Chromatic aberration effect in refraction of spin waves

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We numerically investigated the refraction property of spin waves (SWs) at thickness step in films with out-of-plane magnetization, in which the SWs propagate isotropically in the film plane. It was confirmed the isotropic SWs were refracted at a thickness step by following the Snell's law. We also found that the refraction angle of SWs of the dipole-exchange mode depends on the resonant frequency, indicating that the chromatic aberration effect should be taken into account in designing magnonic devices.

Key words: magnonics, spin waves

Magnonics is a rapidly developing research field, in which spin waves (SWs) are utilized to transfer and process information¹⁻⁴⁾. In magnonics, controlling the propagation direction of SWs is one of the crucial issues⁵⁻⁹⁾. It has been recently demonstrated that refraction is also useful for controlling the propagation direction of SWs^{12,13,16-18)}. In optics, the dispersion relation of light is isotropic, and thus, the relation between the incident and refraction angles is determined by the ratio of the refractive indices. On the other hand, for spin waves in thin films with in-plane magnetization the dispersion relation is anisotropic, and thus, deviations from Snell's law in optics has been reported^{13, 16)}. This deviation from Snell's law in optics results in the complicated design for magnonic devices¹¹⁾. Thus, isotropic SWs, rather than anisotropic SWs, are expected to be more suitable for application. Here, we numerically investigate the refraction property of SWs in thin films with perpendicular magnetization, in which the isotropic SW propagation in the film plane is expected from the symmetry. It is found that the refraction of SWs follows the Snell's law in optics with the chromatic aberration effect.

The micromagnetic simulation is performed utilizing mumax3¹⁹⁾. In simulation, we use the following material parameters reported for yttrium iron garnet (YIG): the saturation magnetization $M_s = 139$ kA/m, the exchange stiffness $A_{ex} = 4.15$ pJ/m, and the Gilbert damping constant 1×10^{-4} ^{20,21)}. In this study, two types of samples, labeled as sample A and B, are designed. Both samples are shaped as displayed in Fig. 1(a) and consist of thicker and thinner regions. The thicknesses of thicker and thinner areas are set to be 800 nm and 400 nm in sample A, and set to be 100 nm and 50 nm in sample B, respectively. The cell size for the calculation is $200 \times 200 \times 200$ nm³ in sample A and $50 \times 50 \times 50$ nm³ in sample B. Note that the cell size is set to be much smaller than the wavelength of excited SWs. To avoid the SW reflections at the edges of calculation area, the damping

constants in the areas of 6 μ m width from the left and right edges are set to increase gradually to 1. An external magnetic field $\mu_0 H_{ext} = 200$ mT is applied to +z direction, which is enough to saturate the magnetization to z direction, and SWs are excited by locally applied rf field $\mu_0 H_{rf} \sin(2\pi f t)$ along x direction at an antenna in the thicker area. In this work, we use $\mu_0 H_{rf} = 1$ mT and the frequency f of rf field is varied as $f = 0.9, 1.0$, and 1.1 GHz. We analyze the results recorded at 100 ns after the SW excitation.

Fig. 1(b) shows a typical result, which displays the phase of magnetization precession at each position in xy-plane. A SW with wavenumber k_1 propagates from the antenna to the thickness step with the incident angle θ_1 and it is refracted into the other SW with wavenumber k_2 at the thickness step with the refraction angle θ_2 . The estimation of k_1 , k_2 , and θ_2 is accomplished as follows. Firstly, we define two areas (A1 and A2) and perform two-dimensional fast Fourier transform (2D-FFT) on the data in each area. Results of 2D-FFT are shown in Figs. 1(c) and 1(d). Secondly, we estimate k_1 , which is the incident wavenumber in +x direction, from the data on $k_y = 0$ (Fig. 1(c)). k_1 is given as the second highest peak, as shown in Fig. 1(e). Thirdly, we linearly fit the 2D-FFT results of A2 and obtain θ_2 from the slope (Fig. 1(d)). Finally, k_2 is estimated from the second highest peak on the fitted line, as shown in Fig. 1(f).

To confirm that SWs propagate isotropically in both samples, we plot k_1 and k_2 as a function of θ_1 , as shown in Fig. 2. Both k_1 and k_2 are almost constant with respect to θ_1 even if the resonant frequency is varied from 0.9 GHz to 1.1 GHz. If the SW dispersion relation is anisotropic in the film plane, the wavenumber should depend on the SW propagating directions¹³⁾. Therefore, the results shown in Fig. 2 indicates that the SWs propagate isotropically in both thicker and thinner regions, and Snell's law in optics, $\sin \theta_1 / \sin \theta_2 = k_2 / k_1$, can be applied in the present system.

It is known that the dispersion relation of SW

propagating in a film with perpendicular magnetization can be described as

$$\omega^2 = \left(\omega_H + \frac{2A_{\text{ex}}\gamma}{M_s} k^2 \right) \left[\omega_H + \frac{2A_{\text{ex}}\gamma}{M_s} k^2 + \omega_M \left(1 - \frac{1-e^{-kd}}{kd} \right) \right] \quad (1)$$

where ω is the angular frequency of the excited SW, $\omega_H = \mu_0\gamma(H_{\text{ext}} - M_s)$, $\omega_M = \mu_0\gamma M_s$, μ_0 is the permeability of vacuum, γ is the gyromagnetic ratio, and d is the film thickness^{10, 22}). Here, both dipole-dipole

interaction and exchange interaction are taken into account. In the small- k limit (i.e. for magnetostatic SWs), where k^2 is comparatively smaller than kd , the influence of exchange interaction can be ignored. In that case, the SW dispersion relation for out-of-plane magnetized films, named as magnetostatic forward volume wave (MSFVW), is expressed as

$$\omega^2 = \omega_H \left[\omega_H + \omega_M \left(1 - \frac{1-e^{-kd}}{kd} \right) \right]. \quad (2)$$

Fig.1 Example of process of simulation and analysis. The shown results are obtained from sample B when we set $f = 1$ GHz and $\theta_1 = 45^\circ$. (a) Typical simulation setup. The black and white areas are respectively the thicker and thinner regions. The thickness step (boundary between two regions) is tilted with the angle θ_1 . The rf magnetic field is applied at the antenna (orange line), which has one-cell width. k and θ of incident and refracted SWs are described by the blue and green arrows, respectively. (b) Simulation result of the SW propagation. The phase of magnetization precession, ϕ , is displayed. The areas A_1 and A_2 surrounded by white frames are utilized for analysis. (c,d) Results of 2D-FFT from (c) A_1 and (d) A_2 . The black dashed line L_1 in (c) is $k_y = 0$ and L_2 in (d) is the result of linear fitting of intensity in the yellow frame. (e,f) Intensity on (e) L_1 and (f) L_2 .

Fig.2 Incident angle dependent k_1 and k_2 in sample A (a,b,c) and sample B (d,e,f). The SWs are excited at three different resonant frequencies; 0.9 GHz (a,d), 1.0 GHz (b,d), and 1.1 GHz (c,f). The green markers show k_1 and the blue markers show k_2 .

Fig.3 The resonant frequency as a function of wavenumber. Dispersion relation of DEFVW and MSFVW are respectively described by the solid and broken lines and the simulation results are plotted by the markers. Purple and green markers represent the SWs in the film of $d = 800$ nm and 400 nm, referring to the data of k_1 and k_2 in sample A, respectively. The blue and red markers represent the SWs in the film of $d = 100$ nm and 50 nm, referring to the data of k_1 and k_2 in sample B, respectively.

Fig. 3 shows the resonant frequency as a function of wavenumber. Symbols show the results by the micromagnetic simulation, and the calculation results by Eq. (1) and Eq. (2) are shown by broken lines and solid

lines, respectively. While the simulation results in the regions of 400 and 800 nm thicknesses (sample A) are well reproduced by Eq. (2), those in the regions of 50 and 100 nm thicknesses (sample B) show deviation from Eq. (2) and can be explained by Eq. (1), indicating that not only the dipole interaction but also the exchange interaction should be taken into account to describe the refraction of spin waves with high wavenumbers.

Figs. 4(a) and 4(b) show the relation between the incident angle θ_1 and the refraction angle θ_2 observed in the samples A and B, respectively. The refraction angle is independent of the resonant frequency in the sample A. This is due to the specific nature of MSFVW. According to Eq. (2) for MSFVW, the wavenumber varies by keeping kd constant, resulting in the special condition of $k_1/k_2 = d_2/d_1$. Therefore, Snell's law, $\sin \theta_1/\sin \theta_2 = k_2/k_1$, is given solely by the thickness ratio of two regions. On the other hand, in the case of the dipole-exchange forward volume wave (DEFVW) described by Eq. (1), the ratio k_2/k_1 depends not only on the thickness ratio but also on the frequency. Note that similar effect is also well known in optics as the chromatic aberration due to the dependence of refractive index on wavelength of light. The lines in Figs. 4(a) and 4(b) show the calculation results of Snell's law based on Eq. (1) and on Eq. (2), respectively, and the simulation results well reproduce the calculation results, confirming that the refraction of SWs follows the Snell's law in optics with the chromatic aberration effect.

Fig.4 Refraction angle versus incident angle of the SWs propagating (a) in sample A and (b) in sample B. In both figures, the pink, green, and blue markers correspond to the simulation results of $f = 0.9$, 1.0 , and 1.1 GHz, respectively. (a) Snell's law of MSFVW is expressed by the black dashed line. (b) The frequency dependent Snell's law of 0.9 , 1.0 , and 1.1 GHz are shown by the pink, green, and blue solid lines, respectively.

In summary, we performed the micromagnetic simulations to investigate the refraction of SWs across the thickness step in magnetic thin films with out-of-plane magnetization. It was confirmed that the SWs propagate isotropically and the Snell's law in optics can be applied in the present system, leading to the simple design of magnonic devices. It was also found that while the refraction angle is independent of the resonant frequency in MSFVW it depends on the resonant frequency in the case of DEFVW, suggesting that the chromatic aberration effect should be taken into account in designing magnonic devices using spin waves with high wavenumbers.

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