Characteristics of Pressure Fluctuation Caused by Self-Induced Flow Oscillation of Under-Expanded Impinging Jet

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The numerical investigation of the self-induced flow oscillation caused by the interaction of a supersonic impinging jet is carried out in this paper. The self-induced flow oscillation occurs at the specific conditions and causes the noise problem. The characteristic of the oscillated flow and the mechanism of the oscillation, therefore, have to be cleared to control the oscillation. This paper aims to clear the characteristic of a pressure fluctuation during the under-expanded supersonic jet impinges on the perpendicular plate. The TVD numerical method is used in the numerical analysis. From the results of numerical analysis, it is concluded that the pressure fluctuation on the plate surface depends on the pressure ratio and the position of a plate.

1. INTRODUCTION

The phenomena of the interaction between a supersonic jet and an obstacle is a very important problem relating to the aeronautical and other industrial engineering. Especially, when the under-expanded supersonic jet impinges on the obstacle, it is well known that the self-induced flow oscillation occurs at the specific conditions, namely the shape or configuration of an obstacle and the pressure ratio in the flow field and so on. This oscillation causes the noise problem so that the many studies have carried out to clear the characteristic of the oscillated flow and the mechanism of an oscillation. But, it seems that the pressure fluctuation on the surface of an obstacle caused by the oscillation and the factor which dominates the oscillation have not been detailed in the past papers. To clear the characteristic of the noise caused by the oscillation, it is needed to measure the pressure fluctuation on the flow field and the surface of an obstacle. This paper aims to clear the characteristics of pressure fluctuation and the influence of some parameters during the under-expanded supersonic jet impinges on the perpendicular plate using the TVD numerical method.

2. PROCEDURE OF NUMERICAL CALCULATION

The typical flow field and the symbols used in this study are shown in Fig.1. The name of the shock wave in this figure is based on the Henderson's report. The supersonic flow exhausted from the nozzle with the diameter $D$ decelerates to the subsonic flow by the Mach disk and the plate shock wave. The decelerated subsonic flow impinges on the perpendicular plate located at the distance from the nozzle exit $x_p$. When the position $x_p/D$ is sufficiently short, the plate shock wave is not formed and the Mach disk oscillates along the flow axis. The flow structure, furthermore, is influenced by the pressure ratio $\phi$ ($=p_a/p_0$, here, $p_0$ is the reservoir pressure, $p_a$ is the back pressure) and the cell structure is formed when the ratio $\phi$ is sufficiently low.

The $x$-$y$ cylindrical coordinates system originated on the center of the nozzle exit is considered as
shown in Fig.2. The basic equations used in this study are the compressible unsteady axisymmetric Euler’s equation so that the inviscid gas is assumed. It can be written in the Eq.(1) by the non-dimensional conservation forms.

\[
\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + W = 0
\]

(1)

Where,

\[
U = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho e \end{bmatrix}, \quad F = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ (e + p) u \end{bmatrix}, \quad G = \begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \\ (e + p) v \end{bmatrix}, \quad W = \begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 \\ (e + p) v \end{bmatrix}
\]

Those parameters are nondimensionalized by the nozzle exit diameter \(D\) and the reservoir pressure \(p_o\), the density in reservoir \(\rho_o\) and the sonic velocity in reservoir \(a_o\).

In the previous reports\(^{21,9,19,10}\), the result of numerical analysis using the Euler equation is good agreement with the experimental result for the jet structure and the fundamental characteristic of self-induced flow oscillation when the under-expanded supersonic jet impinges with the perpendicular plate or cylindrical body. In this numerical analysis, therefore, the influence of viscosity is neglected because the influence of viscosity is not concerned for the fundamental characteristic of pressure fluctuation for impingement with the perpendicular plate. Furthermore, it is cleared that the numerical calculation is enough accuracy to discuss the characteristics of oscillation from a comparison with the experimental result.

Eq.(1) is solved by the TVD method\(^{21}\) with the operator splitting technique\(^{19}\). The square grids such as \(\Delta x = \Delta y\) is used and the time step \(\Delta t\) is decided by the CFL condition. The initial conditions are as follows; the exit Mach number \(M_e = 1\), the ratio of the specific heats \(\kappa = 1.4\), the position of plate \(x_p/D = 2.6\). The pressure ratio of reservoir pressure \(p_o\) and the vacuum chamber pressure \(p_b\) is varied in the range of \(4 \leq \phi (= p_o/p_b) \leq 16\).

3. NUMERICAL RESULTS AND DISCUSSIONS

Typical isopycnics are shown in Fig.3 and the numerical condition is indicated in the figure. These figures are the isopycnics at several times in the one periodic time \(T\) of the oscillation. In Fig.(a), the plate shock most approaches to the nozzle exit and begin to move toward the perpendicular plate. The size of the plate shock becomes large and in Fig.(d), the plate shock disappears. It is noticed that the Mach disk is stationary and the plate shock wave oscillates along the jet axis.

The pressure distribution on the jet axis is shown in Fig.4. The numerical condition is indicated in the figure. The pressure on the jet axis decreases with the increasing of distance \(x/D\) as like the isentropic
change and recovers by the Mach disk. In Fig.(a), the Mach disk oscillates along the jet axis and the pressure at behind the Mach disk $p_m$ and the center of a plate $p_c$ fluctuates with time. The pressure $p_m$ decreases with the increasing of distance $x/D$. In Fig.(b), the Mach disk is a stationary and the plate shock oscillates along the jet axis. The value of $p_m$ decreases with the increasing of a distance $x/D$ because of the self-induced flow oscillation. The pressure at behind plate shock $p_p$ fluctuates with time. From these results, it is noticed that the pressure at behind plate shock $p_p$ or at the center of a plate $p_c$ instantaneously exceeds the pressure at behind the Mach disk $p_m$ so that the plate shock moves toward the nozzle exit and turns back toward the plate when the pressure $p_p$ or $p_c$ decreases than the pressure $p_m$. The maximum pressure at the plate center, furthermore, does not exceed the nozzle exit pressure and the pressure distribution from the nozzle exit to the Mach disk does not change as if the self-induced flow oscillation occurs.

The relation between the nondimensional pressure at behind the Mach disk $p_m/p_0$ and the pressure ratio $\phi$ is shown in Fig.5. The plotted symbol indicates the numerical result for $x_j/D=4$ and the solid line shows the result for the free jet obtained by the numerical calculation. The value of $p_m/p_0$ shows the averaged value during the oscillation occurs. It is remarkable that the value of $p_m/p_0$ decreases with the increasing of a pressure ratio $\phi$ and increases than the case of the free jet.

The relation between the nondimensional pressure at behind the Mach disk $p_m/p_0$ and the nondimensional position of a plate $x_j/D$ is shown in Fig.6. The solid line shows the result for the free jet obtained by the numerical calculation. The value of $p_m/p_0$ decreases with the increasing of the position $x_j/D$ and approaches to the result of the free jet. It is noticed that the pressure at behind the Mach disk during the oscillation occurs exceeds the value of free jet and approximates to it for large pressure ratio $\phi$.

The figure 7 shows the relation between the nondimensional position of the Mach disk during the oscillation occurs $x_j/D$ and the nondimensional position of a plate $x_j/D$. The solid line shows the result for the free jet obtained by the Addy’s equation\((4)\), which shows the nondimensional position on the free
jet. The value of $x_p/D$ indicates the averaged value during the oscillation occurs and decreases for the case of a free jet, namely the Mach disk approaches to the nozzle exit. In the case without the oscillation, the value of $x_p/D$ approximates to the constant value for large $x_p/D$ and this tendency is observed in the case with the oscillation.

The typical pressure variation with time at the center of a plate is shown in Fig.8. The amplitude and periodic time of a pressure variation are not uniform and the amplitude is smaller than the reservoir pressure $p_0$ in this case.

The relation between the peak frequency $f_p$ and the pressure ratio $\phi$ is shown in Fig.9. The peak frequency $f_p$ means the frequency in which the local spectra peak of the pressure oscillation is obtained and is evaluated using the FFT analysis. For reference, the experimental results are indicated for the case of the interaction with the cylindrical body in this figure and were obtained in our past investigation. In the experiment, the diameter of cylindrical body $d_c$ is 4 mm and corresponds to the nozzle exit diameter. It is noticed that in the case of $x_p/D=3$ and 4, an obvious peak of the spectrum is observed, but the two peaks of the spectrum are contained in the case of $x_p/D=7$ in the experiment. In this figure, symbol $f_{p1}$ and $f_{p2}$ indicate the low and high peak frequency, respectively. From the comparison with the experimental result, it is remarkable that the peak frequency $f_p$ obtained by the interaction with the perpendicular plate

![Fig.5](image1.png)  ![Fig.6](image2.png)  ![Fig.7](image3.png)  ![Fig.8](image4.png)

**Fig.5** Relation between nondimensional pressure at behind Mach disk $p_a/p_0$ and pressure ratio $\phi$

**Fig.6** Relation between nondimensional pressure at behind Mach disk $p_a/p_0$ and nondimensional position of plate $x_p/D$

**Fig.7** Relation between nondimensional position of Mach disk $x_a/D$ and nondimensional position of plate $x_p/D$

**Fig.8** Typical pressure fluctuation on center of plate ($x_p/D=5$, $\phi=10$)
good agrees with that of the interaction with the cylindrical body. In the previous study\(^{(10)}\), it is cleared that the mechanism of oscillation depends on the relation between the pressure at behind the Mach disk \(p_m\) and the pressure at behind plate shock \(p_c\) or center of a plate \(p_c\). It seems, therefore, that the peak frequency does not change because the pressure distribution on the jet axis little change under the same condition, although the reflector changes. The value of \(f_{p1}\), furthermore, is agreement with other paper\(^4\) and slightly increases with the increasing of the pressure ratio \(\phi\) and decreasing of position \(x_p/D\) so that the periodic time of oscillation changes with the pressure ratio \(\phi\) and position \(x_p/D\). On the other hand, the secondary peak \(f_{p2}\) is observed at relatively larger distance \(x_p/D\) and it seems that this peak is occurred because of the difference of flow structure.

Figure 10 shows the relation between the time averaged pressure \(p_{av}/p_0\) at the center of a plate and the nondimensional position of a plate \(x_p/D\). The plotted symbols indicate the time averaged pressure \(p_{av}/p_0\) and the solid line shows the impact pressure of the free jet \(p_c/p_0\)\(^{16}\) which is evaluated by the following equation.

\[
\frac{p_8}{p_0} = \left( \frac{\kappa + 1}{\kappa - 1} \right)^{\frac{\kappa}{2}} \left( \frac{\kappa + 1}{2 \kappa} \right)^{\frac{1}{\kappa - 1}} A^{\frac{2}{\kappa + 1}} \left( \frac{x}{D} - 0.13 \right)^{2} / \phi \tag{2}
\]

Where, \(A=3.65\).

Equation (2) is obtained by using the experimental result and the experiential result. The time averaged pressure \(p_{av}/p_0\) slightly decreases with the increasing of position \(x_p/D\) and the pressure ratio \(\phi\) and exceeds the value of \(p_c/p_0\). It is remarkable that the time averaged pressure \(p_{av}/p_0\) depends on the pressure ratio \(\phi\) and the nondimensional position of a plate \(x_p/D\). The value of \(p_{av}/p_0\), furthermore, corresponds to the result of interaction with a cylindrical body\(^{(15)}\). As mentioned in Fig.4, the mechanism of oscillation depends on the relation between the pressure at behind the Mach disk \(p_m\) and the pressure at behind plate shock \(p_c\) or center of a plate \(p_c\). From these results, it is concluded that the mechanism of oscillation dose not change because the peak frequency and the time averaged pressure do not change, as if the reflector changes from plate to cylindrical body.

4. CONCLUSIONS

The self-induced oscillation during the under-expanded supersonic jet impinges on the perpendicular plate is investigated by the numerical calculation in this study. The characteristics of the pressure fluctuation in the flow field are discussed. The conclusions are summarized as follows.

(1) The wave structure is a similar to the case of the free jet and the Mach disk or plate shock oscillates
along the jet axis. The periodic time of a pressure fluctuation on the plate center is not constant and changes with the pressure ratio \( \phi \) and position \( x_p/D \). The amplitude of a pressure fluctuation changes with time and the time-averaged pressure at this point is affected by the pressure ratio \( \phi \) and position \( x_p/D \).

(2) The pressure behind at behind the Mach disk \( p_m/p_0 \) decreases with the increasing of the position \( x_p/D \) and approaches to the result of the free jet. It is noticed that the pressure at behind the Mach disk during the oscillation occurs exceeds the value of free jet and approximates to it for large pressure ratio \( \phi \).

(3) The peak frequency of the pressure fluctuation \( f_p \) obtained by the interaction with the perpendicular plate good agrees with that of the interaction with the cylindrical body. The obvious peak frequency \( f_p \) slightly increases with the increasing of the pressure ratio \( \phi \).

(4) The time averaged pressure on the plate center \( p_m/p_0 \) slightly decreases with the increasing of position \( x_p/D \) and the pressure ratio \( \phi \).

REFERENCES