Development of Design Support Tool Using 3D Topology Optimization with Multilevel Voxel Analysis

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In this paper, a useful design tool that deals with 3-D topology optimization is being developed. Traditionally topology optimization is a tool for CAE engineers, but here we aim to develop a tool that gives design engineers inspiration of structure in conceptual design stage in structural mechanics sense. To help designers, the interactivity and flexibility against the user operation is necessary. To achieve the goal, topology optimization using the adaptive multi level voxel analysis is used. With 3 dimensional examples it was demonstrated that several possible designs can be obtained interactively.

1. INTRODUCTION

The idea to design the structure optimally is natural demand. With the recent progress, the structural optimization has been widely used in the engineering practice. Especially in last 15 years topology optimization techniques based on the material distribution approach has attracted large attention of researchers as well as practical engineers, and have made rapid progress. Several general purpose topology optimization codes such as OPTISHAPE or OptiStruct have been put on the market, and many engineers use them to reduce weight, or reduce cost. The methodology seems to be matured. However, the topology optimizations have not shown its true capability, because they are mainly used by the analysts, not by designers. In general, the time product design comes to analyst is almost final design stage, and in this stage it is hard to make big design modification. Obviously, the topology optimization is suitable tool to be used in the early design stage where designers make conceptual designs such as overall shapes or layout of members. In this paper, authors consider how to change the topology optimization from the tools for analysts to the tools for designers.

Design is usually carried out with the support of logical thinking and sensibility (what is called “Kansei”), and good balance of them is important. Topology optimization can help logical thinking by giving mathematically optimal (or semi-optimal) structures, as well as stimulating the imagination of designer by producing unexpected designs. The tool for designers should not be something just to give single optimal design, but gives several possible candidates to stimulate imagination. For the tool to
be used like the word processor for writer, key issue is interactivity. The results should appear in no more than several minutes. To achieve this goal, we have developed adaptive topology optimization method in which we begin from very rough mesh, and subdivide the area where we need more resolution, as shown in Fig. 1. This concept is also used in the multilevel topology optimization by Kim et al.\textsuperscript{11} using wavelet functions. In this paper we employed multilevel voxel analysis method for analysis that is described in chapter 2. In chapter 3 the topology optimization strategies are discussed. Finally, in chapter 4, 3 dimensional examples to demonstrate the capability to stimulate the imagination of designer are shown.

![Adaptive Topology Optimization](image)

Fig. 1 Adaptive Topology Optimization

2. MULTILEVEL VOXEL ANALYSIS

Currently the finite element method (FEM) has been widely used in the design process of industrial products as CAE tools. As the design process moves to 3D, the model generation of FEM has been bottle neck of the process, which often takes several days to several weeks. Under these circumstances, the mesh free analysis methods that do not require the mesh in the analysis have been emerging in the academic world.

The authors have been proposing the mesh free method called Finite Cover Method (FCM). The FCM aims at analysis of complicated 3D solid geometry and utilizes voxel concept which was proposed by Kikuchi et al.\textsuperscript{39} By splitting geometric representation and approximate function domain in voxel analysis, single scale voxel is used in the PU version of FCM (FCM-PU)\textsuperscript{39}, which employs voxel cover for the cover of Manifold Method\textsuperscript{49}. By changing the polynomial order of each cover function it became possible to control the accuracy locally (p-type adaptive). Also we extended the method to utilize multi-level sized voxel to control the accuracy by mesh size (h-type adaptive) using Cover Least Square Approximation (CLSA, FCM-CLSA)\textsuperscript{31}.

Fig. 2 shows the cover distribution for each case. In this research, we employed the FCM-CLSA for the analysis used in adaptive topology optimization.
In FCM-PU, the displacement functions are approximated as follows.

$$ u(x) = \sum_{i=1}^{n} f_i(x)w_i(x) $$  \hfill (1)

where $f_i(x)$ is cover function and $w_i(x)$ is weight function. The weight function needs to satisfy following conditions, where $U_i$ is common domain of mathematical cover and physical domain.

$$ \begin{cases} 
  w_i(x) \geq 0 & x \in U_i, \\
  w_i(x) = 0 & x \notin U_i
\end{cases} $$ \hfill (2)

$$ \sum_{i} w_i(x) = 1 $$ \hfill (3)

Equation (3) is called Partition of Unity (PU) condition that guarantees the reproducibility of the function in cover functions. Jin et. al. also discuss the weight function that guarantee the linear independency of approximation functions for arbitrary degree of polynomial order of cover functions, which allow accuracy control by changing the order of polynomial. The displacement function (1) is substituted into Galerkin formulation to derive the linear set of equations.

When the size of voxel is changed as in Fig. 2 right, it is not easy to find the weight function that satisfies PU condition. Jin et. al. proposed cover least square Approximation (CLSA) that is similar to the moving least square approximation (MLSA) but instead of evaluating the function by nodes, CLSA evaluate the function by cover. In CLSA, the approximation function $u(x)$ is defined as equation (4), which is same function as the one used in MLSA.

$$ u(x) \equiv u(x,\bar{x}) = \sum_{j=1}^{n} a_j(\bar{x})\phi_j(x - \bar{x}) = \phi a $$ \hfill (4)

Where $\bar{x}$ is certain point in the domain.

In the CLSA, to derive approximation function the functional $J$ in equation (5) is minimized. The
evaluation of the functional to be minimized is carried out on each cover, while in MLSA the evaluation is evaluated at each node.

\[ J = \sum_{i} w_i(\bar{x}) \int_{\Omega_i} \phi_i(x) \phi_i(x, \bar{x}) dx \]  

(5)

where \( \phi_i(x, \bar{x}) = \phi_i - \phi_0 \) and \( \Omega_i \) is each cover area, \( w_i(\bar{x}) \) is weight function, \( \phi_i(x) \) is localization factor function, \( \phi \) is basis functions, \( d_i \) is coefficients vector for cover \( i \).

It has been proved that this CLSA approximate function has reproducibility of the approximation function \( \phi \) as MLSA has, and the linear independency of those functions are mathematically guaranteed, while most mesh free method does not have guaranteed linear independency, which are big advantage of FCM-CLSA.

Using this approximation function for displacement, the structural analysis can be carried out using Galerkin method\(^9\).

3. ADAPTIVE TOPOLOGY OPTIMIZATION TECHNIQUE

There are several approaches for topology optimization. Most promising and widely used approach is to consider the problem as the optimal distribution of the material. The material distribution approach can be categorized into 2 types, namely density method\(^9\) and homogenization method\(^10\). Density method uses the densities of each element as design variables. The relation of material property and density is artificially assumed. For example, the when the element stiffness matrix of full material is \([K_0]\), the stiffness matrix of the element with density \(\rho\) can be assumed as follows, where \(n\) is parameter to be assumed.

\[ [K] = \rho^n[K_0] \]  

(6)

On the other hand, the homogenization approach considers microstructure and takes the sizes of the microstructure as design variables. For example, 2 sizes of rectangular hole are used in FEA. The equivalent material property of microstructure can be computed using homogenization method. In general, topology optimization based on the homogenization method gives better results than that of density method. The reason why homogenization approach gives better answer comes from the capability to represent anisotropy in material property.

However, in 3D the analysis soon becomes quite large scale. When voxel mesh is used, density approach has big advantage over homogenization approach in large scale problem, since in density approach the element stiffness matrix can be computed as the multiplying scalar value to element stiffness matrix of full material as equation (6), because in the voxel analysis shape of each element is same. When the iterative method is used to solve linear system of equations, there is no need to store global stiffness matrix in memory, but just storing element stiffness matrices of several element patterns is enough, and they can be used in the process of iteration. This property reduces the
necessary memory in the analysis dramatically, and often utilized in the voxel analysis. Because of this property, we employed density approach, and for iterative solver conjugate gradient method with diagonal preconditioner is used.

Optimization problem is stated in the same manner as stated in 10). Objective function is defined as minimizing mean compliance of the structure under given loading condition, while maximum volume of the material is set as constraints. Since minimizing mean compliance can be stated as maximizing potential energy as well, we state the problem follows.

\[
\text{maximize } \Pi(a) = \frac{1}{2} d^T K(a) d - f^T d
\]

subject to \[\sum_{i=1}^{N} \int_{V_i} V_i a_i d\Omega \leq m, \]

\[0 < a_i \leq 1 \quad (i = 1, 2, \cdots, N)\]  

Where \( \Pi \) is mean compliance, \( a \) is density of each element, \( K \) is stiffness matrix, \( d \) is displacement vector, \( f \) is load vector, \( V_i \) is volume of each element, \( m \) is upper limit of the volume constraints. Using Lagrange multiplier, the necessary condition for the optimality of the problem can be stated as the stationary condition of following Lagrangian.

\[
L = \frac{1}{2} d^T K(a) d - f^T d - \lambda \left( \sum_{i=1}^{N} \int_{V_i} V_i a_i d\Omega - m \right)
\]

By taking partial differentiation with respect to \( a_i \), the necessary condition becomes as follows.

\[
\frac{\partial L}{\partial a_i} = \frac{1}{2} d^T \frac{\partial K}{\partial a_i} d - \lambda V_i = 0 \quad (i = 1, 2, \cdots, N)
\]

From this condition, we update the density based on the optimality criteria method as follows.

\[
a_i^{\text{new}} = \max \left[ 0, \min \left[ 1, \left( \frac{1}{2 \lambda V_i / \partial a_i} \left( \frac{\partial f}{\partial a_i} \right)^T d \right)^p \right] \right]
\]

where \( p \) is the parameter to be defined a priori, and \( \lambda \) is set so that volume constraints are satisfied exactly using bi-section method. The update of the densities and structural analysis are carried out alternatively until convergence.

To obtain clear shape and topology dividing the domain in small meshes is necessary. However, dividing the 3D domain into small meshes require quite a lot of element and hence a lot of computational time. To overcome this difficulty we employed adaptive topology optimization, in which the voxels around where finer resolution is necessary are subdivided into smaller voxels. For refinement strategy, 2 indexes are used. First, the voxels around the boundary of material area and no material area need to be refined to represent the boundary clearer. Also the voxels with intermediate value (gray) need to be refined to obtain clear shape. The refinement is carried out after iteration of optimization has reached certain level. For the timing of refinement, following contrast index \((CI)^{11}\) is used.
\[ CI = 1 - \frac{\sum \min \left( \frac{\rho_i}{\bar{\rho}}, \frac{1 - \rho_i}{1 - \bar{\rho}} \right) V_i}{V} \]  \hspace{1cm} (11)

where \( \bar{\rho} \) is average density. This CI becomes 1 when all area is black or white, and becomes 0 when all area is average density. When this CI reached certain value, the refinement of the voxels is carried out.

After refinement, there are 2 possibilities of initial condition of material distribution. One is to begin from uniform density distribution again. The other is to begin from the density distribution obtained by the optimization before refinement. Since topology optimization is highly initial condition dependent, they can give different final topology (and they do, as shown in the example in chapter 4).

When you consider this as mathematical optimization problem this is kind of drawback since one or both of the solution is local optimum. When you consider this situation in design stage, being able to get various design possibilities is rather advantage than drawback. Designers can obtain various imagination by obtaining various possibility of better (may not be best) solutions. They may not be best in the sense of minimizing mean compliance, but for designers mean compliance is only part of the aspect to consider, and they may judge to employ either of them. So we employ 2 strategies for the initial condition after refinement

- **#1**: Density is set back to uniform distribution after refinement
- **#2**: Density obtained with previous subdivision is used after refinement

Using this adaptive topology optimization technique, the topology optimization of 3D solid domain can be carried out in less than 15 minutes, beginning from very rough voxel subdivision, and subdivide the voxels where finer resolution is necessary. Using this, designer can perform try and error by assigning several possible loading conditions or support conditions, or applying other design conditions as described in chapter 5.

4. NUMERICAL EXAMPLES

For numerical example a shape of hooked support to hang on the wall is considered. The design domain is set to L shape as shown in Fig. 3 is used. Left hand side is fixed to the wall and point load is applied at the top of the domain at the angle 30 degree with respect to the x axis. Initial voxel subdivision is \( 8 \times 4 \times 8 \) and voxel refinement is carried out in 4 stages, until finest voxel size is the size that divide the domain to \( 64 \times 32 \times 64 \). Volume constraint is set to 40 % of the volume of overall domain.
First, 2 results by the strategy of initial condition # 1 (start from initial condition) and # 2 (start from previous result) are compared. Fig. 4 shows the result with strategy #1, while Fig. 5 shows the result with strategy #2. With strategy #1 the topology consisting of bar like structure with void area inside is obtained, while with strategy #2 topology with plate like structure with no void area are obtained. When we compare the mean compliance the results of Fig. 4 is better than that of Fig. 5. However, for designer both of them are good designs and designer may choose either of them considering other factors.
The computational time is about 2 hours using strategy #1 and 30 minutes using strategy #2 using PC with Pentium 4 2GHz, which is longer than expected. However, it is almost guaranteed that within several years PC will becomes several times faster, and even with PC of these days using PC clusters speed up of several times is quite easy.

The results obtained here shows material concentration at the corner part of L shape. Let us assume that for some reason designer do not want to (or cannot) put material around the corner. In that case, the designer can set no design area around the corner as shown in Fig. 6, and obtain different results as shown in Fig. 7 using strategy #1. The hook is separated into 2 plate like structure.
Finally, if designer want to change the support condition of the wall from overall fix to roller condition as shown in Fig. 8, the optimal results as shown in Fig. 9 is used 1st strategy #1. These results demonstrate the capability of this tool to perform try and error by defining boundary conditions, volume constraints, design domain, and strategies.

5. CONCLUSION

To use topology optimization in conceptual design stage, adaptive topology optimization was developed. For the analysis of multi level voxel subdivision, FCM–CLSA is used. It was demonstrated that topology optimization can stimulate the imagination of designer by giving different good shape and topology interactively, setting volume constraint, boundary conditions, refinement strategy, and no design area.
REFERENCES


