stress states, the strain is non-uniform.

Fig. 9(b) shows the corresponding alterations to spatial distribution of relative permeability inside the Galfenol rods caused by compressive and tensile stresses. The relative permeability of the compressed rod tends to decrease and the relative permeability of the stretched rod tends to increase. It can be seen that the relative permeability of Galfenol ranges between 30 and 250.

Fig. 9(c) demonstrates the corresponding variations in spatial distribution of the x-component of the magnetic flux density inside the Galfenol rods as the flux density decreases in the compressed rod and increases in the stretched rod.

By applying the bending moment, the energy harvesting device is deflected and the free end of the device is displaced. The average flux densities B_{x-ave} inside the Galfenol rods versus the displacement of the free end are presented in Fig. 10. The average flux density in the stretched rod increases from the bias point of 0.18 to 0.66 T, while it decreases from the bias point of 0.18 to 0.1 T in the compressed rod. It is observed from the results that the variation in magnetic flux density caused by tensile stress is much higher than the one caused by compressive stress. This behavior can be explained in terms of the orientation of the magnetic moments under a given magnetic field and stress. When a bias magnetic field is applied along one of the six easy axes, most of the magnetic moments flip from other five easy axes and orient along this direction. Application of a compressive stress collinear to the bias field rotates the magnetic moments perpendicular to the direction of the applied field and stress, however, the bias field resists the rotation of the magnetic moments.

Fig. 8 (a) Energy harvester in 3D depicting magnetic flux lines. (b) Spatial distribution of stress σ_{xx} in the rods.

This results in relatively small changes in the stress induced magnetization for compressive stress. On the other hand, application of a tensile stress collinear to the bias field helps the magnetic moments to rotate from other easy axes directions towards the direction of the bias field and stress. This results in a higher response and larger change in the stress induced magnetization for tensile stresses compared to the compressive stresses.

It is important to note that the magnetic bias has a big impact on the behavior of Galfenol as shown in Fig. 4. Under low magnetic biases, the contribution of tension to the change in magnetic flux density would be higher than the one caused by compression, whereas, under high magnetic biases, the contribution of compression would be dominant.

The average flux densities do not almost vary by applying bending moments larger than 0.12 Nm due to magnetic and magnetostrictive saturation.

Experimental implementation was carried out by vibrating the device at the resonance frequency of 333 Hz, hence the bending moment leads to changes in magnetic flux density in both the compressed and stretched rods and results in the time-varying magnetic flux density. The displacement of the free end of the device was measured by laser sensor, and as mentioned earlier, on each Galfenol rod a coil of 312 turns is wound which were used to measure the linkage magnetic flux. It is noteworthy that the coils measure the differential of the magnetic flux density between the two parallel rods. Fig. 11 shows the experimental measurements of the tip displacement of the energy harvester, open-circuit voltage differential induced in the coils and the corresponding magnetic flux density¹⁾.

Fig. 12 shows the measured flux density versus the corresponding displacement caused by rods deflection. The presence of hysteresis in curves is mainly due to the

back-iron of the device while the Galfenol rods present negligible hysteresis. The calculated magnetic flux density differential between two Galfenol rods is compared to the measured one, which are in good agreement as depicted in Fig. 12.

5. Conclusion

The 3-D static finite element modeling that includes Armstrong's implementation to consider the multiaxial magneto-elastic behaviour of Galfenol, presented here highlights the spatial variations in magnetic field and relative permeability due to the corresponding stress distribution in the Galfenol rods subjected to transverse load. The compressive and tensile stresses contributions to change in magnetic flux densities in the Galfenol rods were calculated and about 1.1 T change in magnetic flux density is achieved which demonstrates the effectiveness of the inspected vibration-driven generator in voltage generation and energy harvesting applicable to wireless sensor networks. The model predictions agree with the experimental results and are coherent with the magnetostriction phenomenon.

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Fig. 9 Spatial distribution of magnetostriction, relative permeability and magnetic flux density of the rods in the same direction [001].

Fig. 10 Average flux density B_{x-ave} inside the two Galfenol rods.

Fig. 12 Comparison between experimentally determined and model predicted magnetic flux density differential between the two Galfenol rods.

Fig. 11 Measured deflection and open-circuit voltage responses by vibrating the energy harvester at the resonance frequency of 333 Hz.

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