Spectral Rendering of Interference Phenomena Caused by Multilayer Films Under Global Illumination Environment

Sho Ikeda †, Shin Watanabe †,††, Bisser Raytchev †, Toru Tamaki †, Kazufumi Kaneda †

Abstract  Interference phenomena caused by multilayer films generates beautiful scenery. When rendering the phenomena in computer graphics, we should take into account both spectral distributions and global illumination. The paper proposes a method for rendering optical phenomena caused by multilayer films based on wave optics under global illumination environment. The method is an extension of a multilayer-film model, where the traditional model is based on a local illumination model and has limitations of rendering a scene with caustics and inter-reflection of light. In the proposed method, the multilayer-film model is reconstructed taking into account global light transport to render optical phenomena caused by interference of light under global illumination environment.

Key words: Computer Graphics, Rendering, Global Illumination, Wave Optics, Spectrum of Light

Fig. 1: Comparison of rendered images of five silicon rings coated with thin films. Each ring has a SiO₂ film and different metal films, and generates different colors due to the interference of light. Only the proposed method can render iridescent caustics and reflected caustics correctly.

1. Introduction

Multilayer films are utilized in many industrial products, such as optical lenses, filters, lighting equipments, and so on. A thin film causes interference of light, and consequently we see iridescent colors on the film. It is desired for industrial designers to visualize the optical phenomena caused by thin films, when designing products utilizing multilayer films. Computer graphics have enabled the designers to satisfy their requirement.

Hirayama, et al.†† developed a multilayer-film model to visualize the optical phenomena caused by thin films, taking into account wave optics and interference of light. The model is able to render interference colors on the films realistically, but the reflected light from the films does not illuminate the surrounding objects, because the multilayer-film model has only a capability of a local illumination (LI). Therefore, optical phenomena such as caustics and inter-reflection of light caused by multilayer films, that are vital to the industrial design, cannot be visualized yet (see Fig. 1(a)).

In computer graphics, a global illumination (GI) model have been developed. A GI model is able to calculate indirect illumination based on physical laws, and generates photo-realistic images including caustics and inter-reflection of light. A photon mapping‡‡ and
a progressive photon mapping\(^3\) are classified into the GI model. These methods, however, cannot deal with multilayer films, because the multilayer-film model and the GI model have been developed individually.

In this paper, the multilayer-film model meets a global illumination model. We propose a method for rendering optical phenomena caused by multilayer films under global illumination environment. The proposed method can generate photo-realistic images including caustics and inter-reflection of light caused by multilayer films.

To render interference phenomena, we should take into account wave optics and spectral distribution of light. Traditionally, rendering methods in the global illumination model usually compute only R, G and B color components. However, when rendering interference phenomena, that greatly depend on the wavelength of light, we should calculate the spectral distribution of light. Figure 1(b) shows the resultant image rendered by a traditional global illumination renderer, where only three color components were computed. The color on the silicon rings coated with thin films is not the actual color of the interference phenomena. In Fig. 1(c), the image was rendered by the proposed method that takes into account both the spectral distribution and global illumination. Iridescent colors on the rings, iridescent caustics on the floor, and reflected caustics on the rings are rendered accurately by the proposed method.

This paper is an extended version of our paper\(^4\) presented at the conference IWAIT 2014. Based on the discussion at the conference, we have added several new resultant images and showed the accuracy of the spectral rendering in this paper, compared with a traditional RGB rendering.

2. Related Work

2.1 Multilayer-Film Model

Multilayer films generate iridescent light due to the interference of light. As shown in Fig. 2, reflected and refracted light on the boundary of each layer are interfered each other in the propagation of light in multilayer films. Incident light with different wavelengths passes through different routes inside the films, but the outgoing rays from the surface of the films proceed in the same direction. That is, reflected light from multilayer films has a certain spectral distribution unlike dispersive phenomena.

\[\gamma_{N-j} = \gamma_0 = r_N,\]  
\[\tau_{N-j} = \tau_0 = t_N,\]  
\[\gamma_{N-j} = \frac{r_j + \gamma_{N-j-1}e^{2i\varphi_{j+1}}}{1 + r_j\gamma_{N-j-1}e^{2i\varphi_{j+1}}},\]  
\[\tau_{N-j} = \frac{t_j\gamma_{N-j-1}e^{2i\varphi_{j+1}}}{1 + r_j\gamma_{N-j-1}e^{2i\varphi_{j+1}}},\]

where \(i\) denotes an imaginary number, \(r_j\) and \(t_j\) are reflectivity and transmissivity at a single boundary between \(j\)-th and \((j+1)\)-th layers, respectively. \(\varphi_j\) is a phase difference on the both sides of \(j\)-th layer, and is expressed by the following equation.

\[\varphi_j = \frac{2\pi}{\lambda}n_jd_js_{zj},\]

where \(\lambda\) is the wavelength of light in a vacuum, \(d_j\) is the thickness of \(j\)-th layer, \(s_{zj}\) is a \(z\)-component of the unit vector, \(s_j\), that indicates the direction of light in \(j\)-th layer, and \(n_j\) is a complex refractive index of \(j\)th layer consisting of a refractive index, \(n_j\), and an extinction coefficient, \(k_j\).

\[n_j = n_j + ik_j.\]
The unit direction vector, \( s_j = (s_{xj}, 0, s_{zj}) \), and reflectivity, \( r_j \), and transmissivity, \( t_j \), are derived from Snell’s law and Fresnel formulae.

The reflectivity and transmissivity represent the ratio of amplitudes of an incident electromagnetic wave versus reflected and transmitted electromagnetic waves, respectively. On the other hand, reflectance and transmittance represent the ratio of energies of reflected and transmitted electromagnetic waves, respectively. Converting the amplitude into energy, reflectance and transmittance coefficients are expressed by the following equations, respectively.

\[
R = \frac{1}{2}(|\gamma_\parallel|^2 + |\gamma_\perp|^2), \quad (7)
\]

\[
T = \begin{cases} 
\frac{n_{N+1} s_{xj} s_{zj}}{n_0 s_{xj} s_{zj}} (|\tau_\parallel|^2 + |\tau_\perp|^2), & c \text{ : real} \\
0, & c \text{ : complex} 
\end{cases} \quad (8)
\]

where \( c = \hat{n}_{N+1} s_{zj} s_{xj} N+1 \).

2.2 Global Illumination Algorithms

A photon mapping (PM)\textsuperscript{2} is categorized into a two-pass global illumination algorithm, and is able to render the effects of caustics. In the first pass, photons are traced from light sources and photon maps are created on non-specular surfaces. In the second pass, viewing rays are traced from a viewpoint until the rays hit a non-specular surface. The radiance at the intersection between the ray and surface is calculated by using photon maps near the intersection. The quality of images rendered by PM, however, is limited by the amount of memory for storing photon maps.

A progressive photon mapping (PPM)\textsuperscript{3} is an improved method of PM, and categorized into the multipass algorithm that solves the rendering equation\textsuperscript{3} by updating the radiant power of photons statistically. In PPM, viewing rays are traced from a viewpoint, and the intersections between the rays and non-specular surfaces are recorded in the initial pass. Next, photons are traced from light sources, and the radiant power of photons are stored in photon maps. The radiance is estimated by using the photon maps in the same manner of PM. Then, PPM iterates the photon tracing pass, updating the radiant power of photons statistically at the intersections obtained in the initial pass. The more the number of iterations is, the more accurate the radiance is, that is, the quality of images rendered by PPM is no longer limited by the amount of memory.

2.3 Rendering Structural Colors

Diffraction generates iridescent colors similarly to interference effects caused by a thin film. Stam\textsuperscript{6} developed a diffraction shader, where diffraction effects are rendered by using Kirchhoff’s diffraction theory and integral theorems. Ward\textsuperscript{7} measured the spatial reflectance distributions of surfaces and modeled the anisotropic reflections using an elliptical Gaussian model. Recently, Cuypers, et al\textsuperscript{8} proposed a method for representing the diffraction property as an extended BSDF, called Wave BSDF. Because diffracted light exits in all directions from the incident point, they have to calculate and store the reflectance in a hemispherical region. On the other hand, interference of light caused by the thin films whose surfaces are perfectly smooth exits the specified direction that obeys the physical law. The proposed method calculates the reflectance and transmittance efficiently and stores them compactly.

To render the structured colors described above, we should employ a spectral rendering instead of a RGB rendering. Elek, et al\textsuperscript{9} introduced spectral ray differentials to reduce the problems of chromatic noise and spatial aliasing in spectral rendering. Their method, however, applies to only dispersive phenomena such as a glass prism. The proposed method solves the problem and accelerates the spectral rendering by using improved sampling strategies.

3. Proposed Method

A traditional multilayer-film model developed by Hi-rayama, et al.\textsuperscript{1} has a limitation of rendering a scene with caustics and inter-reflection of light, because the method is based on a local illumination model. We propose a method based on a global illumination model to render optical phenomena caused by multilayer films considering wave optics. In the proposed method, the multilayer-film model is reconstructed and implemented in a global illumination renderer.

We also extend the method in the following three
important elements: First, we extend a conventional global illumination renderer to a spectral renderer, that is, not RGB color components but spectral distributions of light are calculated in the spectral renderer, because interference of light is greatly depends on the wavelength of light. When displaying rendered images, we convert the spectral distributions into RGB color components taking into account color-matching functions. We describe the details in Section 3.1.

Second, a set of multilayer films is converted into an attribute of surface, that is, composite reflectance and transmittance coefficients of multilayer films are calculated in advance, and the coefficients are stored in a table. When rendering an object coated with multilayer films, we use the pre-computed composite reflectance and transmittance coefficients as a surface attribute to reduce the computation time of interference of light in the multilayer films. We describe the details in Section 3.2.

Third, we further accelerate the global illumination renderer by employing an improved importance sampling strategy, because it is computationally expensive for the renderer to calculate spectral distributions of light. We describe the details in Section 3.3.

Figure 4 shows the process flow of the proposed method. In the precomputation, we calculate composite reflectances and transmittances and store in tables (Section 3.2). In rendering, we first sample wavelengths from the range of visible light (Section 3.1). The sampled wavelengths are assigned to all the emitted photons and rays in every photon and ray tracing path. In the photon tracing path, the continuation of the photon trace is determined by taking into account both surface reflectances and photon power. (Section 3.3).

3.1 Spectrum of Light

In computer graphics, most of traditional rendering methods use RGB color components to calculate the intensity of light. That is, the intensity of light sources and the reflectance of object surfaces are specified in RGB color components. Then, the intensities of reflected and/or refracted rays are calculated in a RGB color space when the ray hits the surface. However, we cannot compute the accurate intensity of light using the RGB color space, because the interaction with light and material follows the wavelength of light. In physics, reflected light from a surface is determined by the spectral distribution of an incident light and the spectral reflectance of a surface. Dispersion and interference of light also greatly depend on the wavelength of light. Therefore, the spectral rendering in which the intensity of light is calculated by using spectral distributions of light and spectral reflectances/transmittances is vital for photo-realistic image synthesis based on a physically based rendering.

Spectral representation in global illumination algorithms such as a PPM can be classified into two types: a single-wavelength photon and a multi-wavelength photon (see Fig. 5). A single-wavelength photon has an advantage in the case of dispersion of light, because the refracted light has only a single wavelength in a specified direction. On the other hand, a light ray has usually a certain spectral distribution in the case of the other interactions with light and materials including multilayer films as described in section 2.1. We use a multi-wavelength photon to represent a spectral distribution.

We need a further consideration when introducing a multi-wavelength photon into a PPM. That is a sampling method of the wavelength of light, and we have three possible strategies: a regular sampling, a random sampling, and an improved random sampling such as a stratified sampling.

A regular sampling literally samples the wavelength of a visible light at a constant interval. It is a simple...
method, but it has several weak points for a bright line spectrum, such as the spectrum of a fluorescent lamp. The bright lines easily drop out of the samples.

A random sampling is one of the solutions of the problem mentioned above. We can sample the bright lines or neighbors of the bright lines. The method, however, requires a lot of samples to represent an accurate spectral distribution, and it results in a high computational cost. In the sampling theory, we need four times the number of samples to reduce the standard deviation half.

One of the variance reduction methods is a stratified sampling\(^{11}\), and the stratified sampling offers advantages of both the regular and random samplings. We first divide the region of the wavelength of a visible light into several subregions, and then, sample the wavelength randomly in each subregion (see Fig. 6). The sampling method reduces the standard deviation, that is, we can generate photo-realistic images in a shorter time than that of the regular and random samplings.

### 3.2 Composite Reflectance and Transmittance

Given a set of refractive indices and the thickness of each film layer, the proposed method calculates composite reflectance and transmittance coefficients of the multilayer films, using the multilayer-film model developed by Hirayama, et al.\(^ {11}\). The calculation is carried out in advance of a photon tracing process, because we can accelerate the photon tracing process by substituting a set of multilayer films coating on a surface for a simple attribute of the surface. That is, we don’t need to trace the huge number of rays inside the multilayer films, each time a photon hits a surface coated with the multilayer films.

Pre-calculated composite reflectance and transmittance coefficients are stored in a table, separately. As shown in Fig. 7, each table is a two-dimensional table whose indices are the incident angle and the wavelength of light. The incident angle and the wavelength are discretized at small intervals, and composite reflectance and transmittance coefficients are calculated and stored in the two-dimensional tables.

When a photon hits a surface coated with multilayer films in the photon tracing process, the incident angle and the sampled wavelengths of the collided photon are examined. Composite reflectance, \(\rho(\theta, \lambda)\), corresponding to the incident angle \(\theta\) and the sampled wavelength \(\lambda\) are obtained by using the following bilinear interpolation:

\[
\rho(\theta, \lambda) = (1 - \alpha_\theta)(1 - \alpha_\lambda)R(\theta_i, \lambda_j) + \alpha_\theta(1 - \alpha_\lambda)R(\theta_{i+1}, \lambda_j) + (1 - \alpha_\theta)\alpha_\lambda R(\theta_i, \lambda_{j+1}) + \alpha_\theta\alpha_\lambda R(\theta_{i+1}, \lambda_{j+1}),
\]

(9)

where \(\theta_i \leq \theta < \theta_{i+1}\), \(\lambda_j \leq \lambda < \lambda_{j+1}\), \(\alpha_\theta = \frac{\theta - \theta_i}{\theta_{i+1} - \theta_i}\), \(\alpha_\lambda = \frac{\lambda - \lambda_j}{\lambda_{j+1} - \lambda_j}\), and \(R(\theta, \lambda)\) is the reflectance corresponding to \(\theta_i\) and \(\lambda_j\) retrieved from the table. In the same way, we calculate composite transmittances.

### 3.3 Improved Importance Sampling on Reflection and Refraction

In a photon tracing process, a reflected photon is traced continuingly obeying an importance sampling. That is, the traditional photon tracing method determines the continuation of the photon trace taking into account only the reflectance of a surface. In the proposed method, the importance sampling is further improved by taking into account both the composite reflectance of multilayer films and the radiant power of the photon\(^{12}\).

Let’s assume the radiant power of an incident photon is \(I\), the radiant power of the light source originating the incident photon is \(I_L\), and the spectral composite reflectance of multilayer films calculated by using Eq. 9 is \(\rho\). The relative radiant power of the incident photon is defined by \(I_r = I/I_L\), where the division symbol means a component-wise division. The continuation of photon trace is determined by using the following probability:

\[
p = \max_{1 \leq i \leq N} (\rho \odot I_r)_i = \max(p_1I_{r1}, p_2I_{r2}, \ldots, p_NI_{rN}),
\]

(10)
Table 1: Rendering parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>reflectance table size</td>
<td>6.9 KB</td>
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<tr>
<td>interval of incident angle $\theta_i$</td>
<td>5 degrees</td>
</tr>
<tr>
<td>interval of wavelength $\lambda_j$</td>
<td>5 nm</td>
</tr>
<tr>
<td>number of photons</td>
<td>100,000</td>
</tr>
<tr>
<td>number of sampled wavelengths</td>
<td>16</td>
</tr>
<tr>
<td>refractive indices</td>
<td>see reference</td>
</tr>
</tbody>
</table>

where $N$ is the number of sampled wavelengths, and the symbol $\odot$ means a component-wise multiplication. Note that we use the relative radiant power to determine the continuation of a photon trace. The relative radiant power also contains the tracing history of the photon. If the relative radiant power has a larger value, the rendering scene may contain more surfaces coated with multilayer films whose spectral composite reflectances contribute to the radiant power of the incident photon. This strategy can decide the continuation of photon trace more efficiently.

A reflected photon is generated probabilistically based on the probability $p$ as shown in Eq. 10. Based on the theory of Monte-Carlo sampling, the radiant power of a reflected photon is $I_0 = \rho \odot I/p$. Using the method, photons having a larger contribution to the rendering scene have higher possibility to survive.

We also introduce the same strategy in a refracted photon tracing. In this case, $\rho$ in Eq. 10 implies the spectral composite transmittance of multilayer films.

4. Result

In this section, several examples are shown to demonstrate the usefulness of the proposed method. Especially, the comparisons of rendered images with a traditional and the proposed methods are shown in Sections 4.1 and 4.2. In Section 4.3, we describe the time measurement of the acceleration method described in Section 3.3.

All the experiments were done on a computer with Mac OS X 10.8, 2.0 GHz Intel Core i7 (2-core, 4-threads), 8 GB memory, and without GPU acceleration. Table 1 shows the rendering parameters used in our experiment.

4.1 RGB Rendering vs Spectral Rendering

Figure 1(b) shows an image rendered by a traditional global illumination renderer, where only three color components were computed, and Fig. 1(c) shows an image rendered by the proposed method that takes into account the spectral distribution. We can perceive the difference of color on the rings. The proposed method based on both the spectral rendering and the wave optics can render the actual color of interference. Figure 8 shows an image of difference between Figs. 1(b) and 1(c), where the pixel values are multiplied by 5 to visualize the difference clearly. The RMS error between Figs. 1(b) and 1(c) was 13.57 in 255 levels. This clearly shows that when rendering interference phenomena, we should employ the spectral rendering.

4.2 Lighting Effects of a Ring Object

A silicon ring coated with two-layer films (10 nm copper and 500 nm SiO$_2$) was rendered by using a traditional local illumination model (LI model) and the proposed global illumination model (GI model). Figure 9 shows the comparison of the rendered images, as well as Figs. 1(a) and 1(c). The proposed method is able to render not only the interference color on the ring but also the caustics on the floor and the reflection of the caustics on the ring.

Figure 10 shows rendered images of a silicon ring with the same structure as that in Fig. 9 under differ-
Fig. 11: A silicon ring coated with different configurations of films

Fig. 12: Close-up view of Fig. 11

Fig. 13: A silicon ring coated with different materials of films

Fig. 14: Close-up view of Fig. 13

4.3 Performance Evaluation

We measured the rendering time and compared to a non-accelerated version without the improved importance sampling described in Section 3.3. Figure 15 (a) shows a ground truth image rendered with a sufficient number of photons. A planar silicon board coated with a SiO₂ film (500 nm) is lit by a small area light source with a directional luminous intensity distribution. Figures 15 (b), (c), and (d) are rendered images in different computation times. RMS errors decrease as the number of rendering passes increases. The close-up views of Fig. 15 are shown in Fig. 16.

The computation time in respect to the RMS errors is shown in Table 2, where RMS errors were calculated in the spectral domain. For the proposed method, the computational costs were reduced by about 19%.
multilayer films as a surface attribute, and by employing the improved importance sampling on reflection and refraction. We have demonstrated that the proposed method is able to generate photo-realistic images in a lower computational cost.

In our multilayer-film model, the surface of objects and films is assumed to be perfectly smooth, and there is no scattering inside the objects and films. In order to further enhance the reality of rendered images, it is important to extend our method to be able to handle the diffuse reflection of boundary surfaces between the objects and films. Light scattering inside the objects and films is also a challenging work, and it makes our method more useful for rendering various kinds of materials. Acceleration using a GPU is also mentioned as a future work.

5. Conclusions

We proposed a method for rendering optical phenomena caused by multilayer films under global illumination environment. In the proposed method, a multilayer-film model is combined with a global illumination model, introducing a multi-wavelength photon, where the wavelength of visible light is sampled by using a stratified sampling. The proposed method is also powered by using the composite reflectance and transmittance of

Table 2: The computation time

<table>
<thead>
<tr>
<th>RMS error</th>
<th>Time (Proposed method) [s]</th>
<th>Time (Without acceleration) [s]</th>
<th>Reduction rate [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
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<td>12</td>
<td>0.0</td>
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<td>0.04</td>
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<td>50</td>
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<tr>
<td>0.014</td>
<td>736</td>
<td>913</td>
<td>19.4</td>
</tr>
</tbody>
</table>

References


13) FILMETRICS Refractive Index Database: http://www.filmetrics.com/refractive-index-database

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