Investigation of Noise Characteristics of Multivalued Signals and Estimation of Single-page Storage Density in Holographic Data Storage

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Abstract We have investigated the noise characteristics in the intensity-modulated and phase-modulated signals and estimated how much the single-page storage density and data transfer rate increase by using the multivalued signals. In our simulation, the diffracted image was calculated considering two important aspects in actual holographic data storage systems. One is the aperture inserted in the signal optical path, and the other is the oblique hologram shape formed in the recording medium. Our numerical simulation revealed that in the case of the intensity-modulated signal, the single-page storage density was almost unchanged even if the number of multivalued signals was increased. In contrast, when an 8-level phase multivalued signal was employed in the system, the single-page storage density was estimated to be increased by a factor of 2.4 -3.0 compared to the intensity binary signal.

Keywords: Holographic data storage, holographic simulator, Nyquist aperture, phase-modulated signal, multivalued signal.

1. Introduction

Importance of the storage device has been significantly increased because most information around us is electronically treated and stored as digital data in the present day. One of the biggest issues in such an information society is that quite large energy is consumed by hard disk drives running in the data centers. In contrast, optical data storage (ODS) systems, like CD, DVD, and Blu-ray, have been developed as another archival storage device with long lifetime and low energy consumption. Thus, there is a great advantage from the environmental point of view if the ODS system is partially replaced with the hard disk drive in archival data center. However, the current optical disk system does not have a sufficient recording performance, and its technology has already reached to its theoretical limit at the Blu-ray disk system. Thus, a major shift, from the conventional optical disk to alternative new architecture, is necessary for further development of the ODS system.

A holographic data storage system^{1, 2)} is one of the most promising candidates for the next ODS system. In this system, information is stored in the form of volume

Fig. 1. A schematic diagram of the angle multiplexed holographic data storage system.

holographic gratings within a photosensitive material, and the readout is performed by utilizing the principle of holography. High data transfer rate can be achieved by parallel processing of two-dimensional data bits called page data³⁾. Large storage capacity is expected owing to the multiplex recording⁴⁻¹²⁾ where different holograms can be recorded at the same medium location and their information can be readout independently. Figure 1 shows schematic diagram of an angle-multiplex recording method⁶⁾. Another hologram with different page data is recorded while slightly changing the reference beam angle. In this case, the storage density D

Received March 20, 2021; Revised May 21, 2021; Accepted June 15, 2021

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is determined by three-factors: single page capacity C_{1page} , multiplexed page number M , and the exposed area on the recording media $S_{holo.}$, that is written by,

$$D = \frac{C_{1page} M}{S_{holo.}}. \quad (1)$$

In order to increase the storage density, a lot of study have been conducted so far. Utilizing a multi-valued signal¹³⁻¹⁸⁾ instead of the binary intensity signal is one of the examples to increase the page capacity C_{1page} . Another example is to insert an aperture in the Fourier plane of the signal image to limit the exposed area^{12, 19-24)}. However, usually it will degrade the reconstructed signal quality due to the crosstalk noise and result in large bit error rate. Therefore, finding their adequate recording parameters is quite essential to design a realistic memory system. In this case, numerical simulation could be a powerful and effective tool for saving time and costs as compared to conducting a lot of blind experiments. Recently, we have developed a holographic simulator considering a realistic hologram shape formed in the recording media for accurate evaluation of the reconstructed image quality²⁵⁻²⁸⁾. In addition, the effect of inter-pixel crosstalk due to the aperture in the Fourier plane can be taken into their calculation. The aim of our numerical simulation is to know how much the storage density and transfer rate will be increased by using the multivalued signals. In the multivalued signals, in general, the code rate C_{1bit} , which represents the average number of bits carried by one pixel, always increases, and thus it has a great possibility to improve the data transfer rate and storage density, simultaneously. However, as a matter of course, the storage density cannot be increased infinitely. This is because increasing the number of multivalued signals increases crosstalk between symbols and increases detection errors. Since errors must be corrected by an error correction method²⁹⁻³¹⁾ that the system adopted, the error rate should be kept less than a specific value allowed by the system. For example, when the error rate increased by increasing the number of multivalued signals, it should be compensated by loosening other parameters, such as the aperture size and the angular spacing for multiplexing, in order to keep it within the allowable error rate. Therefore, in order to maximize the storage density and transfer rate, it is necessary to optimize each parameter within the allowable error rate. In this study, we limited the situation to a one-page recording and evaluated how much the theoretical limits of storage density could be improved with the number of

multivalued signals and aperture size as optimization parameters.

This paper is organized as follows. Section 2 describes the simulation model. The characteristic features of our simulator, such as, a plane-wave expansion method, the Born approximation in an oblique rectangular hologram, and how to incorporate the effect of the aperture, are explained. Section 3 describes our simulated results. We analyzed the noise characteristic in the intensity- and phase modulated signals and estimated how much the storage density can be increased by adopting multivalued signals as compared to the binary intensity signal. Finally, Section 4 concludes this paper.

2. Simulation Method

In the numerical analysis of system characteristics, it is very important to evaluate the actual diffracted light wave accurately. In this section, the basic calculation principle of our holographic simulator will be explained. The typical optical path of the signal light wave in holographic data storage is shown in Fig. 2. The input image is usually Fourier-transformed by the lens, and that image is recorded in the recording medium. In order to limit the exposure area on the recording medium, an aperture is often inserted in the first Fourier plane F1. This is because the recording medium position F2 is a conjugate plane of F1. In this case, a created hologram has similar distribution with the amplitude of the signal wave just after the aperture. Furthermore, since a part of the high spatial frequency component of the input image is blocked by the aperture, the resultant image on the plane I2 loses the sharpness of the edge and becomes blurred. This is known as inter-pixel crosstalk and has a significant effect on the quality of the detected signal image. In the following subsections, we describe how to calculate the diffracted light wave in our holographic simulator and the process of extracting the signal from the obtained diffracted images.

Fig. 2. A typical optical path of signal wave in holographic data storage system.

2.1. Calculation for diffracted light waves

2.1.1 Plane wave expansion method

In this subsection, the basic principle of our simulator, the plane wave expansion method³²⁾, is described. In holographic data storage systems, a recorded hologram has usually a very complicated structure because the signal wave includes various wave vectors depending on the input signal pattern. The basic idea of the plane wave expansion method is to consider this complicated hologram as a sum of the plane-wave holograms formed by every signal- and reference plane waves. In the recording process, the signal wave leaving from the pixel on the spatial light modulator (SLM) spreads to some extent by diffraction and reaches the lens. After that, the curved wave-front is transformed into a pseudo-plane wave by the Fourier transform lens, and proceeds to the Fourier plane F1, as shown in Fig. 3. Therefore, a signal wave coming from a certain pixel can be regarded as a pseudo plane wave having a wave vector $\mathbf{k}_s$ corresponding to its pixel position. If the pixel position is (ξ, η) , the wavelength is λ , the focal length of the lens is f , and the refractive index of the medium is n , then, the main wave vector $\mathbf{k}_s$ is represented by,

$$\mathbf{k}_s = \left(-\frac{2\pi\xi}{\lambda f}, -\frac{2\pi\eta}{\lambda f}, \frac{2\pi}{\lambda} \sqrt{n^2 - \left(\frac{\xi}{f}\right)^2 - \left(\frac{\eta}{f}\right)^2} \right). \quad (2)$$

Furthermore, the optical field of the signal wave at F1 is expressed by the Fourier transform of a rectangular function related to the optical field just after the SLM pixel. Therefore, except for the time-dependent term, the signal wave at the Fourier domain is written by,

$$E_s(\mathbf{r}) = E_{s0} \text{sinc}\left(\frac{\pi a}{\lambda f} x\right) \text{sinc}\left(\frac{\pi a}{\lambda f} y\right) \exp(i\mathbf{k}_s \cdot \mathbf{r}), \quad (3)$$

where E_{s0} is the constant complex amplitude, a is the pixel size of the SLM, $\mathbf{r}$ is the position vector in Fourier domain, and $\text{sinc}(x)$ is the sinc function defined as $\sin(x)/x$. This signal wave having the pseudo plane wavefront interferes with the reference wave of

Fig. 3. The basic concept of the plane wave expansion model.

$E_{r0}\exp(i\mathbf{k}_r \cdot \mathbf{r})$ and forms a simple plane wave hologram, which we call an elementary grating. The elementary grating has a grating vector represented by $\mathbf{K}_g = \mathbf{k}_s - \mathbf{k}_r$. The light wave diffracted from such an elementary grating can be easily calculated analytically or numerically. Since the actual hologram consists of a lot of the elementary gratings, the diffracted waves from every elementary grating are summed up to calculate the light wave reaching at one detector pixel. In these calculations, the diffracted wave is calculated by using the Born approximation and considering the influence of the aperture inserted in the Fourier plane, as is described in the following subsection.

2.1.2 The Born approximation

In holographic data storage, the diffraction efficiency at each page is usually smaller than 1%, so the first Born approximation can be a very good approximation. Thus, when the elementary grating with the refractive index distribution of $n(\mathbf{r}) = \Delta n(\mathbf{r})\exp(i\mathbf{K}_g \cdot \mathbf{r})$ is illuminated with the probe light wave of $E_{p0}\exp(i\mathbf{K}_p \cdot \mathbf{r})$, the amplitude of the diffracted wave having the wavevector $\mathbf{k}_d$ may be expressed as³³⁾,

$$E_{d0} \propto \int_V E_{p0} \Delta n(\mathbf{r}) \exp(i\delta\mathbf{K}_{goff} \cdot \mathbf{r}) dV, \quad (4)$$

where V is the hologram volume and $\delta\mathbf{K}_{goff}$ is the off-Bragg vector defined by,

$$\delta\mathbf{K}_{goff} = \mathbf{k}_s - \mathbf{k}_r - \mathbf{k}_d + \mathbf{k}_p. \quad (5)$$

Now, for simplicity, it is assumed that the amplitude of the refractive index grating is proportional to the amplitude of the signal wave and also assumed that the signal wave is normally incident on the recording medium. Then, as shown in Fig. 4, a signal wave reaching at the recording material has an amplitude distribution represented by a sinc function with both ends forced to zero. If the reference wave has sufficiently large beam width, it can be considered that the hologram will be created along the wave vector $\mathbf{k}_s$ while

Fig. 4. The role of the aperture inserted in the first Fourier plane F1 and the resulting hologram shape formed in the recording media.

maintaining the amplitude distribution of the signal wave. Considering above, using the wave vector $\mathbf{k}_s = (k_{sx}, k_{sy}, k_{sz})$, the Eq. (4) is rewritten by,

$$E_{d0} \propto \int_V E_{s0} E_{r0}^* E_{p0} \text{sinc} \left[\frac{\pi a}{\lambda f} \left(x - \frac{k_{sx}}{k_{sz}} z \right) \right] \times \text{sinc} \left[\frac{\pi a}{\lambda f} \left(y - \frac{k_{sy}}{k_{sz}} z \right) \right] \exp(i\delta \mathbf{K}_{goff} \cdot \mathbf{r}) dV. \quad (6)$$

The above integral can be separated into x , y , and z integral terms by appropriately transforming the variables. After some algebraic calculations, the Eq. (6) can be reduced to,

$$E_{d0} \propto E_{s0} E_{r0}^* E_{p0} w^2 L_z \text{PxSF}(R_{Nyq}, \delta K_{goffx}) \times \text{PxSF}(R_{Nyq}, \delta K_{goffy}) \text{sinc} \left(\frac{\delta \mathbf{K}_{goff} \cdot \mathbf{k}_s}{2k_{sz}} L_z \right), \quad (7)$$

where w is the Nyquist size defined as $w = \lambda f / a$, L_z is the media thickness, R_{Nyq} is the Nyquist ratio that defined as the aperture size normalized by the Nyquist size w , and $\text{PxSF}(R_{Nyq}, \delta K_{goff})$ is a pixel spread function which expresses how a light wave leaving from the pixel of the SLM spreads out at a detector plane I2, that is defined by,

$$\text{PxSF}(R_{Nyq}, \delta K_{goff}) \equiv \int_{-\frac{R_{Nyq}}{2}}^{\frac{R_{Nyq}}{2}} \text{sinc}(\pi X) \exp(iw\delta K_{goff} X) dX. \quad (8)$$

When the aperture size is kept constant, this function depends only on the off-Bragg component, not the position in the detected image. Therefore, in the simulator, this value is tabulated and stored in the memory at the start of calculation, and then, the table is referred when calculating the diffracted wave reaching at each detector pixel.

2.2. Extraction of the signal information

After calculating the diffracted image, it is necessary to extract the signal information from the image. This subsection describes the extraction process in our simulator.

2.2.1 Extracting intensity modulated signals

In the calculation of the diffracted image, in order to simulate the effect of blurring due to the aperture, the data pixels were further divided and the calculation was performed with finer calculation pixels. Therefore, the signal intensity of each data pixel was calculated by averaging the light intensities of all the calculation pixels contained in that data pixel. Then, the field amplitude was calculated by taking the square root of that signal light intensity. This field amplitude was used to output the statistical values of each symbol, such as a

histogram, mean value, and standard deviation. The average value of each symbol is employed to calculate the threshold value that defines the boundary between the adjacent symbols. In this study, the threshold value was set as the mean value of the two symbol averages. The standard deviation of each symbol can be a measure of the noise level of the corresponding symbol. This is because the symbol with larger standard deviation has a higher possibility to show the field amplitude beyond the threshold value and to be recognized as a wrong symbol. If the detected symbol was recognized as a symbol different from the original one, then, the corresponding pixel was recorded as an error pixel. These analyses were performed on all data pixels within the diffracted image, and the pixel error rate (PxER) was calculated as the ratio of the number of error pixels to the number of all analyzed data pixels.

2.2.2 Extracting phase modulated signals

In experiments, interference measurements such as the 4-step phase shifting method are often used to determine the phase of the diffracted light wave. In our simulator, in order to faithfully simulate the phase detection in the experiments, the phase of the diffracted wave was determined by virtually performing the 4-step phase shifting method. As mentioned earlier, each data pixel was divided into several calculation pixels, which individually possesses the information about the field amplitude and phase of the diffracted wave. Each calculation pixel was virtually interfered with the phase detection reference wave, and the resultant light intensity was averaged over all calculation pixels within the data pixel, which gave the interfered light intensity acquired by the corresponding data pixel. From the four light intensities obtained by changing the phase of the phase detection reference wave, the phase of the diffracted wave was determined using the formula of the 4-step phase shifting method. After the phase was determined, that data was accumulated for each symbol, and the statistical values, such as mean value, standard deviation, and error rate were calculated, as in the case of the intensity-modulated signal.

2.3. Simulation condition

Figure 5 shows the optical system assumed in our simulation. In this study, in order to evaluate intra-page crosstalk, all simulations were performed assuming the single page recording. The number of data pixels in the input image was 200×200 pixels. One data pixel was further divided into 7×7 calculation pixels. Thus, the total number of calculated pixels per page was $1400 \times$

Fig. 5. The optical configuration assumed in our simulation.

Table 1. The calculation parameters used in our simulation.

Wavelength λ	405 nm
NA	0.5
Focal length of lens f	4 mm
Pixel pitch of SLM a	20 μ m
Number of data pixels	200 $\times$ 200 pix
Number of calculation pixels	1400 $\times$ 1400 pix
Nyquist size ($= f\lambda/a$)	81 μ m
Nyquist ratio R_{Nyq} (= Aperture size / Nyquist size)	0.2 - 2.0
Thickness of medium	1.5 mm
Refractive index	1.5
Incident angle of signal wave	-30°
Incident angle of reference wave	45°
Number of multiplexed pages	1

1400 pixels. In this study, simulations were performed at various Nyquist ratio R_{Nyq} in order to see the effect of the aperture inserted in the Fourier plane F1. Other calculation parameters are summarized in Table 1.

3. Results and Discussion

3.1. Intensity modulated signals

The intensity modulated signal is a signal modulation method that utilizes the magnitude of the diffracted light intensity as information. The binary intensity signal that distinguishes the presence (ON) and absence (OFF) of the diffracted light wave is the most basic intensity modulated signal. Furthermore, in addition to ON and OFF, the intensity multivalued signal utilizes the middle intensity values as the symbol. In this simulation, the number of intensity multivalued signals I_b were changed from 2 to 5 for evaluation of the storage density. In these intensity multivalued signals, the light intensity of each symbol was chosen so that the maximum light intensity sets to unity, the minimum to zero, and the rest to the equally spaced light intensity

Fig. 6. The simulated results of the pixel error rate for the intensity modulated signals.

according to the number of multivalued signals. For example, in a multivalued signal with three intensity levels, i.e., $I_b = 3$, each symbol has a light intensity level of 0, 0.5, or 1. In addition, as is often done in experiments, a 4-level random phase was added to each signal. Such a random phase is essential in the actual experiment for avoiding sharp intensity peaks on the recording medium. When the number of intensity multivalued signals is I_b , the code rate C_{1bit} can be expressed as $C_{1bit} = \log_2(I_b)$.

The PxER was evaluated at various aperture sizes for each multivalued signal. The results are shown in Fig. 6. Note that the PxER is plotted as a function of the Nyquist ratio R_{Nyq} that represents an aperture size normalized by the Nyquist size. From the graph, it can be seen that PxER increases with decreasing the Nyquist ratio R_{Nyq} in all the multivalued signals. Such a behavior is mainly originated from two factors. One is the increases of inter-pixel crosstalk, i.e., the leakage of the light waves to adjacent pixels. At smaller R_{Nyq} , the higher spatial frequency components of the image cannot pass through the aperture, and thus the resultant output image becomes blurred and shows an indistinct border, as shown in Figs. 7(a)-(c). Due to the leakage of light waves from the adjacent pixels, the field amplitude in each symbol acquired by the detector has a large variance, as can be seen from the histograms in Figs. 7 (d)-(f). The other factor is the decreased light intensity passing through the aperture. As the aperture size becomes smaller, more high spatial frequency components are blocked, resulting that the total light energy reaching the detector becomes smaller. In order to see the relationship between the aperture size and the signal amplitude received by the detector in more detail, Figure 8 shows the R_{Nyq} dependence of the field amplitude and standard deviation of each symbol for the binary intensity signal ($I_b = 2$). In this figure, each plot

Fig. 7. Calculated diffracted images (a)-(c), and the histograms of the detected signal amplitude (d)-(f) for the Nyquist ratio $R_{Nyq} = 0.8, 1.0$, and 1.2 in the binary intensity signal ($I_b = 2$). The colors in (a)-(c) represents the phase of the diffracted wave.

Fig. 8. The average $\mu_A(m, n)$ and standard deviation $\sigma_A(m, n)$ of the detected field amplitude for each symbol, where the parameters of m and n are the number of multivalued signal and the symbol identification number, respectively. The threshold is determined by the middle point of $\mu_A(2, 2)$ and $\mu_A(2, 1)$. The $S_{\pm}(m, n)$ represents the symbol region reserved by the n -th symbol.

point represents the average field amplitude of the symbol, and the length of the error bar represents the standard deviation. The symbols $\mu_A(m, n)$ and $\sigma_A(m, n)$ means the average value and standard deviation of the detected field amplitude for the n -th symbol in the m -level multivalued signal, respectively, and the $S_{\pm}(m, n)$ represents the symbol region reserved by the n -th symbol. Note that n is assigned in ascending order of the intensity. In the present case, m should be 2 because of a binary signal and the OFF and ON symbols correspond to $n = 1$ and 2 , respectively. As described in Section 2.2.1, determination of the symbol was done by judging the symbol region to which the received field amplitude belongs. For example, in the present case, a threshold is set at the center between the average values as shown in Fig. 8, and then the determination of the symbols was done by judging whether the detected field amplitude was larger or smaller than the threshold. Therefore, if

Fig. 9. The I_b dependence of average $\mu_A(n, m)$ and standard deviation $\sigma_A(n, m)$. The Nyquist ratio R_{Nyq} is 1.3.

the standard deviation is constant, the larger the symbol region, the lower the probability of detection error. The simulation results show that reducing the aperture size leads to not only increasing the standard deviation, but also narrowing the symbol region. In addition, the average value of the OFF symbol $\mu_A(2, 1)$ does not go to zero even when R_{Nyq} is large, resulting in a smaller symbol region. These are the main drawbacks of intensity modulated signals.

Next, consider the effect of the number of multivalued signals I_b on PxER. In Fig. 6, when the R_{Nyq} is fixed, PxER monotonically increases with increasing I_b . To clarify this reason, Figure 9 plots μ_A and σ_A for each I_b . From Fig. 9, it was found that the number of multivalued signals I_b affects only the symbol region $S_{\pm}(m, n)$. While the $S_{\pm}(m, n)$ is decreased inversely proportional to I_b , the magnitude of the standard deviation was almost unchanged by I_b . In addition, the largest and smallest average values, i.e., $\mu_A(m, 1)$ and $\mu_A(m, m)$, remained constant with varying I_b . This is the noise characteristic determining the PxER in intensity multivalued signals.

For achieving the larger storage density, the larger I_b and the smaller R_{Nyq} should be adopted in the system. However, as was seen in the above discussion, in both cases, we have to pay a penalty of increased PxER. Therefore, these two parameters, I_b and R_{Nyq} , must be chosen appropriately so as to maximize the storage density while keeping the PxER within the allowable PxER by the system. Here, we introduced a storage density factor ξ_{1page} as a figure of merit for the storage density, which is defined as,

$$\xi_{1page} = \frac{C_{1bit}}{R_{Nyq}} \cdot \quad (9)$$

The ξ_{1page} was calculated as a function of the allowable PxER and plotted in Fig. 10. From the figure, it can be seen that the theoretical limit of the storage density

Fig. 10. The storage density factor ξ_{1page} for intensity modulated signals.

hardly increases even if I_b is increased. This result indicates that the increase of ξ_{1page} by increasing I_b was almost offset by increasing R_{Nyq} that was required to keep the PxER constant. Therefore, when an intensity multivalued signal is adopted, while the data transfer rate is expected to be improved, the theoretical limit of the storage density is almost unchanged.

3.2. Phase modulated signals

In the phase modulated signal, the phase of the diffracted wave is utilized as the information instead of their intensity. In this simulation, the number of phase multivalued signals P_b were changed from 2 to 16 for evaluation of the storage density. In these phase multivalued signals, the representative phase of each symbol was determined so that the distances to adjacent symbols were equally spaced. When preparing the page data for the phase multivalued signal, the phase at each data pixel was randomly selected from the representative phases of the symbol, while all the field amplitudes were set to unity. In this phase multivalued signals, the code rate C_{1bit} can be expressed as $C_{1bit} = \log_2(P_b)$.

The simulated results for PxERs as a function of R_{Nyq} was shown in Fig. 11. From the results, it can be seen that PxER increases with decreasing R_{Nyq} at all P_b values, as is in the case of the intensity modulated signal. Figure 12 shows the R_{Nyq} dependence of the average value and standard deviation of detected phase for each symbol at $P_b = 4$. As the aperture size decreases, the phase is strongly modulated owing to the interference with the light waves that leaked from the adjacent pixels, and as a result, the standard deviation of detected phase increases more as in the case of the intensity modulated signals. On the other hand, in the case of a phase modulated signal, even if the diffracted wave is modulated by the crosstalk, the symbol region, that is, the interval between each phase symbol does not

Fig. 11. The pixel error rate for the phase modulated signals.

Fig. 12. The Nyquist ratio dependence of the detected average phase and standard deviation.

change by R_{Nyq} . This is a decisive difference in the noise characteristics of the phase modulated signal compared to that of the intensity modulated signal.

Next, consider the influence of the number of phase multivalued signals P_b on PxER. Figure 13 shows a two-dimensional histogram where the frequencies of the detected complex amplitudes are plotted on the complex plane for each P_b . Note that these histograms were plotted on a logarithmic scale for better visibility. As can be seen from the figure, the distribution of the detected complex amplitude of each symbol has a fairly characteristic shape according to the number of multivalued signals P_b . When the P_b is small, the distribution becomes far from circular symmetry. In particular, the detected phase for $P_b = 2$ lies only in the real axis on the complex plane, which is quite different from the distribution of other phase multivalued signals. These features can be explained as follows by using a phaser on the complex plane. First, the complex amplitude for the light wave that is finally detected at a certain pixel is obtained by the superposition of the signal wave and a lot of noise waves leaked from surrounding pixels. Since each data pixel is assigned a specific phase according to the symbol definition, the

Fig. 13. The two-dimensional histograms for detected complex amplitude of the phase modulated signals. The Nyquist ratio R_{Nyq} is 1.0.

Fig. 14. The symbol region S and the standard deviation σ_p as a function of number of multivalued signals P_b . The Nyquist ratio R_{Nyq} is 1.0. Note that the symbol region S and standard deviation σ_p are almost same at each symbol, so we have omitted the identification parameters for each symbol.

phaser for each noise wave should be oriented in the corresponding direction. For example, in the 4-level phase multivalued signal, the phaser points in one of the four directions of the angle, 0 , $\pi/2$, π , and $3\pi/2$. In this case, the resultant complex amplitude after the superposition should be determined by a random walk with these four distinct directions. This is the reason why the shape of the distribution differs depending on P_b . Fig. 14 shows the average value μ_P and standard deviation σ_P of detected phase for each P_b . Even though the light wave with various phases are leaked and interfered at the large P_b , the standard deviation of the detected phase is almost independent on P_b , except for $P_b = 2$. Therefore, the decreasing the symbol region S is the main reason for increasing PxER at large P_b , as is similar to the intensity multivalued signal. It should be noted that the small standard deviation for $P_b = 2$ is due to the fact that the detected phase is limited to 0 or π by a one-dimensional random walk.

Fig. 15. The storage density factor ξ_{1page} for phase modulated signals.

Fig. 16. The P_b dependence of the storage density factor ξ_{1page} at the fixed PxER.

Figure 15 shows the results of the storage density factor ξ_{1page} estimated from the calculated PxERs. Unlike the case of intensity multivalued signals, the storage density factor ξ_{1page} clearly depends on the number of phase multivalued signals P_b . To see more clearly, in Fig. 16, the storage density factor ξ_{1page} 's are plotted as a function of P_b , assuming the allowable PxER to be 10%, 1%, and 0.1%, respectively. From the graph, we can see that ξ_{1page} does not increase monotonically with P_b and there is an optimal P_b to get the target ξ_{1page} . Therefore, although larger P_b results in higher data transfer rate, the larger P_b may be not always preferable from the viewpoint of storage density. Compared with ξ_{1page} for the binary intensity signal shown in Fig. 10, it was found that the storage density can be improved by a factor of 2.4 - 3.0 by adopting the phase multivalued signals.

4. Conclusion

In this paper, the noise characteristic and the theoretical limit of the single-page storage density in the intensity- and phase modulated signals have been investigated using the holographic simulator. Our simulator considers the effect of the aperture inserted in

the Fourier plane and also considers the actual hologram shape formed in the recording medium. Our numerical simulations revealed that, in the case of the intensity modulated signals, the large number of multivalued signals will improve only the data transfer rate, not the theoretical limit of the single-page storage density. On the other hand, the phase multivalued signals can improve both the single-page storage density and data transfer rate, simultaneously. Assuming the allowable PxER of 10%, the 8-level phase multivalued signal can be expected to improve the single-page storage density by a factor of 2.4 compared to the binary intensity signal.

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