Magnetic thin films with large magnetostriction coefficients have been studied for magnetic thin films. Fe-Co alloys with bcc structure are typical soft magnetic materials and have recently attracted much attention as one of magnetostrictive materials, since they show large magnetostrictive properties, it is useful to prepare epitaxial thin films, since the crystallographic orientation relationship between film and substrate can be controlled by the substrate orientation. Fe-Co epitaxial films have been prepared on single-crystal substrates of GaAs[11–10], MgO[17–23], MgAl2O4[22–24], SrTiO3[22–24], etc. However, the magnetostriction has not been investigated by employing Fe-Co epitaxial films, though there exist reports on the magnetostriction of polycrystalline films[8,25–31]. In the present study, FeCo films (x = 0–50 at. %) were prepared on MgO substrates of (001), (110), and (111) orientations. The influences of film orientation and composition on the magnetostriction are systematically investigated.

2. Experimental Procedure

An ultra-high vacuum system consisting of two chambers equipped with radio-frequency (RF) magnetron sputter deposition and reflection high-energy electron diffraction (RHEED) facilities was employed. The base pressure of deposition chamber was lower than 4 × 10–7 Pa. MgO(001), MgO(110), and Al2O3(0001) single-crystal substrates were used. Before film formation, substrates were heated at 600 °C in the deposition chamber to obtain clean surfaces, which were confirmed by RHEED (not shown here). MgO and FeCo alloy films were respectively fixed at 100 nm. The RF powers for MgO, Fe, FeCo, and FeCo targets were respectively adjusted to be 200, 50, 51, and 52 W. Under these conditions, the deposition rate was 0.015 nm/s for MgO, whereas it was 0.020 nm/s for the other materials. The substrate temperature during sputter deposition was kept constant at 300 °C.

FeCo films were formed on MgO(001) and MgO(110) substrates and MgO(111) underlayers hetero epitaxially grown on Al2O3(0001) substrates. The crystallographic orientation relationship between MgO underlayer and Al2O3 substrate was determined by RHEED as MgO(111)[110] and (111)[110] || Al2O3(0001)[110]. The MgO underlayer consisted of two (111) variants whose orientations were rotated around the film normal by 180° each other. The surface atomic arrangements of the two variants are the same. Therefore, only the crystallographic orientation of MgO(111)[110] || Al2O3(0001)[110] is used below. The thicknesses of MgO(001), MgO(110), and MgO(111)/Al2O3(0001) substrates were respectively 0.30, 0.30, and 0.43 mm, whereas that of FeCo film was fixed at 100 nm.

The crystallographic orientation relationship between film and substrate was determined by RHEED. The resulting film
structure was investigated by 2θ/0-θ scan out-of-plane and 2θ/θ-scan in-plane X-ray diffractions (XRDs) with Cu-Kα radiation (wave length: 0.15418 nm). The magnetocrystalline anisotropy was measured by vibrating sample magnetometry.

The magnetostriction was observed by using a cantilever method under a rotating magnetic field of 1.2 kOe. The bending was measured by using a laser displacement meter fixed on a vibration isolation table. The details of our measurement system and the Young’s modulus, Poisson’s ratio of Fe and MgO single crystals.

The relative length change, $\Delta l/l$, was calculated from the following formula,

$$\Delta l/l = \frac{\Delta S \cdot t^2 \cdot E_o \cdot (1 + \nu)}{3 \cdot L^2 \cdot t^2 \cdot E_o \cdot (1 + \nu)},$$  

where $\Delta S$ was the measured bending, $L$ was the distance between laser beam points ($12.5 \text{ mm}$), $t$ was the thickness, $E$ was the Young’s modulus, $\nu$ was the Poisson’s ratio, and the $x$ and $y$ subscripts of $f$ and $s$ respectively referred to film and substrate.

The $E$ and $\nu$ values of single crystal vary depending on crystallographic direction, though $E$ and $\nu$ are usually defined in an isotropic elastic body. In the present study, $E$ and $\nu$ are respectively defined as $\sigma_{[001]}$ and $-\epsilon_{[001]}$, where $\sigma$ is the uniaxial stress applied along $[g_1, g_2, g_3]$, and $\epsilon$ is the strain occurred along $[g_1, g_2, g_3]$. The elastic stiffness values of MgO and Fe single crystals calculated with Young’s modulus, Poisson’s ratios of MgO and Fe single crystals were used in the calculation of $\Delta l/l$ for Fe-Co alloy films, since the elastic stiffness values of Fe-Co alloys were unknown. For hexagonal Al$_2$O$_3$ crystal, the reported values of $E$ = 407.5±62.5 GPa (345–470 GPa) and $\nu$ = 0.285±0.015 (0.27–0.30) were used.

3. Results and Discussion

3.1 Film growth and structure

Figures 2(a)–(c) show the RHEED patterns observed for Fe$_{100}$-Co films with different compositions formed on MgO(001), MgO(110), and MgO(111)/Al$_2$O$_3$(0001) substrates, respectively. Figures 2(d)–(f) illustrate the diffraction patterns simulated for bcc(001) single-crystal, bcc(211) bi-crystal, and bcc(110) crystal with Nishiya-Wasserman (NW) and Kurdumov-Sachs (KS) variants, respectively. The details of the simulations have been shown in our previous papers.

The observed patterns of Figs. 2(a)–(c) are respectively in agreement with the simulated patterns of Figs. 2(d)–(f). Therefore, Fe$_{100}$-Co(001) single-crystal, (211) bi-crystal, and (110) crystal are respectively epitaxially grown on MgO(001), MgO(110), MgO(111)/Al$_2$O$_3$(0001) substrates for all the compositions of $x = 0–50$ at. %. The crystallographic orientation relationships are determined as

$$\text{Fe}_{100}\text{-Co}(001)[110]_{\text{boc}} || \text{MgO}(001)[100],$$  

$$\text{Fe}_{100}\text{-Co}(211)[0\overline{1}1]_{\text{boc}} || \text{MgO}(110)[001],$$

(type A)

$$\text{Fe}_{100}\text{-Co}(211)[0\overline{1}1]_{\text{boc}} || \text{MgO}(110)[001],$$

(type B)

Table 1 Young’s moduli of Fe and MgO single crystals.

<table>
<thead>
<tr>
<th>Crystal plane &amp; direction</th>
<th>MgO(001)[110]</th>
<th>MgO(001)[110]</th>
<th>MgO(110)[001]</th>
<th>MgO(111)[110]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{MgO}}$ [GPa]</td>
<td>245</td>
<td>308</td>
<td>336</td>
<td></td>
</tr>
<tr>
<td>$E_{\text{Fe}}$ (= $E_{\text{Fe}(\text{MgO})}$) [GPa]</td>
<td>132</td>
<td>221</td>
<td>284</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Poisson’s ratios of Fe and MgO single crystals.

<table>
<thead>
<tr>
<th>Poisson’s ratio</th>
<th>MgO(001)[100]</th>
<th>MgO(110)[001]</th>
<th>MgO(111)[110]</th>
<th>MgO(111)[110]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_{\text{MgO}}$</td>
<td>0.04</td>
<td>0.23</td>
<td>0.33</td>
<td>0.29</td>
</tr>
<tr>
<td>$\nu_{\text{Fe}}$ (= $\nu_{\text{Fe}(\text{MgO})}$)</td>
<td>0.27</td>
<td>-0.05</td>
<td>0.18</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Fig. 1 In-plane (a) $E$ and (b) $\nu$ distributions of Fe(110) single-crystal film.

$\gamma_1, \gamma_2, \gamma_3$ are respectively the cosines of angles [001] and $[d_1, d_2, d_3]$ with respect to the three crystallographic axes (a, b, c). Tables 1 and 2 summarize the $E$ and the $\nu$ values of MgO and Fe single crystals calculated with the reported values of ($C_{11}$, $C_{12}$, $C_{44}$) $\approx (286, 87, 148)^{35}$ and ($C_{11}$, $C_{12}$, $C_{44}$)$_{\text{Fe}}$ = (237, 141, 116)$^{30}$. Figures 1(a) and (b), respectively, show the in-plane $E$ and $\nu$ distributions of Fe(110) single-crystal film. The $E$ and $\nu$ values of Fe(110) film with multiple variants are respectively regarded as the averages of Fig. 1(a) ($E = 217$ GPa) and Fig. 1(b) ($\nu = 0.34$). In the present study, the $E$ and the $\nu$ values of Fe crystal were used in the calculation of $\Delta l/l$, since the elastic stiffness values of Fe-Co alloys were unknown. For hexagonal Al$_2$O$_3$ crystal, the reported values of $E = 407.5\pm62.5$ GPa (345–470 GPa) and $\nu = 0.285\pm0.015$ (0.27–0.30) were used.
Fig. 2 (a)–(c) RHEED patterns observed for (a-1)–(c-1) Fe, (a-2)–(c-2) Fe$_{70}$Co$_{30}$, and (a-3)–(c-3) Fe$_{50}$Co$_{50}$ films formed on (a) MgO(001), (b) MgO(110), and (c) MgO(111)/Al$_2$O$_3$(0001) substrates. (d)–(f) Schematic diagrams of RHEED patterns simulated for (d) bcc(001) single-crystal, (e) bcc(211) bi-crystal, and (f) bcc(110) multi-crystal with NW and KS variants. The incident electron beam is parallel to (a) MgO[100], (b) MgO[001], (c) MgO[110], (d) bcc[110], (e) bcc[110]+[001], or (f) bcc[001]+[111].

Figures 3(a-1)–(c-1) show the out-of-plane XRD patterns of the Fe$_{100-x}$Co$_x$ epitaxial films with different orientations, bcc(002), bcc(211), and bcc(110) reflections are observed for the films formed on MgO(001), MgO(110), and MgO(111)/Al$_2$O$_3$(0001) substrates, respectively. Figures 3(a-2)–(c-2) show the in-plane XRD patterns of the epitaxial films formed on MgO(001), MgO(110), and MgO(111)/Al$_2$O$_3$(0001) substrates measured by making the scattering vector parallel to MgO[110], MgO[001], and MgO[111] (∥ Al$_2$O$_3$(1100), respectively. bcc(200) reflection is observed in the patterns of Fe$_{100-x}$Co$_x$(001) single-crystal films [Fig. 3(a-2)], bcc(011) reflection from the A-type variant and bcc(011) reflection from the B-type variant.

Fe$_{100-x}$Co$_x$(110)[001]$_{bcc} ||$ MgO(111)[110], (type NW1)
Fe$_{100-x}$Co$_x$(110)[001]$_{bcc} ||$ MgO(111)[011], (type NW2)
Fe$_{100-x}$Co$_x$(110)[001]$_{bcc} ||$ MgO(111)[101], (type KS1)
Fe$_{100-x}$Co$_x$(110)[111]$_{bcc} ||$ MgO(111)[100], (type KS2)
Fe$_{100-x}$Co$_x$(110)[111]$_{bcc} ||$ MgO(111)[110], (type KS3)
Fe$_{100-x}$Co$_x$(110)[111]$_{bcc} ||$ MgO(111)[110], (type KS4)
Fe$_{100-x}$Co$_x$(110)[111]$_{bcc} ||$ MgO(111)[011], (type KS5)
Fe$_{100-x}$Co$_x$(110)[111]$_{bcc} ||$ MgO(111)[110], (type Kβ5)

which are similar to the case of Fe$_{100-x}$Co$_x$ film growth by molecular beam epitaxy.

Fig. 3 (a-1)–(c-1) Out-of-plane and (a-2)–(c-2) in-plane XRD patterns of Fe, Fe$_{70}$Co$_{30}$, and Fe$_{50}$Co$_{50}$ films formed on (a) MgO(001), (b) MgO(110), and (c) MgO(111)/Al$_2$O$_3$(0001) substrates. The scattering vector of in-plane XRD is parallel to (a-2) MgO[110], (b-2) MgO[110], and (c-2) MgO[110] (∥ Al$_2$O$_3$(1100)). The intensity is shown in logarithmic scale. (d)–(f) Compositional dependences of out-of-plane and in-plane lattice spacings of Fe$_{100-x}$Co$_x$ films formed on (d) MgO(001), (e) MgO(110), and (f) MgO(111)/Al$_2$O$_3$(0001) substrates.
small.

Epitaxial orientation relationships determined by RHEED. Figures 4(a-1)–(c-1) in-plane magnetization curves and (a-2)–(c-2) $M_r/M_s$ distributions measured for (a) Fe, (b) Fe$_{50}$Co$_{50}$, and (c) Fe$_{50}$Co$_{50}$(001) single-crystal films formed on MgO(001) substrates. The applied magnetic field directions are shown by using the crystallographic directions of Fe$_{50}$Co$_{50}$ film.

Figures 4(a-1)–(c-1) in-plane magnetization curves and (a-2)–(c-2) $M_r/M_s$ distributions measured for (a) Fe, (b) Fe$_{50}$Co$_{50}$, and (c) Fe$_{50}$Co$_{50}$(211) bi-crystal films formed on MgO(110) substrates. The applied magnetic field directions are shown by using the crystallographic directions of two Fe$_{50}$Co$_{50}$(211) variants.

Fig. 5(a-1) shows the hysteresis curves of the Fe(211) film. The applied magnetic field directions are shown by using the crystallographic directions determined by RHEED. Figures 3(d)–(f) summarize the out-of-plane and in-plane lattice spacings of Fe$_{50}$Co$_{50}$ films, which are estimated from the XRD data. The lattice parameters of Fe$_{50}$Co$_{50}$ films agree with small differences within ±0.7% with those of bulk Fe$_{50}$Co$_{50}$ crystal, which indicates that the lattice strains of Fe$_{50}$Co$_{50}$ films are small.

3.2 Magnetic anisotropy

Figures 4(a-1) and (b-1) show two typical examples of in-plane magnetization curves measured for the Fe and the Fe$_{50}$Co$_{50}$(001) single-crystal films. The distributions of normalized remnant magnetization, $M_r/M_s$, are summarized in Figs. 4(a-2) and (b-2). The applied magnetic field directions are shown by using the crystallographic directions of Fe$_{50}$Co$_{50}$ film. The films show four-fold symmetric in-plane magnetic anisotropies. The easy magnetization directions are observed along [100], [010], [100], and [010] (blue solid lines in Figs. 4(a-2) and (b-2)), which is reflecting the magnetocrystalline anisotropy of Fe$_{50}$Co$_{50}$ crystal with the easy magnetization axes parallel to <100>. Figure 4(c) shows the magnetization property of the Fe$_{50}$Co$_{50}$(001) single-crystal film. Although a four-fold symmetry in in-plane magnetic anisotropy is recognized, the easy magnetization directions are parallel to [110], [110], [110], and [110] (orange dotted lines in Fig. 4(c-2)), which are different from those observed for Fe and Fe$_{50}$Co$_{50}$ films. It is known that the easy magnetization axes of bulk Fe$_{50}$Co$_{50}$ crystal vary from <100> to <111> when the Co content increases beyond about 40 at. %$^6$. Therefore, the in-plane magnetic anisotropy observed for Fe$_{50}$Co$_{50}$ film seems to be reflecting the magnetocrystalline anisotropy of Fe$_{50}$Co$_{50}$ crystal with the easy axes parallel to <111> and the demagnetization field. The magnetic anisotropy of Fe$_{50}$Co$_{50}$(001) film varies depending on the composition, similar to the case of bulk crystal.

Figures 5(a-1) shows the hysteresis curves of the Fe(211) bi-crystal film. The distributions of $M_r/M_s$ and saturation field ($H_s$) are respectively summarized in Figs. 5(a-2) and (a-3). The applied field directions are shown by using the crystallographic directions of two (211) variants. The Fe film shows a two-fold
symmetric in-plane magnetic anisotropy. The easy magnetization directions are observed along \([215]_A, [215]_B, [251]_A,\) and \([215]_B\) (pink dotted lines in Figs. 5(a-2) and (a-3)), which are respectively obtained by projecting \([010]_A, [001]_A, [010]_B,\) and \([001]_B\) on the \((211)\) surface as shown in Fig. 6. Therefore, the magnetic anisotropy is interpreted to be reflecting the magnetocrystalline anisotropy of \(\text{Fe}_{50}\text{Co}_{50}\) crystal with the easy axes parallel to \(<111>\) and the demagnetization field. Figure 5(b) shows the in-plane magnetic anisotropy is observed. However, the easy magnetization directions are parallel to \([011]_A+\langle 011\rangle_B\) and \([011]_A+\langle 011\rangle_B\) (blue solid lines in Figs. 5(c-2) and (c-3)). It is also noted that the \(H_i\) values measured along \([111]_A+[111]_B\) and \([111]_A+[111]_B\) are not so high (orange dotted lines in Fig. 5(c-2)). Therefore, the film is moderately easily magnetized along \([111]_A+[111]_B\) and \([111]_A+[111]_B\). Moreover, the magnetization curves measured along \([251]_A, [215]_A, [251]_B,\) and \([215]_B\), which are respectively obtained by projecting \([010]_A, [001]_A, [010]_B,\) and \([001]_B\) on the film plane, saturate at higher magnetic fields (pink dotted lines in Fig. 5(c-3)). When the Co content increases up to 50 at. %, the easy and hard magnetization axes are respectively considered to be parallel to \(<111>\) and \(<100>\). Therefore, the \(\text{Fe}_{50}\text{Co}_{50}\) film shows the in-plane magnetic anisotropy reflecting the magnetocrystalline anisotropy of \(\text{Fe}_{50}\text{Co}_{50}\) crystal with the easy magnetization axes parallel to \(<111>\) and the demagnetization field. Figure 5(b) shows the magnetization property of the \(\text{Fe}_{50}\text{Co}_{50}\) film. The \(M_i/M_s\) and the \(H_i\) distributions of \(\text{Fe}_{50}\text{Co}_{50}\) film are similar to those of \(\text{Fe}_{50}\text{Co}_{50}\) result. The tilt composition where the easy magnetization axes change from \(<100>\) to \(<111>\) seems to be delicately influenced by the film orientation.

Figure 7 shows the in-plane magnetic properties of the \(\text{Fe}_{50}\text{Co}_{50}\) (110) epitaxial films with NW and KS variants formed on \(\text{MgO}(111)/\text{Al}_2\text{O}_3(0001)\) substrates. The films show almost isotropic magnetization properties. Nine \((110)\) variants are coexisting in the \((110)\) epitaxial films and the respective magnetic anisotropies are overlapped. Therefore, isotropic magnetization properties are considered to be observed.

3.3 Magnetostriiction of \((001)\) single-crystal films

The relative length change, \(\Delta l/l\), of a cubic crystal caused by magnetostriiction \((\gamma)\) is shown as

\[
\Delta l/l = \frac{3}{2} \lambda_{100} (\alpha_1^2 \beta_2^2 + \alpha_2^2 \beta_1^2 + \alpha_3^2 \beta_3^2 - \frac{1}{3}) + 3 \lambda_{111} (\alpha_1 \alpha_2 \beta_3 + \alpha_2 \alpha_3 \beta_1 + \alpha_3 \alpha_1 \beta_2),
\]

where \(\lambda_{100}\) and \(\lambda_{111}\) are the magnetostriiction coefficients, \((\alpha_1, \alpha_2, \alpha_3)\) and \((\beta_1, \beta_2, \beta_3)\) are respectively the cosines of the angles of magnetization and observation directions with respect to the three crystallographic axes \((a, b, c)\).

When the magnetization rotates in a \((001)\) plane under in-plane rotating magnetic field as shown in Fig. 8(a), the crystallographic direction of magnetization is shown as \((\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta)\), where \(\phi\) is the angle of magnetization direction with respect to \([100]\). The \((\alpha_1, \alpha_2, \alpha_3)\) values are thus expressed as

\[(\alpha_1, \alpha_2, \alpha_3)_{(001)} = (\cos \phi, \sin \theta, 0).\]

When the observation directions are parallel to \([100]\) and \([110]\), the \((\beta_1, \beta_2, \beta_3)_{(100)}\) and \((\beta_1, \beta_2, \beta_3)_{(110)}\) values are respectively expressed as

\[(\beta_1, \beta_2, \beta_3)_{(100)} = (1, 0, 0),\]

\[(\beta_1, \beta_2, \beta_3)_{(110)} = (0, 1/\sqrt{2}, 1/\sqrt{2}).\]

By substituting Eqs. (5)–(7) into (4), the relative length changes measured along \([100]\) and \([110]\) under in-plane rotating
The phases of observed waves are in agreement with that of calculated wave of Fig. 8(c-2), whereas the phase of wave observed for Fe50Co50 film is in agreement with that of wave of Fig. 8(c-1). Therefore, the $\lambda_{111}$ value is negative for the Fe and the Fe50Co50 films, while that is positive for the FeCo50 film.

Figure 8(e) shows the $\lambda_{100}$ and the $\lambda_{111}$ values plotted as a function of Co content. The $\lambda_{100}$ and the $\lambda_{111}$ values increase with increasing the Co content. The Fe50Co50 film shows a large $\lambda_{100}$ value of $+234 \times 10^{-6}$ and a small $\lambda_{111}$ value of $-5 \times 10^{-6}$. On the contrary, a large $\lambda_{100}$ value of $+274 \times 10^{-6}$ and a moderately large $\lambda_{111}$ value of $+78 \times 10^{-6}$ are observed for the FeCo50 film.

### 3.4 Magnetostriiction of (211) bi-crystal films

When the magnetization rotates in a (211)$_\lambda$ plane as shown in Fig. 9(a), the crystallographic direction of magnetization is shown as $\{ -\sin\chi/\beta^2 \cos\phi/\beta^2 + \sin\chi/\beta^2 \cos\phi/\beta^2 - \sin\phi/\beta \}$. Here, $\chi$ is the angle of magnetization direction with respect to $[011]_\lambda$ (// MgO[001]). The $(\alpha_1, \alpha_2, \alpha_3)_{211A}$ values are thus expressed as

$$ (\alpha_1, \alpha_2, \alpha_3)_{211A} = ( -\sin \phi/\beta, \cos \phi/\beta, -\sin \phi/\beta ). $$

When the observation directions are parallel to $[011]_\lambda$ and $[111]_\lambda$, the $(\beta_1, \beta_2, \beta_3)_{011A}$ and the $(\beta_1, \beta_2, \beta_3)_{111A}$ values are respectively expressed as

$$ (\beta_1, \beta_2, \beta_3)_{011A} = ( -\sin \phi/\beta, \cos \phi/\beta, -\sin \phi/\beta ). $$

$$ (\beta_1, \beta_2, \beta_3)_{111A} = ( -\sin \phi/\beta, \cos \phi/\beta, -\sin \phi/\beta ). $$
\[ \lambda_{000} = \frac{1}{4} \lambda_{111} \cos 2 \chi + \left( \frac{1}{8} \lambda_{100} + \frac{1}{8} \lambda_{111} \right) \cos 2 \lambda_{111}, \]  
\[ \lambda_{011} = \frac{3}{4} \lambda_{111} \cos 2 \chi + \frac{1}{4} \lambda_{111}. \]  

By substituting Eqs. (12)–(14) into (4), the \( \Delta l/|l_{111}|(\chi) \) and the \( \Delta l/|l_{111}|(\chi) \) are respectively given as

\[ \frac{\Delta l}{l_{011}}(\chi) = -\left( \frac{1}{4} \lambda_{111} \cos 2 \chi + \frac{1}{8} \lambda_{100} + \frac{1}{8} \lambda_{111} \right), \]

\[ \frac{\Delta l}{l_{011}}(\chi) = -\frac{3}{4} \lambda_{111} \cos 2 \chi + \frac{1}{4} \lambda_{111}. \]

Since the Fe\(_{100-x}\)Co\(_x\)(211) films consist of two types of variants, A and B, it is necessary to take into account the \( \Delta l/|l_{011}|(\chi) \) and the \( \Delta l/|l_{111}|(\chi) \), which are respectively shown as follows,

\[ \frac{\Delta l}{l_{011}}(\chi) = -\left( \frac{1}{4} \lambda_{111} \cos 2 \chi + \frac{1}{8} \lambda_{100} + \frac{1}{8} \lambda_{111} \right), \]

\[ \frac{\Delta l}{l_{111}}(\chi) = -\frac{3}{4} \lambda_{111} \cos 2 \chi + \frac{1}{4} \lambda_{111}. \]

Therefore, when the magnetostriction is measured along MgO[001] (|| Fe\(_{100-x}\)Co\(_x\)[011],x+[0 1 1]A) and MgO[1 0 1] (|| Fe\(_{100-x}\)Co\(_x\)[1 1 1],x+[1 1 1]B), the averages of relative length changes of types A and B variants, \( \Delta l/|l_{011}|(\chi) \) and \( \Delta l/|l_{111}|(\chi) \), are respectively given as

\[ \frac{\Delta l}{l_{011}}(\chi) = -\left( \frac{1}{4} \lambda_{111} \cos 2 \chi + \frac{1}{8} \lambda_{100} + \frac{1}{8} \lambda_{111} \right), \]

\[ \frac{\Delta l}{l_{111}}(\chi) = -\frac{3}{4} \lambda_{111} \cos 2 \chi + \frac{1}{4} \lambda_{111}. \]

which are shown in Figs. 9(b) and (c). Furthermore, the \( \lambda_{100} \) and the \( \lambda_{111} \) values can be estimated by using the following equations,

\[ \lambda_{100} = 8 \left[ \frac{\Delta l}{l_{011}}(\chi = 0^\circ) + \frac{\Delta l}{l_{011}}(\chi = 45^\circ) \right] - \frac{1}{3} \left[ \frac{\Delta l}{l_{011}}(\chi = 90^\circ) + \frac{\Delta l}{l_{011}}(\chi = 135^\circ) \right]; \]  
\[ \lambda_{111} = -\frac{4}{3} \left[ \frac{\Delta l}{l_{111}}(\chi = 90^\circ) + \frac{\Delta l}{l_{111}}(\chi = 135^\circ) \right]. \]
Figure 9(d) shows the Δl/λ100(ψ) and the Δl/λ111(ψ) measured for the Fe100–Co(110) epitaxial films. Figure 9(e) summarizes the χ111 and the χ111 values of the Fe100–Co(001) single-crystal film. The Fe50Co50(211) film shows a large λ100 value of +236×10–6 and a moderately large λ111 value of +97×10–6.

3.5 Magnetostriction of Fe100–Co(110) epitaxial films with NW and KS variants

When the magnetization rotates in a (110) plane as shown in Fig. 10(a), the crystallographic direction of magnetization is shown as [sin ψ/2, –sin ψ/2, cos ψ]. Here, ψ is the angle of magnetization direction with respect to [001]. The (α1, α2, α3) values are thus expressed as

(a1, a2, α3) = (sin ψ/2, –sin ψ/2, cos ψ).

When the angle of in-plane observation direction with respect to [001] is shown as ω, the (β1, β2, β3) values are expressed as

(β1, β2, β3) = (sin ω/2, –sin ω/2, cos ω).

The Δl/λ111 value for Fe100–Co(110) epitaxial films is thus given by substituting Eqs. (22) and (23) into (4) as follows,

\[
\Delta l/\lambda_{111}(\psi, \omega) = \frac{2}{3} \lambda_{100} \left( \frac{\sin^2 \psi \sin^2 \omega}{2} + \cos^2 \psi \cos^2 \omega - \frac{1}{3} \right) + 3 \lambda_{111} \left( \frac{\sin^2 \psi \sin^2 \omega}{4} + \sin \psi \cos \psi \sin \omega \cos \omega \right)
\]

(25)

In order to characterize the magnetostriction of an epitaxial film with multi-variant structure, it is necessary to take into account the volume ratio of each variant and the respective relative length changes. However, there are as many as 9 variants in the Fe100–Co(110) epitaxial films prepared in the present study.

Therefore, the in-plane orientation can be regarded as being random and the average of Δl/λ(ψ) of each variant is expressed as

\[
\frac{\Delta l}{\lambda}(\psi) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Delta l/\lambda(\psi, \omega) \, d\omega
\]

\[
= \frac{3}{16} \left( \lambda_{100} - \lambda_{111} \right) \cos^2 \psi + \frac{1}{16} \left( 3 \lambda_{100} + 3 \lambda_{111} \right),
\]

(26)

which is shown in Fig. 10(b). Although the relationship of λ100 > λ111, λ100 = λ111, or λ100 < λ111 cannot be determined by considering the phase of observed wave, the values of λ100 and λ111 can not be estimated in the case of the Fe100–Co(110) epitaxial film.

The (λ100 – λ111) value is shown as

\[
\lambda_{100} - \lambda_{111} = \frac{16}{3} \left( \frac{\Delta l}{\lambda}(\psi = 0^\circ) - \frac{\Delta l}{\lambda}(\psi = 45^\circ) \right).
\]

(27)

Figure 10(c) shows the Δl/λ(ψ) measured for the Fe100–Co(110) films. The phase of wave observed for Fe film is in agreement with that of calculated wave of Fig. 10(b-2), whereas the phases of waves measured for Fe70Co30 and Fe50Co50 films agree with that of wave of Fig. 10(b-1). The result shows that the λ100 value is smaller than the λ111 value for the Fe film, while the λ100 value is larger than the λ111 value for the Fe50Co50 and the Fe50Co50 films. Figure 10(d) shows the (λ100 – λ111) values plotted as a function of Co content. As the Co content increases, the (λ100 – λ111) value increases. The Fe70Co30 and the Fe50Co50 films show large (λ100 – λ111) values, indicating that large λ100 values are obtained.

Large λ100 values are obtained, even if Fe-Co films are prepared on MgO substrates with different orientations. Therefore, well-defined epitaxial Fe-Co films have potentials to achieve large magnetostriction.

![Fig. 10](image)

(a) Schematic diagram showing the magnetization and the observation directions with respect to the typical crystallographic directions. (b) Δl/λ(ψ) calculated for (110) epitaxial multi-crystal films with (b-1) λ100 > λ111 and (b-2) λ100 < λ111. (c) Δl/λ(ψ) measured for Fe100–Co(110) epitaxial multi-crystal films formed on MgO(111)/Al2O3(0001) substrates. (d) Compositional dependences of (λ100–λ111). The bulk values in (d) are calculated by using λ100 and λ111 values in Refs. 5–8 and 42.
4. Conclusion

Fe\(_{100-}\text{Co}\) \((x = 0-50 \text{ at. \%})\) alloy epitaxial films are prepared on MgO substrates with different orientations. The magnetization and the magnetostriction properties are characterized. Fe\(_{100-}\text{Co}(001)\) single-crystal and (211) bi-crystal films are respectively obtained on MgO(001) and (110) substrates, whereas Fe\(_{100-}\text{Co}(110)\) films with nine variants are epitaxially grown on MgO(111) substrates. The (001) single-crystal and the (211) bi-crystal films, respectively, show four- and two-fold symmetric in-plane magnetic anisotropies, which are reflecting the magnetocrystalline anisotropy of Fe\(_{100-}\text{Co}\) crystal with the easy magnetization axes parallel to the \((001)\) or \((111)\). The easy magnetization directions vary depending on the film composition and orientation. On the contrary, isotropic in-plane magnetization properties are observed for the (110) epitaxial films due to an influence of the variant structure. The magnetostriction behavior under rotating magnetic field is studied. As the Co content increases, the \(\lambda_{100}\) and the \(\lambda_{111}\) values, respectively, increase from \(+10^2\) to \(+10^4\) and from \(-10^5\) to \(+10^5\) for both Fe\(_{100-}\text{Co}(001)\) single-crystal and (211) bi-crystal films. Large \(\lambda_{100}\) values are also indicated for the Fe\(_{50}\text{Co}_{50}\) and the Fe\(_{50}\text{Co}_{50}\)(110) epitaxial films. The present study shows that it is possible to obtain large magnetostriction of \(10^4\) by control of the film orientation and composition.

References

38) G. Wasserman: Arch. Eisenhuettenwes, 16, 647 (1933).