Compressive Inverse Light Transport for Radiometric Compensation in Projection-based Displays

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Abstract We propose a new method that obtains an inverse light transport matrix suitable for radiometric compensation; it creates seamless projection-based displays on unknown 3D structures. We extend the theory of inverse light transport to support uncalibrated projector-camera systems in which the optical axes of the projector and the camera are not aligned. The inverse light transport matrix consists of the inverse matrices of direct light transport matrix and inter-reflection matrix. Given that direct and inter-reflection matrices are separated from the light transport matrix, the inverse inter-reflection matrix is computed by Neumann inverse series. The proposed method introduces compressed sensing to optimally estimate the inverse direct light transport matrix from the sets of projector illumination patterns and the camera responses from which the global components are canceled by the inverse inter-reflection matrix. We visualize the matrix elements of the various light transport matrices generated by our approach to examine its performance. We demonstrate compensation trials that compare the proposed method with existing methods. Our experiments confirm that our method offers excellent projection-based displays on 3D objects.

Key words: compressed sensing, inverse light transport, direct and global reflections, radiometric compensation, projection-based display

1. Introduction

Projectors that offer arbitrary scaling allow 3D objects in real space to be used as display screens with complete control and so provide augmented reality, computer-human interaction, and various projection-based displays^{(1)~(6)}. In particular, the projection-based displays used in amusement parks, entertainment events, and to create huge 3D advertisements are gaining in popularity as projection mapping applications⁽⁷⁾⁽⁸⁾. The common approach to displaying an image on an uneven surface assumes the calibration of the system setup beforehand, as well as the acquisition of a model or depth of the 3D structure. Practical applications, however, demand projection-based display systems that need neither calibration work nor 3D scanning to better support users with enhanced handiness and flexibility.

A projector-camera system is used to display arbitrary images on 3D objects, where the camera senses the display states yielded by the projector. The system needs to adequately correct the brightness output by the projector to ensure that the desired image is finally displayed on the objects as confirmed by the camera. This technique is well known as radiometric compensation^{(4)(5)(9)~(15)}. In order to handle direct and global reflection lights radiated from arbitrary objects⁽¹⁶⁾⁽¹⁷⁾, we have recourse to use light transports^{(18)~(21)}, which describe the geometric and photometric correspon-

dences between all projector pixels and all camera pixels. We often use a light transport matrix in which each column represents the camera response image that corresponds to each point light source; it allows us to handle all light transports in an effective manner⁽²⁰⁾. For instance, given a projector vector as arbitrary illumination, the matrix-vector product of the light transport matrix and the vector yields the virtual image relighted under the illumination. In addition, the inverse matrix of the light transport matrix has potential for radiometric compensation. Although the methods proposed in 9)~12) handle only direct light reflected from 3D structures, the inverse light transport matrix provides image displays that account for not only direct reflection light but also global reflection light.

When we acquire light transports by generating and capturing illumination patterns with both high and low frequencies on 3D objects, the light transport matrix is sparse and high-rank^{(18)(20)(22)~(24)}. Some compensation methods that approximately obtain the inverse light transport matrix are available for projection-based displays^{(25)~(27)}. In contrast, low-rank matrix approximations⁽²⁸⁾⁽²⁹⁾ assume that high frequency projector lights are seldom present in the camera responses and acquire dense light transports from 3D target by using a projector-camera system with custom optical apparatus and a diffuser device⁽³⁰⁾. Calculating the inverse light transport matrix may be tractable due to its low-rank property. However, since the system drastically reduces the rank of the light transport matrix, it is not evident that the low-rank matrix approximation can support the use of real pho-

Received December 20, 2016; Revised May 1, 2017; Accepted June 7, 2017

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tographs or artistic illuminations in projection mapping. In this paper, we tackle radiometric compensation with sparse and high-rank light transport matrix with the goal of diversified video image representations. If the light transport matrix is huge, straightforward compensation via inverse matrix is computationally infeasible. We note that the theory of inverse light transport^{(31)~(35)} provides a more efficient calculation method. The inter-reflection matrix, which transforms direct light transport into global light transport based on a rendering equation, yields series expansions that physically correspond to light bounces. Inverting the inter-reflection matrix creates radiometric compensation based on the inverse light transport. This approach assumes that a first-bounce matrix, which is composed of direct light transports reflected from the 3D structure, is a diagonal matrix. This means that all geometric correspondences based on direct reflection are represented by one-to-one mapping between projector and camera pixels, provided that the optical axis of the projector is aligned to that of the camera in system installation and operation.

Considering real-time processing by the system⁽¹¹⁾⁽³⁶⁾, a key desirable feature is implementing radiometric compensation with simple linear operations. As the inverse light transport matrix promptly yields the compensated image by the matrix-vector product, it has the possibility of providing highly responsive image displays. However, existing methods^{(31)~(35)} demand that the projector-camera systems be installed and operated in compliance with the theory of inverse light transport. Even if users set up the projector-camera system by separately directing the projector and the camera toward the 3D objects, we aim to realize seamless and accurate image displays on uneven surfaces. In our previous work⁽³⁷⁾, we proposed a compensation method that handles the color-mixing matrix reformed from the light transport matrix. In terms of practical use, its application is restricted to static images because the method relies on feedback control. To resolve this technical problem, we propose a new method that attains the inverse light transport matrix; it allows the systems to rapidly and accurately create image displays on complex 3D objects. Based on the theory of inverse light transport, we focus on that the inverse light transport matrix consists of the inverse matrices of direct light transport matrix and inter-reflection matrix. Given that direct and inter-reflection matrices are separated from the light transport matrix, the inverse inter-reflection matrix is computed by Neumann inverse series. The proposed method introduces compressed sensing⁽³⁸⁾⁽³⁹⁾ to optimally estimate the inverse direct light transport matrix from the sets of projector illumination patterns and the camera responses from which the global

components are canceled by the inverse inter-reflection matrix. Accurately computing the inverse light transport matrix without resort to calibration setup is a novel approach in the framework of inverse light transport. Compressed sensing has been used to acquire light transport matrices as compressive light transport⁽²³⁾⁽²⁴⁾. As far as we know, it has never been applied to estimate inverse light transport matrices for practical use. We call this approach “compressive inverse light transport” in this paper.

This paper is organized as follows: Section 2 briefly introduces the theory of inverse light transport and mentions the compressive light transport approach as previous work. In Section 3, we describe our compressive inverse light transport that attains the inverse light transport matrix by employing compressed sensing. Section 4 examines the performance of compressive inverse light transport and demonstrates projection-based displays yielded by our method in a comparison with those yielded by existing methods. Section 5 concludes this study and mentions future work.

2. Previous Work

2.1 Theory of Inverse Light Transport

In this subsection, we describe briefly radiometric compensation based on the theory of inverse light transport^{(33)~(35)}. We assume a projector-camera system that handles projector image with $M \times M$ pixels and camera image with $M \times M$ pixels. Given projector illumination, which is denoted by the M^2 -dimensional vector $\mathbf{P}$, we represent the camera response by the M^2 -dimensional vector $\mathbf{C}$, from which the ambient light components are removed. For an arbitrary camera response, we denote direct reflection response and global reflection response that includes indirect reflection light by M^2 -dimensional vectors $\mathbf{C}_D$ and $\mathbf{C}_G$, respectively. According to Kajiya’s rendering equation⁽⁴⁰⁾⁽⁴¹⁾, the global reflection response $\mathbf{C}_G$ is represented by the direct reflection response $\mathbf{C}_D$. The camera response $\mathbf{C}$ detected by the system is given by

$$\mathbf{C} = \mathbf{C}_D + \mathbf{C}_G = \mathbf{S}\mathbf{C}_D, \quad (1)$$

$$\mathbf{S} = \mathbf{I} + \mathbf{R}, \quad (2)$$

where matrix $\mathbf{I}$ is the $M^2 \times M^2$ identity matrix and matrix $\mathbf{S}$ is called the $M^2 \times M^2$ inter-reflection matrix. Matrix $\mathbf{R}$ is defined as a linear operator that satisfies $\mathbf{C}_G = \mathbf{R}\mathbf{C}_D$.

Meanwhile, given light transport matrix $\mathbf{T}$, the geometric and photometric correspondences between projector vector $\mathbf{P}$ and camera vector $\mathbf{C}$ are described by

$$\mathbf{C} = \mathbf{T}\mathbf{P}, \quad (3)$$

i.e. a linear input/output system^{(18)~(21)}. Eq. (3) assumes

a light transport matrix that represents the direct reflection responses between projector and camera pixels, i.e. first-bounce matrix, given here as matrix $\mathbf{D}$. From Eqs. (1) and (3), we obtain the following equation,

$$\mathbf{T} = \mathbf{SD}, \quad (4)$$

because the direct reflection response is given by $\mathbf{C}_D = \mathbf{DP}$. The inverse light transport matrix yields

$$\mathbf{T}^{-1} = \mathbf{D}^{-1}\mathbf{S}^{-1} = \mathbf{D}^{-1}(\mathbf{I} - \mathbf{R} + \mathbf{R}^2 - \mathbf{R}^3 + \cdots), \quad (5)$$

by applying Neumann inverse series to inverse inter-reflection matrix $\mathbf{S}^{-1}$. When inverse matrix $\mathbf{S}^{-1}$ in Eq. (5) is represented by n -bounce reflection approximation, the compensated projector illumination is given by

$$\mathbf{P} = \mathbf{T}^{-1}\mathbf{C} \approx \mathbf{D}^{-1}\left(\mathbf{I} + \sum_{k=1}^n (-\mathbf{R})^k\right)\mathbf{C}, \quad (6)$$

which is based on Eqs. (3) and (5). Existing methods^{33)~35)} assume that direct light transport matrix $\mathbf{D}$ is a diagonal matrix and obtain inverse matrix $\mathbf{D}^{-1}$ by calculating the inverse number of each diagonal element. Eq. (6) indicates that computing inverse light transport matrix $\mathbf{T}^{-1}$ virtually equals the matrix product of inverse matrix $\mathbf{D}^{-1}$ and series expansion with matrix $\mathbf{R}$. In general, we need to consider not only direct reflection light but also global reflection light because it yields key visual effects in real world scenes¹⁸⁾³⁴⁾⁴¹⁾. The global reflection light is caused by indirect illumination due to other point sources in a scene, e.g. inter-reflection, subsurface scattering, and translucency¹⁶⁾¹⁷⁾. Since inter-reflection matrix $\mathbf{S}$ given by Eqs. (2) and (4) represents the complicated light transports between projector and camera pixels, inverse inter-reflection matrix $\mathbf{S}^{-1}$ can cancel the visual effects created by global reflection lights.

However, the radiometric compensation yielded by Eq. (6) depends on the calibration setup of the projector-camera system, i.e. the geometric correspondence between projector and camera pixels. The fact that direct light transport matrix $\mathbf{D}$ is diagonal assumes that each projector pixel corresponds to one camera pixel with the same 2D coordinates due to one-to-one mapping. When users separately direct the projector and the camera toward the 3D objects, the direct light transport matrix acquired by using the uncalibrated system is not diagonal. In addition, some elements in the light transport matrix are lost for occlusion detection owing to the camera's/projector's orientations and positions in the system. Our goal is to obtain the inverse light transport matrix that yields radiometric compensation in projector-camera systems without the need for dedicated setup in order

to attain seamless and accurate projection-based displays on uneven surfaces.

2.2 Compressive Light Transport

This subsection details the acquisition of light transport matrix by compressed sensing because the descriptions are related to our compressive inverse light transport. We assume that the camera plane has $M \times M$ pixels and the projector plane has $N \times N$ pixels. We denote the r -, g -, and b -channel's projector illuminations by the N^2 -dimensional vectors $\mathbf{P}_r$, $\mathbf{P}_g$, and $\mathbf{P}_b$ and denote the r -, g -, and b -channel's camera responses by the M^2 -dimensional vectors $\mathbf{C}_r$, $\mathbf{C}_g$, and $\mathbf{C}_b$, respectively. The geometric and photometric correspondences between all projector pixels and all camera pixels are given by

$$\mathbf{C}_\alpha = \mathbf{T}_{\alpha\alpha}\mathbf{P}_\alpha + \mathbf{F}_\alpha, \quad \alpha \in \{r, g, b\}, \quad (7)$$

where matrix $\mathbf{T}_{\alpha\alpha}$ represents the $M^2 \times N^2$ light transport matrix that transports projector pixels of α -channel to camera pixels of α -channel. Vector $\mathbf{F}_\alpha$ represents the α -component of ambient light, which includes black offsets from the projector, as detected by the camera response. Although it is found that projector-camera systems actually have color crosstalk that link projector pixels and camera pixels^{9)~11)}, the camera responses yielded via pure color channels are definitely dominant in the observed image, in comparison with those yielded via color mixture channels. Most methods that acquire light transports do not consider the color crosstalk between projector pixel and camera pixel^{18)~24)}. Moreover, since the theory of inverse light transport^{31)~35)} is supported by light transports via pure color channel in projector-camera systems, we do not use color crosstalk in handling light transport in this study.

Compressive light transport²³⁾²⁴⁾ is the breakthrough that efficiently acquires direct and global light transports by using as few illumination patterns as possible. The left side of **Fig. 1** gives an overview of compressive light transport. The system outputs the K ($< N^2$)-kinds of projector illuminations: $\{\mathbf{P}_1, \mathbf{P}_2, \dots, \mathbf{P}_K\}$ from r -, g -, and b -channels, simultaneously, and then obtains the camera responses of α -channel: $\{\mathbf{C}_1^{(\alpha)}, \mathbf{C}_2^{(\alpha)}, \dots, \mathbf{C}_K^{(\alpha)}\}$, $\alpha \in \{r, g, b\}$. Eq. (7) yields

$$\begin{bmatrix} \mathbf{C}_1^{(\alpha)} & \mathbf{C}_2^{(\alpha)} & \cdots & \mathbf{C}_K^{(\alpha)} \end{bmatrix} = \mathbf{T}_{\alpha\alpha} \begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_2 & \cdots & \mathbf{P}_K \end{bmatrix}, \quad (8)$$

in which environmental components are eliminated from each camera response. From Eq. (8), since we have

$$\begin{bmatrix} \mathbf{C}_1^{(\alpha)} & \mathbf{C}_2^{(\alpha)} & \cdots & \mathbf{C}_K^{(\alpha)} \end{bmatrix}^T = \begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_2 & \cdots & \mathbf{P}_K \end{bmatrix}^T \mathbf{T}_{\alpha\alpha}^T, \quad (9)$$

we denote the M^2 columns in the $K \times M^2$ matrix of the left hand by K -dimensional vectors: $\{\mathbf{v}_1^{(\alpha)}, \mathbf{v}_2^{(\alpha)}, \dots, \mathbf{v}_{M^2}^{(\alpha)}\}$, and denote the M^2 columns in matrix $\mathbf{T}_{\alpha\alpha}^T$ by N^2 -dimensional vectors: $\{\mathbf{u}_1^{(\alpha)}, \mathbf{u}_2^{(\alpha)}, \dots, \mathbf{u}_{M^2}^{(\alpha)}\}$. Eq. (9) is thus rewritten as

$$\begin{bmatrix} \mathbf{v}_1^{(\alpha)} & \mathbf{v}_2^{(\alpha)} & \dots & \mathbf{v}_{M^2}^{(\alpha)} \end{bmatrix} = \Phi \begin{bmatrix} \mathbf{u}_1^{(\alpha)} & \mathbf{u}_2^{(\alpha)} & \dots & \mathbf{u}_{M^2}^{(\alpha)} \end{bmatrix}, \quad (10)$$

$$\Phi = \begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_2 & \dots & \mathbf{P}_K \end{bmatrix}^T. \quad (11)$$

Vector $\mathbf{v}_j^{(\alpha)}$ of the j -th column represents the sequential responses on the j -th camera pixel detected in response to the K -kinds of projector illuminations: $\{\mathbf{P}_1, \mathbf{P}_2, \dots, \mathbf{P}_K\}$, and is given by

$$\mathbf{v}_j^{(\alpha)} = \Phi \mathbf{u}_j^{(\alpha)}, \quad j \in \{1, 2, \dots, M^2\}. \quad (12)$$

The light transport matrix generally has the property of sparseness, namely, vector $\mathbf{u}_j^{(\alpha)}$ is also sparse. Eq. (12) means that N^2 -dimensional vector $\mathbf{u}_j^{(\alpha)}$ is compressed into K -dimensional vector $\mathbf{v}_j^{(\alpha)}$ by $K \times N^2$ matrix Φ , which is called a sensing matrix or a measurement matrix in compressed sensing. We use Bernoulli binary patterns that satisfy RIP (Restricted Isometry Property) in the sensing matrix⁽²³⁾⁽²⁴⁾, and then obtain the sparse optimal solutions: $\{\mathbf{u}_1^{(\alpha)}, \mathbf{u}_2^{(\alpha)}, \dots, \mathbf{u}_{M^2}^{(\alpha)}\}$ that satisfy

$$\begin{aligned} & \text{minimize } \|\mathbf{u}_j^{(\alpha)}\|_{l_1} \\ & \text{subject to } \|\mathbf{v}_j^{(\alpha)} - \Phi \mathbf{u}_j^{(\alpha)}\|_{l_2} \leq \epsilon, \end{aligned} \quad (13)$$

where symbols $\|\cdot\|_{l_1}$ and $\|\cdot\|_{l_2}$ represent l_1 -norm and l_2 -norm, respectively. Considering that image noise is randomly added to the camera responses in projector-camera systems, we set permissible error ϵ in Eq. (13). We find the optimal solutions: $\{\mathbf{u}_1^{(\alpha)}, \mathbf{u}_2^{(\alpha)}, \dots, \mathbf{u}_{M^2}^{(\alpha)}\}$ in the underdetermined linear systems and attain the light transport matrix

$$\mathbf{T}_{\alpha\alpha} = \begin{bmatrix} \mathbf{u}_1^{(\alpha)} & \mathbf{u}_2^{(\alpha)} & \dots & \mathbf{u}_{M^2}^{(\alpha)} \end{bmatrix}^T, \quad (14)$$

by arranging all vectors in sequence.

3. Compressive Inverse Light Transport

The right side of Fig. 1 provides a summary of our compressive inverse light transport. As mentioned in Section 2, the inverse light transport matrix consists of the inverse matrices of direct light transport matrix and inter-reflection matrix. Supposing that direct and global light transport matrices are separated from the light transport matrix, the inverse inter-reflection matrix is computed by utilizing Neumann inverse series with n -bounce reflection approximation. Previous methods^{(33)~(35)} cannot obtain inverse direct light transport matrix without calibration of the projector-camera system.

Based on Eq. (6), we focus on the fact that the projector illumination vector basically equals the matrix-vector product of inverse direct light transport matrix and the camera response vector corrected by using the inverse inter-reflection matrix. We introduce compressed sensing to optimally estimate the inverse direct light transport matrix from the sets of projector illumination patterns and the corrected camera responses, as shown in Fig. 1. This approach is innovative and original because the compressed sensing has never been applied to estimate inverse light transport matrices used in radiometric compensation. Accurately computing the inverse light transport matrix without resort to calibration setup is a novel approach in the framework of inverse light transport. Our compressive inverse light transport approach is explained below.

3.1 Direct and Global Light Transports

We extract direct light transports and global light transports from the light transport matrix acquired by uncalibrated projector-camera system. As the theory of inverse light transport implicitly assumes that the intensities of camera responses via direct light transports are stronger than those via global light transports, we find the r -, g -, and b -channel's camera pixels that most strongly respond to each projector pixel. Given light transport matrix $\mathbf{T}_{\alpha\alpha}$, $\alpha \in \{r, g, b\}$, we detect the maximum for every column and then extract the matrix element as a direct light transport. For instance, when we find the maximum on the l -th row for the j -th column, we generate the M^2 -dimensional column vector $\mathbf{d}_j^{(\alpha)}$ which has only the l -th elements. If we cannot find the maximum, we assign an M^2 -dimensional zero vector to the column. By arranging all M^2 -dimensional column vectors extracted from matrix $\mathbf{T}_{\alpha\alpha}$, we construct the $M^2 \times N^2$ matrix $\mathbf{D}_{\alpha\alpha}$ by using

$$\mathbf{D}_{\alpha\alpha} = \begin{bmatrix} \mathbf{d}_1^{(\alpha)} & \mathbf{d}_2^{(\alpha)} & \dots & \mathbf{d}_{N^2}^{(\alpha)} \end{bmatrix} = \begin{bmatrix} \mathbf{e}_1^{(\alpha)} & \mathbf{e}_2^{(\alpha)} & \dots & \mathbf{e}_{M^2}^{(\alpha)} \end{bmatrix}^T, \quad (15)$$

and take it as the direct light transport matrix. Even if we handle a light transport matrix as a square matrix, one or more projector pixels may connect to one camera pixel via the direct light transport; it means that the rows in matrix $\mathbf{D}_{\alpha\alpha}$ have one or more non-zero elements. Eq. (15) represents every row in matrix $\mathbf{D}_{\alpha\alpha}$ by N^2 -dimensional vectors: $\{\mathbf{e}_1^{(\alpha)}, \mathbf{e}_2^{(\alpha)}, \dots, \mathbf{e}_{M^2}^{(\alpha)}\}$.

Next, we obtain the $M^2 \times M^2$ matrix $\mathbf{R}_{\alpha\alpha}$ that satisfies Eqs. (2) and (4). It is not possible to compute straightforwardly the inverse matrix of matrix $\mathbf{D}_{\alpha\alpha}$ to solve Eq. (4) due to the sparsity and the huge computation space. We adopt an alternative approach that uses the data structure of direct

Fig.1 Illustrations of compressive light transport and compressive inverse light transport.

camera response vectors, i.e. the orthogonality of vectors in Eq. (15). Subtracting matrix $\mathbf{D}_{\alpha\alpha}$ from matrix $\mathbf{T}_{\alpha\alpha}$ yields

$$\mathbf{T}_{\alpha\alpha} - \mathbf{D}_{\alpha\alpha} = \mathbf{R}_{\alpha\alpha} \mathbf{D}_{\alpha\alpha}, \quad (16)$$

based on Eqs. (2) and (4). We multiply both sides of Eq. (16) by the transpose of matrix $\mathbf{D}_{\alpha\alpha}$, which yields

$$\mathbf{R}_{\alpha\alpha} \mathbf{D}_{\alpha\alpha} \mathbf{D}_{\alpha\alpha}^T = (\mathbf{T}_{\alpha\alpha} - \mathbf{D}_{\alpha\alpha}) \mathbf{D}_{\alpha\alpha}^T. \quad (17)$$

In direct light transport matrix $\mathbf{D}_{\alpha\alpha}$, all column vectors: $\{\mathbf{d}_1^{(\alpha)}, \mathbf{d}_2^{(\alpha)}, \dots, \mathbf{d}_{N^2}^{(\alpha)}\}$ except for zero column vectors have only one element so that each projector pixel corresponds to at most a single camera pixel. One column vector and another are not always orthogonal because the matrix includes one or more elements in the row direction. Conversely, two column vectors extracted from $\{\mathbf{e}_1^{(\alpha)}, \mathbf{e}_2^{(\alpha)}, \dots, \mathbf{e}_{M^2}^{(\alpha)}\}$ are always orthogonal. Suppose that two vectors $\mathbf{e}_i^{(\alpha)}$ and $\mathbf{e}_j^{(\alpha)}$ correspond to the i -th and j -th camera pixels ($i \neq j$). Since the inner product of vectors $\mathbf{e}_i^{(\alpha)}$ and $\mathbf{e}_j^{(\alpha)}$ yields

$$(\mathbf{e}_i^{(\alpha)})^T \mathbf{e}_j^{(\alpha)} = 0, \quad (18)$$

matrix $\mathbf{D}_{\alpha\alpha} \mathbf{D}_{\alpha\alpha}^T$ becomes an $M^2 \times M^2$ diagonal matrix. From Eq. (17), we determine matrix $\mathbf{R}_{\alpha\alpha}$ as follows,

$$\mathbf{R}_{\alpha\alpha} = (\mathbf{T}_{\alpha\alpha} - \mathbf{D}_{\alpha\alpha}) \mathbf{D}_{\alpha\alpha}^T \begin{bmatrix} \lambda_1^{(\alpha)} & 0 & \dots & 0 \\ 0 & \lambda_2^{(\alpha)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_{M^2}^{(\alpha)} \end{bmatrix}, \quad (19)$$

where each factor $\lambda_j^{(\alpha)}$, $j \in \{1, 2, \dots, M^2\}$ is given by

$$\lambda_j^{(\alpha)} = \begin{cases} 1 / (\mathbf{e}_j^{(\alpha)})^T \mathbf{e}_j^{(\alpha)}, & \text{if } \mathbf{e}_j^{(\alpha)} \neq \mathbf{0}, \\ 0, & \text{if } \mathbf{e}_j^{(\alpha)} = \mathbf{0}. \end{cases} \quad (20)$$

3.2 Inverse Light Transport Matrix

We assume that the theory of inverse light transport always supports light transports via pure color channels in the system. Based on Eq. (6), we define the $N^2 \times M^2$ pseudo-inverse matrix of the light transport matrix by

$$\mathbf{T}_{\alpha\alpha}^\dagger = \mathbf{D}_{\alpha\alpha}^\dagger \mathbf{S}_{\alpha\alpha}^{-1}, \quad (21)$$

$$\mathbf{S}_{\alpha\alpha}^{-1} = \mathbf{I} + \sum_{k=1}^n (-\mathbf{R}_{\alpha\alpha})^k, \quad \alpha \in \{r, g, b\}, \quad (22)$$

where matrix $\mathbf{D}_{\alpha\alpha}^\dagger$ indicates the $N^2 \times M^2$ pseudo-inverse matrix of direct light transport matrix $\mathbf{D}_{\alpha\alpha}$. Although matrix product $\mathbf{D}_{\alpha\alpha} \mathbf{D}_{\alpha\alpha}^\dagger$ ideally demands the $M^2 \times M^2$ identity matrix to obtain the desired image by the camera, some elements in the matrix product are, in practice, lost owing to occlusion caused by the camera's and projector's orientation and position as setup. Given an arbitrary image to the system, the matrix-vector product developed from the inverse light transport matrix and the camera vector $\mathbf{C}_\alpha$ yields the compensated projector vector $\mathbf{P}_\alpha$ as

$$\mathbf{P}_\alpha = \mathbf{T}_{\alpha\alpha}^\dagger (\mathbf{C}_\alpha - \mathbf{F}_\alpha), \quad \alpha \in \{r, g, b\}, \quad (23)$$

for radiometric compensation.

As inverse matrix $\mathbf{S}_{\alpha\alpha}^{-1}$ is given by Eq. (22), our

original approach obtains pseudo-inverse matrix $\mathbf{D}_{\alpha\alpha}^\dagger$ in Eq. (21) by introducing compressed sensing. Assuming that matrix $\mathbf{D}_{\alpha\alpha}^\dagger$ holds the property of sparsity, we estimate it by reusing the K -kinds of projector illuminations: $\{\mathbf{P}_1, \mathbf{P}_2, \dots, \mathbf{P}_K\}$ and the sequential camera responses of α -channel: $\{\mathbf{C}_1^{(\alpha)}, \mathbf{C}_2^{(\alpha)}, \dots, \mathbf{C}_K^{(\alpha)}\}$. According to Eq. (23), we have

$$\begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_2 & \dots & \mathbf{P}_K \end{bmatrix} = \mathbf{D}_{\alpha\alpha}^\dagger \mathbf{S}_{\alpha\alpha}^{-1} \begin{bmatrix} \mathbf{C}_1^{(\alpha)} & \mathbf{C}_2^{(\alpha)} & \dots & \mathbf{C}_K^{(\alpha)} \end{bmatrix}, \quad (24)$$

in which we have removed environmental components from all camera responses. Transposing Eq. (24) yields

$$\begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_2 & \dots & \mathbf{P}_K \end{bmatrix}^T = \mathbf{\Psi}_\alpha (\mathbf{D}_{\alpha\alpha}^\dagger)^T, \quad (25)$$

with the replacements of

$$\mathbf{\Psi}_\alpha = \begin{bmatrix} \mathbf{S}_{\alpha\alpha}^{-1} \mathbf{C}_1^{(\alpha)} & \mathbf{S}_{\alpha\alpha}^{-1} \mathbf{C}_2^{(\alpha)} & \dots & \mathbf{S}_{\alpha\alpha}^{-1} \mathbf{C}_K^{(\alpha)} \end{bmatrix}^T. \quad (26)$$

In Eq. (25), we replace the N^2 columns in the $K \times N^2$ matrix of the left side by K -dimensional vectors: $\{\mathbf{y}_1^{(\alpha)}, \mathbf{y}_2^{(\alpha)}, \dots, \mathbf{y}_{N^2}^{(\alpha)}\}$, and replace the N^2 columns in the $M^2 \times N^2$ matrix $(\mathbf{D}_{\alpha\alpha}^\dagger)^T$ by M^2 -dimensional vectors: $\{\mathbf{x}_1^{(\alpha)}, \mathbf{x}_2^{(\alpha)}, \dots, \mathbf{x}_{N^2}^{(\alpha)}\}$. Hence it follows that Eq. (25) is rewritten as

$$\begin{bmatrix} \mathbf{y}_1^{(\alpha)} & \mathbf{y}_2^{(\alpha)} & \dots & \mathbf{y}_{N^2}^{(\alpha)} \end{bmatrix} = \mathbf{\Psi}_\alpha \begin{bmatrix} \mathbf{x}_1^{(\alpha)} & \mathbf{x}_2^{(\alpha)} & \dots & \mathbf{x}_{N^2}^{(\alpha)} \end{bmatrix}, \quad (27)$$

where the K -dimensional vector $\mathbf{y}_j^{(\alpha)}$ is given by

$$\mathbf{y}_j^{(\alpha)} = \mathbf{\Psi}_\alpha \mathbf{x}_j^{(\alpha)}, \quad j \in \{1, 2, \dots, N^2\}. \quad (28)$$

Eq. (28) states that M^2 -dimensional vector $\mathbf{x}_j^{(\alpha)}$ is compressed into K -dimensional vector $\mathbf{y}_j^{(\alpha)}$ by $K \times M^2$ sensing matrix $\mathbf{\Psi}_\alpha$. Based on Eq. (28), we find the optimal solutions: $\{\mathbf{x}_1^{(\alpha)}, \mathbf{x}_2^{(\alpha)}, \dots, \mathbf{x}_{N^2}^{(\alpha)}\}$ that satisfy

$$\begin{aligned} & \text{minimize } \|\mathbf{x}_j^{(\alpha)}\|_{l_1} \\ & \text{subject to } \|\mathbf{y}_j^{(\alpha)} - \mathbf{\Psi}_\alpha \mathbf{x}_j^{(\alpha)}\|_{l_2} \leq \epsilon, \end{aligned} \quad (29)$$

by a decoding algorithm used in acquiring the light transport matrix. Finally, we obtain the pseudo-inverse matrix

$$\mathbf{D}_{\alpha\alpha}^\dagger = \begin{bmatrix} \mathbf{x}_1^{(\alpha)} & \mathbf{x}_2^{(\alpha)} & \dots & \mathbf{x}_{N^2}^{(\alpha)} \end{bmatrix}^T, \quad (30)$$

by arranging all solutions in sequence.

Based on the theory of inverse light transport^{(33)~(35)}, the inverse inter-reflection matrix plays the important role of canceling global reflection light. In our approach, inverse inter-reflection matrix $\mathbf{S}_{\alpha\alpha}^{-1}$ is used in constructing sensing matrix $\mathbf{\Psi}_\alpha$. Our proposal uses compressed sensing with sensing matrix $\mathbf{\Psi}_\alpha$ to determine inverse direct light transport matrix $\mathbf{D}_{\alpha\alpha}^\dagger$ in the framework of inverse light transport.

3.3 Processing Flow

We summarize here the processing flow of compressive inverse light transport:

- (i) Generate K -kinds of illumination images based on Bernoulli binary patterns. Output the illumination images $\{\mathbf{P}_1, \mathbf{P}_2, \dots, \mathbf{P}_K\}$ from r -, g -, and b -channels in the projector, and capture the camera responses of α -channel: $\{\mathbf{C}_1^{(\alpha)}, \mathbf{C}_2^{(\alpha)}, \dots, \mathbf{C}_K^{(\alpha)}\}$, $\alpha \in \{r, g, b\}$, in sequence.
- (ii) Set sensing matrix $\mathbf{\Phi}$ given by Eq. (11). Find the optimal solutions: $\{\mathbf{u}_1^{(\alpha)}, \mathbf{u}_2^{(\alpha)}, \dots, \mathbf{u}_{M^2}^{(\alpha)}\}$ that satisfy Eq. (13). Construct light transport matrix $\mathbf{T}_{\alpha\alpha}$ according to Eq. (14).
- (iii) Extract direct light transports in matrix $\mathbf{T}_{\alpha\alpha}$ and construct direct light transport matrix $\mathbf{D}_{\alpha\alpha}$ shown in Eq. (15). Compute matrix $\mathbf{R}_{\alpha\alpha}$ by using Eqs. (19) and (20).
- (iv) Obtain the corrected camera responses $\mathbf{S}_{\alpha\alpha}^{-1} \mathbf{C}_i^{(\alpha)}$, $i \in \{1, 2, \dots, K\}$ by employing iterative computation using matrix $\mathbf{R}_{\alpha\alpha}$; refer to Appendix A, and then set sensing matrix $\mathbf{\Psi}_\alpha$ according to Eq. (26).
- (v) Find the optimal solutions that satisfy Eq. (29). Construct pseudo-inverse matrix $\mathbf{D}_{\alpha\alpha}^\dagger$ by arranging all solutions: $\{\mathbf{x}_1^{(\alpha)}, \mathbf{x}_2^{(\alpha)}, \dots, \mathbf{x}_{N^2}^{(\alpha)}\}$ according to Eq. (30).

If users obtain a light transport matrix by using another approach, they can omit (i) and (ii) from the above processing flow. Our approach reuses the sets of illumination patterns and the corresponding camera responses used in acquiring the light transport matrix in order to efficiently obtain the inverse light transport matrix.

Although inverse light transport matrix $\mathbf{T}_{\alpha\alpha}^\dagger$ consists of matrix product $\mathbf{D}_{\alpha\alpha}^\dagger \mathbf{S}_{\alpha\alpha}^{-1}$, we do not compute inverse matrix $\mathbf{S}_{\alpha\alpha}^{-1}$ by using Eq. (22) because the power operations with matrix $\mathbf{R}_{\alpha\alpha}$ need enormous working memory. Instead, we use the iterative computation in (iv) for practical radiometric compensation. As shown in Appendix A, when desired color vector $\mathbf{C}_\alpha$ is given, our radiometric compensation yields a temporary compensated vector, which virtually equals vector $\mathbf{S}_{\alpha\alpha}^{-1} \mathbf{C}_\alpha$ by iterative computation using matrix $\mathbf{R}_{\alpha\alpha}$, and then creates the compensated projector illumination $\mathbf{P}_\alpha$ by using the matrix-vector product of pseudo-inverse matrix $\mathbf{D}_{\alpha\alpha}^\dagger$ and the temporary vector. Once the system has pseudo-inverse matrix $\mathbf{D}_{\alpha\alpha}^\dagger$ and matrix $\mathbf{R}_{\alpha\alpha}$ according to 3D structures, the compensated projector image is generated based on Eq. (23) so that the desired projection-based display is formed on the 3D objects.

4. Experiments and Results

4.1 Experimental Setup

Fig. 2(a) shows our experimental system. It consists of an LCD projector (SONY VPL-FX41) and an IEEE1394b camera (Point Grey GRAS-50S5C-C) controlled by separate computers. Once the computer that handles the camera captures an image, it automatically transfers the observed image to the other computer that controls the projector via LAN. The experiments demand that the projector-camera system offers linear input/output responses to accurately acquire light transports. Since the camera responses exhibited almost linear functions relative to input brightness in each color channel, we measured the color channel's luminance from the projector light by installing a spectroradiometer (PR655 SpectraScan). According to the inverse functions, we linearized the projector responses subject to input brightness⁴⁾.

Fig. 2(b) shows the 3D objects used: a gypsum torso, an accordion folded paper, and plastic boxes. In our experiments, the system measures light transports from each 3D object in an almost totally dark room. On the assumption that the projector and the camera are integrated, we conducted a preliminary experiment to determine the suitable distance between the system and these objects. When illuminating Bernoulli binary patterns, the system observed strong black offset lights at distances of less than 1 m. Compared to direct reflection light, global lights reflected from the 3D structures were dominant in the camera responses at such close distances. We confirmed that the system detected few black offset lights at the distance of about 1.5 m. We found that some defective pixels began to occur in the sequential camera responses at distances in excess of 4 m due to the attenuated illumination patterns. Given these conditions, we determined that the experimental system adequately works in obtaining direct and global light transports within the range of 1.5 – 4 m. In Fig. 2(a), the distance from the projector to the screen was about 2.5 m. The region of 256×256 pixels used in projector illumination is marked by the red frame, and the region of 260×260 pixels detected as camera response is marked by the green frame in Fig. 2(b).

Orthogonal matching pursuit⁴²⁾ is a popular decoding algorithm often used in compressed sensing due to its speed and ease of implementation. However, it has been pointed out that when adding signal coefficients in each iteration to recover the sparse signal, an erroneous addition may significantly degrade the quality of the solution. To avoid this problem, we implemented the regularized orthogonal matching pursuit developed in 43) because it has better perfor-

Fig. 2 Experimental system. (a) Configuration. (b) 3D objects.

Fig. 3 Example of compressive light transports in scene A. (a) Bernoulli binary patterns. (b) Camera responses.

Fig. 4 Visualization of light transport matrix; left: scene A, middle: scene B, and right: scene C. (a) T_{gg} . (b) D_{gg} . (c) R_{gg} .

mance while maintaining both processing speed and usability. The compressed sensing algorithm ran on a desktop computer, OS: Windows 7 (32-bit), CPU: Intel Xeon E5520 (2.26 GHz), Memory: 3.43 GB. We used the spalloc function implemented in MATLAB to allocate working memory for the sparse matrix and efficiently decoded the compressive light transports of Eq. (13) and the compressive inverse light transports of Eq. (29) with $\epsilon = 10^{-3}$.

Fig. 5 Close-ups of matrix structure. (a) Red frame. (b) Green frame.

4.2 Acquisition of Light Transport Matrix

Existing compressive light transport approaches do not determine the optimal number of illumination patterns because it depends on system configuration, object's structure, and so on. We determined the suitable number of illumination patterns from the relighting appearance generated by the light transport matrix. For instance, when the system used N -kinds of illumination patterns ($K = N$), the acquired light transport matrix yielded undesirable relighting images with some defective pixels, which hardly have the camera responses related to the illumination patterns. According to our relighting tests with varying projector resolutions, compressive light transport with $K = 2N \sim 3N$ generated adequate relighting images. In accordance with this result, we used 600 kinds of projector illuminations for the case of 256×256 pixels and measured the camera responses for each scene. For reference, the compressed sensing in 23) also uses the same number of illumination patterns. **Fig. 3** provides examples of the Bernoulli binary patterns and the response images as observed by the camera. We removed environment components from the camera images by using background subtraction and took the camera responses that exceeded pixel value 10 as the threshold to eliminate random image noise. From the sets of illumination patterns and observed responses, we decoded the compressive light transport of each 3D structure by using the acquisition method described in Section 2. When compressed sensing yielded feeble matrix elements (i.e. values under 1), we nullified them in the light transport matrix.

The acquired light transport matrices $\mathbf{T}_{\alpha\alpha}$, $\alpha \in \{r, g, b\}$ had similar matrix elements. In order to visualize the whole structure of the light transport matrix, we generated 260×256 matrix element maps by reducing the entire

Fig. 6 Relighting images. (a) Observed. (b) VHLP. (c) BBP.

$67,600 \times 65,536$ matrix $\mathbf{T}_{gg}$. Sampling the elements in a sparse matrix may not reflect the original structure of the matrix. Accordingly, when each 260×256 sub-matrix in original matrix $\mathbf{T}_{gg}$ had one or more non-zero elements, we replaced the sub-matrix with one white pixel. **Fig. 4(a)** illustrates the element structures of the light transport matrices $\mathbf{T}_{gg}$ acquired from scene A, scene B, and scene C. **Fig. 5(a)** shows the original matrix elements marked by the tiny red frame in Fig. 4(a). Although it seems that the elements in Fig. 4(a) concentrate on the diagonal components, Fig. 5(a) shows that the actual distributions are sparse. In addition, Fig. 4(a) has some matrix structures that appear like vertical line segments. Fig. 5(b) shows the original matrix elements marked by the tiny green frame in Fig. 4(a); various isolated elements are dotted along the vertical axis in each matrix. When a single projector pixel connects with multiple camera pixels, it is well known that the light transport matrix has matrix structures that look like vertical line segments³⁷⁾. It means that these camera pixels have not only direct light transport but also some global light transports which include complicate indirect reflections via the 3D structure. In compressed sensing, when K -dimensional column vector in sensing matrix Φ and some K -dimensional sequential camera vectors are strongly correlated, the sparse optimal solution that satisfies Eq. (13) yields such global light transports.

Next, we formed a relighting image by using Eq. (7) to confirm the accuracy of the light transport matrix. The il-

lumination image used in this test is the checkered pattern image #1 shown in **Fig. 11**. We also tested our previous method³⁷⁾ for a comparison; it obtains camera responses by handling $2N$ -kinds of push broom-type illuminations, i.e. vertical- and horizontal-line patterns (VHLP) and then estimates the light transport matrix by employing compressed sensing. **Fig. 6(a)** shows the observation images captured by the camera. **Fig. 6(b)** and **Fig. 6(c)** exhibit the relighting images yielded by the existing method and the present method using Bernoulli binary patterns (BBP). We calculated the RMS errors between the camera images and the relighting images. From the left to the right in **Fig. 6(b)**, the RMS errors were 17.1, 16.5, and 17.5 levels, while the RMS errors yielded by the present method were 13.7, 11.1, and 14.0 levels in **Fig. 6(c)**. Although we used the same algorithm in decoding the compressive light transports, we found that the present method yielded better relighting images than previous method. A comparison with camera images shows that compressed sensing using Bernoulli binary patterns yields good relighting images.

4.3 Performance of Compressive Inverse Light Transport

This subsection examines the performance of compressive inverse light transport. We constructed direct light transport matrix $\mathbf{D}_{\alpha\alpha}$ according to Eq. (15) and extracted matrix $\mathbf{R}_{\alpha\alpha}$ by using Eqs. (19) and (20) from the light transport matrix $\mathbf{T}_{\alpha\alpha}$. **Fig. 4(b)** and **Fig. 4(c)** visualize the elements of matrix $\mathbf{D}_{gg}$ and matrix $\mathbf{R}_{gg}$ by a reducing scheme similar to that used in **Fig. 4(a)**. By computing matrix product $\mathbf{D}_{\alpha\alpha}\mathbf{D}_{\alpha\alpha}^T$, in fact, we confirmed that it was exactly a diagonal matrix. Matrix $\mathbf{R}_{gg}$ faithfully reproduces global reflection lights in light transport matrix $\mathbf{T}_{gg}$. Incidentally, relighting computation using matrix $(\mathbf{I} + \mathbf{R}_{\alpha\alpha})\mathbf{D}_{\alpha\alpha}$ generated almost the same images as **Fig. 6(b)**. The results ensure that the desired direct and global light transports were accurately separated from various light transports. In the system, the memory limits of the computer that implemented our algorithm prevented it from directly computing the power operations with matrix $\mathbf{R}_{\alpha\alpha}$ by using Eq. (22). Instead of visualizing inverse matrix $\mathbf{S}_{\alpha\alpha}^{-1}$, we provide examples of camera response image yielded by $\mathbf{S}_{\alpha\alpha}^{-1}\mathbf{C}_i^{(\alpha)}$, $i \in \{1, 2, \dots, K\}$ in **Fig. 7**; see Appendix A. **Fig. 7(a)** and **Fig. 7(b)** show the original camera response images and the corrected camera response images by Eq. (22) with $n = 20$, respectively. Since human vision may not be able to recognize the differences between original camera responses and the corrected responses, we measured the RMS error between the corrected response image at the n -th iteration and the previous image; when $n = 1$, the previous image is the original

Fig. 7 Comparison of camera responses. (a) Original. (b) Corrected.

Fig. 8 RMS error versus iteration number n .

camera image $\mathbf{C}_i^{(\alpha)}$. **Fig. 8** shows the RMS errors at each state up to the completion of the 40-th iteration. Note that matrix $\mathbf{R}_{\alpha\alpha}$ represents not only multiple reflections but also global reflections radiated from the entire environment. As shown in **Fig. 8**, significant iteration of matrix $\mathbf{R}_{\alpha\alpha}$ cancels the complicated global reflections in the detected camera response. After the 20-th iteration, the power operation with matrix $\mathbf{R}_{\alpha\alpha}$ yields almost the same images because the RMS errors converge to less than level 1. Thus, we determined that the effective number of iterations is $n = 20$ for constructing sensing matrix Ψ_α in Eq. (26) and compensating the projector illumination images in Eq. (23).

Next, we estimated the inverse direct light transport matrix by using our proposed method. **Fig. 9(a)** exhibits the elements of inverse matrix $\mathbf{D}_{gg}^\dagger$. Compared to light transports around the diagonal elements in **Fig. 4(b)**, each inverse matrix $\mathbf{D}_{gg}^\dagger$ has matrix structure as if the light transports invert the matrix elements centered on diagonal elements. In fact, **Fig. 9(b)** shows that matrix product $\mathbf{D}_{gg}\mathbf{D}_{gg}^\dagger$ achieves

Fig. 9 Inverse direct light transport matrix. (a) $\mathbf{D}_{gg}^\dagger \cdot \mathbf{D}_{gg}^\dagger$. (b) $\mathbf{D}_{gg} \mathbf{D}_{gg}^\dagger$.

Fig. 10 Inverse matrix computed with $\mathbf{S}_{\alpha\alpha}^{-1} = \mathbf{I}$. (a) $\mathbf{D}_{gg}^\dagger$. (b) $\mathbf{D}_{gg} \mathbf{D}_{gg}^\dagger$.

almost complete diagonalization. We confirmed that compressed sensing offers excellent performance in decoding inverse light transport matrix $\mathbf{D}_{\alpha\alpha}^\dagger$. As previous studies^{(31)~(35)} have investigated the effect of inverse matrix $\mathbf{S}_{\alpha\alpha}^{-1}$ in radiometric compensation and confirmed that it cancels n -bounce reflection from 3D structures, we examine here the impact of matrix $\mathbf{S}_{\alpha\alpha}^{-1}$ on compressive inverse light transport. We estimated inverse matrix $\mathbf{D}_{\alpha\alpha}^\dagger$ by approximating matrix $\mathbf{S}_{\alpha\alpha}^{-1}$ as identity matrix $\mathbf{I}$ in Eq. (22). In other words, this computation is not based on the theory of inverse light transport but instead depends on the performance of compressed sensing without resort to inverse matrix $\mathbf{S}_{\alpha\alpha}^{-1}$. Sensing matrix Ψ_α consists of the sequential camera responses of α -channel: $\{\mathbf{C}_1^{(\alpha)}, \mathbf{C}_2^{(\alpha)}, \dots, \mathbf{C}_K^{(\alpha)}\}$ in Eq. (26). **Fig. 10(a)** visualizes inverse matrix $\mathbf{D}_{gg}^\dagger$ yielded by the compressed sensing. **Fig. 10(b)** clearly shows that matrix product $\mathbf{D}_{gg} \mathbf{D}_{gg}^\dagger$ does not achieve sufficient diagonalization. These results ensure that the impact of inverse matrix $\mathbf{S}_{\alpha\alpha}^{-1}$ is to realize diagonalization for radiometric compensation and that compressed sensing needs to use the theory of inverse light transport to accurately solve Eq. (21).

4.4 Comparison of Radiometric Compensation

We demonstrate radiometric compensation tests conducted with existing methods and our method. We use Fujii’s adaptive compensation method⁽¹¹⁾ and our previous method⁽³⁷⁾ as benchmarks. The adaptive compensation handles only direct light reflected from 3D structures and compensates projector brightness based on the color variation detected on 3D objects. We use the geometric correspondences between projector and camera pixels extracted from direct light transport matrix, and obtain a color-mixing matrix for each camera pixel by using 4 kinds of color images⁽¹¹⁾. In our previous work⁽³⁷⁾, we demonstrated that the reformed color-mixing matrix is an effective approach to radiometric

Fig. 11 Illumination images used in our experiments.

compensation for creating desired image displays on 3D objects. Given a light transport matrix with color crosstalk, the approach reforms the color-mixing matrices^{(9)~(11)} of various combinations of all projector pixels and all camera pixels and uses feedback control to complete the projection-based display. We decode the light transports with color crosstalk by using color Bernoulli patterns and the corresponding response images, and then reform the color-mixing matrices between each camera pixel and the multiple projector pixels via the light transports. Hereafter, the adaptive compensation method, the compensation method by using the reformed color-mixing matrix, and our compressive inverse light transport are abbreviated as AC, RCM, and CILT, respectively.

Each method corrected geometrically and photometrically the test images shown in Fig. 11 so that the system created the new appearance images within the green frame in Fig. 2(b). **Fig. 12** displays the new appearance images yielded by AC, RCM, and CILT in scene A. Existing methods and our method create good images which are geometrically correct, and also favorably compensate the image texture on the surfaces that correspond to breast and belly parts. **Fig. 13** displays the new appearance images observed from scene B. AC yields image displays that are somewhat different from the desired images. RCM creates almost seamless image displays that follow the convexity of the accordion shape. However, RCM could not adjust some camera pixels even after 10 iterations of feedback control in the system.

Fig. 12 New appearance images observed from scene A. (a) AC. (b) RCM. (c) CILT.

Fig. 13 New appearance images observed from scene B. (a) AC. (b) RCM. (c) CILT.

In contrast, CILT yields good image displays as a whole on the concave-convex surface and attains adequate radiometric compensation on the accordion folded paper. **Fig. 14** shows image displays provided by each method in scene C. AC displays inadequate images on the convex shape and generates glaring color gaps among multiple boxes. While RCM well compensates the surface textures of the boxes, CILT

achieves complete seamless image displays on the complicated surfaces.

We summarize the RMS errors and NCC (normalized correlation coefficient) between desired images and the new appearance images observed from each scene in **Table 1**, **Table 2**, and **Table 3**. Values in parentheses indicate NCC yielded by each method. CILT output image displays that

Fig. 14 New appearance images observed from scene C. (a) AC. (b) RCM. (c) CILT.

Table 1	Scene A: RMS error [unit: level] and NCC.		
	AC	RCM	CILT
Image #1	44.3 (0.971)	23.2 (0.986)	24.3 (0.976)
Image #2	36.6 (0.953)	26.8 (0.968)	27.4 (0.949)
Image #3	33.7 (0.937)	20.2 (0.969)	20.6 (0.952)
Image #4	36.4 (0.938)	23.9 (0.963)	25.6 (0.941)
Image #5	29.3 (0.816)	26.7 (0.870)	27.1 (0.859)
Image #6	23.2 (0.867)	23.7 (0.896)	24.2 (0.890)

Table 2	Scene B: RMS error [unit: level] and NCC.		
	AC	RCM	CILT
Image #1	47.0 (0.975)	27.8 (0.984)	27.3 (0.985)
Image #2	36.3 (0.956)	30.2 (0.969)	30.6 (0.968)
Image #3	32.3 (0.938)	20.3 (0.967)	18.8 (0.972)
Image #4	37.1 (0.945)	25.6 (0.962)	25.8 (0.957)
Image #5	32.8 (0.713)	31.1 (0.777)	32.9 (0.749)
Image #6	28.0 (0.767)	27.8 (0.809)	31.0 (0.770)

Table 3	Scene C: RMS error [unit: level] and NCC.		
	AC	RCM	CILT
Image #1	56.5 (0.963)	27.9 (0.990)	22.7 (0.991)
Image #2	38.8 (0.952)	29.8 (0.977)	29.0 (0.974)
Image #3	34.3 (0.935)	25.0 (0.967)	16.7 (0.974)
Image #4	40.6 (0.921)	28.0 (0.970)	23.8 (0.972)
Image #5	32.3 (0.717)	28.7 (0.845)	29.7 (0.830)
Image #6	25.4 (0.795)	23.5 (0.873)	27.7 (0.861)

were approximately equivalent to those of RCM in scene A and scene B. RCM strongly relies on feedback control to achieve the desired image display, while CILT virtually realizes similar displays by computing the simple matrix-vector product. In scene C, CILT yields accurate image displays with better quality than those of RCM for image #1 ~ image #4. CILT provides excellent radiometric compensation performance even with the presence of complicated global reflection lights on the multiple boxes. These results confirm that our compensation method based on compressive inverse light transport works well to provide desired image displays on various uneven surfaces.

The radiometric compensation approach used in RCM minimizes the brightness error between desired image and new appearance image via feedback control so that projector brightness levels are adjusted based on direct and global light transports³⁷⁾. It exhibited generally good performance in all scenes. However, the approach always needs camera observation whenever projector brightness levels are updated. Obviously, since the feedback requirement hinders sequential processing in projection-based displays, its application is restricted to static images in practice. In contrast, CILT dispenses with iterative feedback by using camera observation and provides image displays that compare favorably with the images yielded by RCM in these scenes, even though it uses just the matrix-vector product. As the goal of this study, our method performed radiometric compensation using a sparse and high-rank light transport matrix. Once the system determines the inverse light transport matrix specific to the 3D structures, the compensation approach can rapidly create projection-based displays on the 3D objects according to desired images in sequence. By attaining high-speed cal-

culation of the matrix-vector product, our proposed method will be able to compensate sequential images as with real-time compensation methods⁽¹⁾⁽³⁶⁾.

Although we obtained inverse light transport matrices that approximately equal a square matrix in our experiments, Eq. (21) supports inverse light transport matrices of various sizes depending on the system configuration. We discuss two cases of inverse light transport matrix on the assumption that the size of projection-based display on the 3D object is constant. If the distance between the projector and 3D object is farther than that between the camera and the object, our method creates the inverse light transport matrix with $M > N$. As one projector pixel connects to many camera pixels via the direct light transport, multiple camera pixels are simultaneously compensated by single projector pixel. Namely, the inverse light transport matrix may yield low-resolution image displays. In the case that the distance between the camera and 3D object is farther than that between the projector and the object, conversely, our method creates the inverse light transport matrix with $M < N$. Unfortunately, accurate light transports will not be decoded even with compressed sensing due to camera responses with low-resolution. The inverse light transport matrix cannot create good image displays if that happens. In order to execute effectively and accurately radiometric compensation, it is desirable that users obtain the inverse light transport matrix with $M \approx N$.

5. Conclusion

We proposed a new method that obtains inverse light transport matrices to realize radiometric compensation for projection-based displays. The theory of inverse light transport demands that the inverse light transport matrix consists of the inverse matrices of a direct light transport matrix and an inter-reflection matrix. Assuming that the inverse inter-reflection matrix is approximately calculated by Neumann inverse series, previous methods calculated the inverse direct light transport matrix by using dedicated projector-camera system in which the optical axes of the projector and the camera are aligned. Our proposed method utilizes the inverse inter-reflection matrix that cancels global reflection lights for constructing the sensing matrix and optimally decodes inverse direct light transports by introducing compressed sensing with the sensing matrix. Our proposed method is a novel approach that dispenses with calibration setup in the framework of inverse light transport; it expands the application scope of inverse light transport in real environments.

We visualized the matrix elements of the various light

transport matrices yielded in the process of compressive inverse light transport and examined its performance in detail. We confirmed that compressed sensing offers excellent performance in decoding the inverse light transport matrix because the matrix product created from the light transport matrix and the estimated inverse matrix basically satisfies the diagonalization requirement. We conducted radiometric compensation tests on existing methods and our method, and showed that our method provided excellent image displays on various 3D surfaces. Since our new approach executes radiometric compensation with simple linear operations, its application is not restricted to static images and will offer real-time compensation processing.

Compressed sensing suppresses the sets of projector illumination patterns and camera response images to the requisite minimum in constructing the inverse light transport matrix. However, its decoding approach processes the compressive inverse light transport for each projector pixel in sequence. Future work is to improve the speed of processing in order to decode the compressive inverse light transports as quickly as possible to handle high resolution projection-based displays.

Acknowledgment

The constructive comments and advice from the editor and reviewers are gratefully acknowledged.

6. Appendix A: Iterative Computation Using Matrix $\mathbf{R}_{\alpha\alpha}$

We extract the i -th camera response of α -channel,

$$\mathbf{q}_0^{(\alpha)} = \mathbf{C}_i^{(\alpha)}, \quad i = \{1, 2, \dots, K\}, \alpha \in \{r, g, b\}, \quad (31)$$

from the sequential camera images, and then hold the M^2 -dimensional vector $\mathbf{q}_0^{(\alpha)}$ at initial state $t = 0$. At state t , we get vector $\mathbf{q}_t^{(\alpha)}$ by updating with

$$\mathbf{q}_t^{(\alpha)} = \mathbf{q}_0^{(\alpha)} - \mathbf{R}_{\alpha\alpha} \mathbf{q}_{t-1}^{(\alpha)}, \quad t \geq 1. \quad (32)$$

When we repeat the iterative computation in Eq. (32) to $t = n$, we obtain

$$\begin{aligned} \mathbf{q}_n^{(\alpha)} &= \mathbf{q}_0^{(\alpha)} - \mathbf{R}_{\alpha\alpha} \mathbf{q}_{n-1}^{(\alpha)} \\ &= (\mathbf{I} - \mathbf{R}_{\alpha\alpha}) \mathbf{q}_0^{(\alpha)} + \mathbf{R}_{\alpha\alpha}^2 \mathbf{q}_{n-2}^{(\alpha)} \\ &\vdots \\ &= (\mathbf{I} - \mathbf{R}_{\alpha\alpha} + \mathbf{R}_{\alpha\alpha}^2 - \dots + (-\mathbf{R}_{\alpha\alpha})^n) \mathbf{q}_0^{(\alpha)} \\ &= (\mathbf{I} + \sum_{k=1}^n (-\mathbf{R}_{\alpha\alpha})^k) \mathbf{q}_0^{(\alpha)}. \end{aligned} \quad (33)$$

In the same way, we calculate all vectors $\mathbf{q}_n^{(\alpha)}$ in the sequential order: $i \in \{1, 2, \dots, K\}$. Thus, we obtain sensing ma-

trix Ψ_α shown in Eq. (26) by arranging all corrected camera response vectors in sequence. The computation yielded by repeatedly executing Eq. (32) achieves the same solution as that yielded by Eq. (26).

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