A Numerical Study on the Effect of Thermal Cycling on Monotonic Response of Cast Aluminium Alloy-SiC Particulate Composites

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The microscopic deformation of a particulate reinforced metal matrix composite was modeled by a unit cell model with Mises and Gurson yield conditions based on metal plasticity. The numerical predictions were compared and calibrated with the results of monotonic experiments of a cast aluminium alloy discontinuously reinforced with SiC particulates with and without thermal cycling. The responses of monotonic work hardening and the microscopic fracture mechanisms were investigated numerically. The micromechanical considerations of monotonic deformation in the particulate reinforced metal matrix composite were examined. (1) Whether there is a plastic deformation band penetrating a matrix or not, is the controlling factor of the magnitude of a work hardening rate for particulate reinforced composites; with or without thermal cycling. (2) For TT treated composites, the maximum stress in the particle was larger than that of the interfacial average stress. For the composite with thermal cycling, the maximum particle stress was less than that of the interfacial average stress.

1. INTRODUCTION

In order to characterize and understand mechanical properties and performance of metal matrix composites (MMCs), and to assess the importance of variables such as reinforcement morphology and volume fraction, many authors have made use of the analytical and computational methods in the past years. One method based on achieving representations of the material microstructure is the computational method. By linking the microstructural behavior with the macrostructural behavior, using assumptions on the deformation and stress fields within the materials, computational micromechanics can be used to relate the microstructural performance in terms of deformation mechanisms, strengthening mechanisms and failure mechanisms to the macroscopic mechanical performances of the material\(^1\)\(^-\)\(^5\). Many descriptions of composites are based on periodic unit cell approaches or embedded cell methods, due to the fact that a complete analysis of a material system is currently beyond computational capabilities, regarding those in which the matrix and reinforcement are represented separately, with realistic reinforcement geometries and distributions. Excellent reviews of investigations on these aspects of MMCs can be found in references\(^6\)\(^-\)\(^8\).

In infinite periodic microstructures, stress and strain fields in the elastic state can be studied analytically by using series expansions that make use of the periodicity of the phase arrangement by Sangani and Lu\(^9\). In the periodic unit cell approach; however, the microstructural geometry is divided into periodically repeating unit cells to which the investigation may be limited without greatly reducing its generality. For particulate reinforced MMCs, a large body of work exists in the literature showing how the periodic unit cell methodology has been very successfully used to study the monotonic tensile loading behavior of the materials, based on 2D models with idealized and regular microstructural geometries\(^9\)\(^-\)\(^16\). Models of this type have also been used to study cyclic loading\(^17\)\(^-\)\(^18\). 3D unit cell models have been used to study particle arrangement and loading state effects\(^19\). Recently, unit cells describing a more complex arrangement of particles have been reported in literature. In analysis using 3D cells, cubic shaped unit cells containing up to 64 spherical inclusions were used to describe the elastic behavior of particle reinforced polymers, and unit cells containing approximately 10 particles were used to study particle distribution effects in MMCs\(^20\). 2D unit cell models with realistic microstructural arrangements within the cell, determined from actual composite sections, have been used. 3D studies of realistic particle distributions have also been published on a particle reinforced ceramic matrix composite using periodic unit cell models\(^21\). As regards
to embedded cell methods, a powerful method for predicting tensile flow stress of two phase materials was presented\(^{20}\).

The potential application of MMCs includes a kind of component machinery which suffers cyclic thermal and mechanical loads. Although a large number of finite element calculations have been carried out on SiC particulate reinforced composites, few studies are reported on the microscopic evolution of stress or strain, together with its relationship with the macroscopic deformation properties and the possible fracture mechanism of MMCs with and without thermal cycling. Besides this, this paper is concerned with the study on monotonic response of a particulate reinforced metal matrix composite material. Recently the effect of thermal cycling on the monotonic and cyclic deformation behaviours of MMCs was clarified experimentally\(^{21}\).

The objective of the present work was to apply micromechanical modeling techniques to investigate the microscopic deformations in MMCs and to predict the overall monotonic deformation behavior of the MMCs based on an axisymmetric model. Of specific interest was a casted MMC of an AC4CH alloy reinforced with 10% SiC particles in the T7 condition and thermal cycling, which was subjected to monotonic mechanical loading.

2. FINITE ELEMENT FORMULATION AND Constitutive RELATION

Based on finite element analysis, where body forces are neglected; the requirement of equilibrium is specified in terms of the principle of virtual work\(^{24}\),

\[
\int_V \tau^{ij} \delta E_{ij} dV = \int_S T^i \delta u_i dS.
\]  

(1)

Here, \(\tau^{ij}\) are the contravariant components of the Kirchhoff stress (\(\tau = J \sigma\), where \(\sigma\) is the Cauchy stress and \(J\) is the ratio of current to reference volume of a material element), \(u_i\) are the displacement components, \(i, j = 1, 2, 3\), the summation convention is assumed. The \(V\) and \(S\) are the volume and surface areas; respectively. The nominal traction components, \(T^i\), and the Lagrange strain components, \(E_{ij}\), are given by

\[
T^i = (\tau^{ij} + \tau^{kj} u^i_k) u_j,
\]

(2)

\[
E_{ij} = \frac{1}{2} (u_{ij} + u_{ji} + u^k_i u^k_j),
\]

(3)

where \(u_j\) are the normal surface direction in the reference configuration, and \((\_).i\) denotes the covariant differentiation in the reference frame.

In order to consider the effects of micromovoid nucleation and growth, the yield condition of Gurson is introduced as follows,

\[
\Phi = \frac{\sigma^2}{\sigma_y^2} + 2q_1 f \cosh(\frac{3q_2 \sigma_h}{2\sigma_y}) - 1 - q_1^2 f^2 = 0,
\]

(4)

where \(f\) is the void volume fraction, \(\sigma_y\) is the yield strength of the matrix material\(^{25}\).

\[
\hat{\sigma}^2 = \frac{3}{2} \tau' : \tau', \quad \sigma_h = \frac{1}{3} \tau : I, \quad \tau' = \tau - \sigma_y I,
\]

(5)

A:B denotes \(A_{ij} B_{ji}\), I is the identity tensor. The parameters \(q_1\) and \(q_2\) were introduced by Tvergaard\(^{26}\). When \(f = 0\), eq.(4) reduces to the Mises yield condition. In general, the evolution equation for the rate of void volume fraction, \(\dot{f}\), consists of two terms; growth of existing voids and nucleation of new voids,

\[
\dot{f} = \dot{f}_{\text{growth}} + \dot{f}_{\text{nucleation}}.
\]

(6)

The void growth, \(\dot{f}_{\text{growth}}\), is proportional to the rate of plastic volume dilatation,

\[
\dot{f}_{\text{growth}} = (1 - f) D^p : I.
\]

(7)

In the present calculations, attention is confined to strain-controlled void nucleation so that the contribution to eq.(4) from the nucleation of new voids is given by
\[ f_{\text{nucleation}} = D \beta^p. \]  

\[ \beta^p \] is the equivalent plastic strain. In this study, the strain induced nucleation mechanism proposed by Chu and Needleman\(^27\) was used:

\[ D = \frac{f_N}{s_N \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\beta^p - \varepsilon_N}{s_N} \right)^2 \right], \]

where \( \varepsilon_N \) and \( s_N \) are mean value and standard deviation of the nucleation-strain normal distribution, respectively. The plastic part of the rate of deformation, \( \dot{D}^p \), is taken in a direction normal to the flow potential

\[ \dot{D}^p = \Lambda \frac{\partial \Phi}{\partial \tau}. \]

By setting the plastic work rate equal to the matrix dissipation

\[ \tau : \dot{D}^p = (1 - f)\dot{\varepsilon}, \]

the plastic flow proportionality factor, \( \Lambda \), is determined to be

\[ \Lambda = \frac{(1 - f)\dot{\varepsilon}}{\tau : \frac{\partial \Phi}{\partial \tau}}. \]

During the loading process, the reinforcing particles are taken as elastic bodies and the matrix is taken to be elastic-plastic that obeys the Gurson or Mises yield criterion. All the materials were heat treated by the T7 treatment. In addition to the T7 heat treatment, some of the materials are subjected to a thermal cycle (the maximum temperature was 400°C, cooling in air, 1000 times). The term 'TC' in this paper stands for 'Thermal Cycling'. The experimentally measured flow stresses of the cast aluminum alloy(JIS-AC4CH\(^28\)) with and without thermal cycling, as a function of the plastic strain, are shown in Fig. 1. The material parameters of the cast aluminum alloy with and without thermal cycling and SiC were decided to reproduce the experimental data shown in Table 1.

![Image of graph comparing flow stresses](image)

**Fig. 1** The relation of the flow stress and the plastic strain for T7 and T7+TC aluminum alloy (experimental results).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>E (GPa)</th>
<th>( \nu )</th>
<th>( \sigma_{\mu 0} ) (MPa)</th>
<th>( q_1 )</th>
<th>( q_2 )</th>
<th>( \varepsilon_N )</th>
<th>( s_N )</th>
<th>( f_N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC4CH(T7)</td>
<td>70</td>
<td>0.33</td>
<td>130.0</td>
<td>1.25</td>
<td>1.0</td>
<td>0.05</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>AC4CH(T7+TC)</td>
<td>70</td>
<td>0.33</td>
<td>46.8</td>
<td>1.25</td>
<td>1.0</td>
<td>0.05</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>SiC</td>
<td>450</td>
<td>0.17</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

\( \sigma_{\mu 0} \) is the stress value in the case of \( \beta^p = 0.0 \).
3. UNIT CELL MODELS AND BOUNDARY CONDITION

In order to investigate the effects of thermal cycling on the response of monotonic work hardening and the trend of fracture mechanisms, a classical unit cell model was used shown in Fig. 2. In general, the stress-strain analysis of both the MMC and the individual cells within it is a 3D plasticity problem. However, in this study, as a first approach, the inclusions were approximated to a spherical or cylindrical body surrounded by a cylinder of matrix material. The cylinder represented one cell of the MMC. This approach was first proposed by Tvergaard and it has been used extensively to analyze the mechanical response of particle reinforced composites\cite{10-12,17-18,29-33}. The cylindrical cell is introduced as an approximation to the hexagonal cell for computational simplicity, where the stress distribution is axisymmetric if the particle has axisymmetric geometry. Because of the symmetry of the cell, only a quarter of one unit cell is treated in the analysis.

A cylindrical coordinate system \((r, \phi, z)\) is used as shown in Fig. 2. Attention is confined to axisymmetric deformation so that all field quantities are independent of \(\phi\) within each cell. Symmetry is assumed around the center line. Furthermore, the circular cylindrical cell surrounding each particle should remain a circular cylinder throughout the deformation history; which means the following constraint on the \(r = R_0\) face of the cell. The forced displacement allowing slip tangentially is applied to the top and bottom boundary. The expressions about the boundary conditions for the axisymmetric region analyzed numerically are

\[
\begin{align*}
  u_z &= 0 \quad \text{on} \quad z = 0 \quad \text{and} \quad u_z = U \quad \text{on} \quad z = H, \\
  u_r &= 0 \quad \text{on} \quad r = 0 \quad \text{and} \quad u_r = U^* \quad \text{on} \quad r = R_0, \\
  T_r &= 0 \quad \text{on} \quad z = 0 \quad \text{and} \quad z = H, \quad r \neq R_0, \\
  T_z &= 0 \quad \text{on} \quad r = 0 \quad \text{and} \quad r = R_0, \quad z \neq H.
\end{align*}
\]

Here, \(U\) is a prescribed displacement, \(u_r\) and \(u_z\) stand for the displacement components and \(T_r\) and \(T_z\) are the tractions in the \(r\) and \(z\) direction; respectively. \(U^*\) is determined from the condition that the average lateral traction vanishes, i.e.,

\[
\int_0^H T_r dz = 0 \quad \text{on} \quad r = R_0.
\]

The true stress-true strain response of the unit cells was calculated for use in the macroscopic considerations. The global strain, \(\varepsilon\), was determined from the displacement at the top face of the cell, \(U\). The global stress, \(\sigma\), is taken to be the average stress normal to the displaced face at \(z = H\):

\[
\sigma = \frac{1}{A} \int_A \sigma_{zz} dA.
\]

The convergence of the finite element discretization was checked by a calculation on two mesh configurations with 1743 8-node and 2286 6-node axisymmetric elements (Fig. 3) and 1139 8-node and 1444 6-node axisymmetric elements. The overall stress-strain curves obtained from the two meshes differed by less than 0.064% up to a failure initiation load. The stress distribution along the interface between the matrix and the particle and the stress distribution in the particle are investigated. The normal and shear stress of the interface between the particle and the matrix, and the normal stress in the particle are defined as illustrated in Fig. 4. The interfacial average stress is defined as follows,

\[
(\sigma_n)_{\text{int}} = \sqrt{[\sigma_{Rn}]_{\text{int}}^2 + [\tau_{RN}]_{\text{int}}^2}.
\]

\[
(\sigma_n)_{\text{mat}} = \sqrt{[\sigma_{Rn}]_{\text{mat}}^2 + [\tau_{RN}]_{\text{mat}}^2}.
\]

\[
(\sigma_n)_{\text{part}} = \sqrt{[\sigma_{Rn}]_{\text{part}}^2 + [\tau_{RN}]_{\text{part}}^2}.
\]
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Fig. 2 Composite idealization as a cylindrical unit cell, each containing a spherical (a) or cylindrical (b) reinforcement at the centre. The volume fraction of the particle is 10%.

Fig. 3 FEM discretization of the unit cell for 10 vol.% of particle.

Fig. 4 Definition of interfacial stresses, \((\sigma_R)_{\text{int}}\) and \((\tau_{R\theta})_{\text{int}}\) in Spherical particle model, \((\sigma_z)_{p}\) in Spherical particle model, \((\sigma_z)_{\text{int}}\) and \((\tau_{zr})_{\text{int}}\) in Cylindrical particle model, and particle stress, \((\sigma_z)_{p}\).
4. STRESS-STRAIN RESPONSE AND MICROSCOPIC DEFORMATION

In order to investigate the effect of the particle shape on the stress-strain response, the numerical computations based on the two unit cell models illustrated in Fig. 2 were carried out. Figure 5 indicates that the global stress-strain curve of the cylindrical particle model lies above the one of the spherical particle model for the same material properties of particle and matrix. In this section, the results of the spherical and cylindrical particle models based on the Mises yield condition are compared to investigate the monotonic global stress-strain response and the microscopic deformation of a ceramic particle reinforced aluminium alloy. The analytical models are calibrated through the macroscopic stress-strain response by experimental results which lie between the predictions of the two models as shown in Fig. 5.

Figure 6 shows the relation of the work hardening rate and the global plastic strain for the composite with and without thermal cycling including the numerical and experimental results. The white and black dots represent the spherical and cylindrical model results, while the solid lines show the experimental results. The predicted values are slightly larger than those of the experimental ones in the scope of the small plastic strain for the composite with and without thermal cycling. With increasing the plastic strain, the FEM results lie above the experimental ones for the T7 treated composite, while the predicted work hardening rate is smaller than the measured values for the thermally cycled composite. Although the tensile properties of the matrix of MMC might not be perfectly the same as the ones of the monolithic aluminum alloy which were experimentally measured and used in the computation (effect of the reinforcing particle on aging of the matrix alloy is pointed out(5)), the tendency of the predicted results for MMC are reasonably same as that of the experimental one in Fig. 6. Furthermore, it should be noted that two characteristics are found in the results. One is that the work hardening rate decreases abruptly with increasing plastic strain for the T7 heat treated composite. The other is that the experimental and predicted work hardening rate of the composite with thermal cycling at 0.001 plastic strain is significantly lower than that of the composite without thermal cycling. The two features will be explained by the following microscopic equivalent plastic strain distributions around the reinforcing particle.

![Fig. 5](image1.png)

Fig. 5 The comparison of the numerical and the experimental stress-strain response for the thermally cycled and the received composite.

![Fig. 6](image2.png)

Fig. 6 The comparison of the numerical and the experimental results about work hardening rate of the SiC particle reinforced composites.

Figure 7 indicates the distribution of the equivalent plastic strain around the reinforcing particle at the 0.001 global plastic strain for the composite without thermal cycling in the case of spherical and cylindrical particle models. There is an elastic band (the plastic strain less than 0.001) between the two plastic deformation bands in the matrix for the models of both types of particle shapes, which results in the matrix being more stiff. Especially, for the cylindrical model, local plastic strain concentration is found around the particle edge. This can be the reason for higher work hardening rate under this global plastic strain condition in the T7 heat treated composite. However, plastic deformation happens in the whole matrix in the case of a 0.0035 global plastic strain as shown in Fig. 8. This means that it is difficult
for the material to resist an external load, i.e., a smaller load increment leads to a larger deformation. So the abrupt decrease of the work hardening rate in the T7 heat treated composite is due to the evolution of an equivalent plastic strain in the matrix at the vicinity of particles\textsuperscript{30}. The equivalent plastic strain distributions of the composite with thermal cycling are shown in Fig. 9, under a 0.001 global plastic strain condition. There is a plastic deformation band from the upper left to the lower right in the matrix, which makes the material deformation easier as shown in Fig. 9 comparing with the T7 treated composite in Fig. 7(a). Whether there is a plastic deformation band penetrating the matrix or not is the controlling factor of the magnitude of the work hardening rate for the particulate reinforced composites with or without thermal cycling.

(a) Spherical particle model
(b) Cylindrical particle model

Fig. 7 The equivalent plastic strain distribution around reinforcing particle at a 0.001 global plastic strain for the composite without thermal cycling with the spherical and cylindrical particle.

Fig. 8 The equivalent plastic strain distribution in the case of a 0.0035 global plastic strain for the composite without thermal cycling with the spherical particle.

Fig. 9 The equivalent plastic strain distribution at a 0.001 global plastic strain for the composite with thermal cycling for the spherical particle.

5. EVOLUTION OF PARTICLE AND INTERFACIAL STRESSES

From the experimental observation it is found that composite shows the reduction of yield strength and the increase in ductility by applying thermal cycling. As shown in Fig. 5 the finite element simulation can reproduce this phenomenon. In the following the microscopic stress and strain distributions are investigated focusing on the effect of thermal cycling. The stress distributions discussed below are in the ranges of the
global strain in which the composites with and without thermal cycling are subjected to failure during the monotonic tensile tests. Figure 10 shows the changes of the interfacial average stress at spherical particle surface depending on the angle \( \theta \) for the T7 heat treated composite. Results from the Gurson (bold lines) and Mises (thin lines) yield conditions are compared. \( s \) indicates the global strain given on the top boundary edge of a mesh. The results of the two yield conditions coincide almost in the figure. The effect of the void exist in the matrix on the interfacial average stress is negligible for each given overall strain. The maximal stress value lies between 70 and 80 degrees, from the normal stress distribution (90 degrees) due to shear stress. The increase of maximum stress with the given global strain increment; however, is not linear to the global strain due to the plastic deformation of the matrix. The distribution of particle stress at \( \theta = 0 \) of the composite without thermal cycling is shown in Fig. 11 in the case of spherical particle model. The maximal normal stress in the particle lies at the position of \( r = r_0 \) which corresponds to the boundary of the particle and the matrix.

The distributions of interfacial average stress are shown in Fig. 12 for the thermally cycled composite. Under the Gurson yield condition, the position of the maximal stress value moves with an increase in the given global strain towards a larger global strain. This shift is related to the evolution of the void nucleation around \( \theta = 30 \sim 60 \) degrees due to the low yield strength induced by the thermal cycling as shown in Fig. 13. This is significant to the initiation of any debonding that may occur on the interface. Furthermore, the stress concentration of the interfacial average stress occurs on a larger global strain at \( \theta = 60 \) degrees for Mises yield condition as shown in Fig. 12. While for the Gurson yield condition, the maximal position is transferred due to the void nucleation and growth. The stress increment due to the increase of global strain at the stress concentrated site is larger than that at \( \theta = 90 \) degrees. Figure 14 indicates the particle stress distribution for each given strain. At a small global strain, the maximal stress in the particle occurs at the position of \( r = r_0 \). With increasing the global strain, the position of the maximum value is shifted to the center of the particle.

The stress concentrations caused by the differences in mechanical properties of the particles and the

![Fig. 10](image1.png)  
Fig. 10 The interfacial average stress distribution along the interface for the composite without thermal cycling (spherical particle model).

![Fig. 11](image2.png)  
Fig. 11 The particle stress distribution along the particle radius direction (\( \theta = 0 \)) for the composite without thermal cycling (spherical particle model).
matrix, and the inhomogeneous plastic deformation in the matrix lead to failure of the material. Fracture initiation is a localized phenomenon, and the peak stress levels developed in the composite will control which failure mechanism (the interface debonding or the particle fracture) are operative as well as the strength of the particle and the interface. The maximum values of the particle stress and the interfacial average stress as a function of the global strain are shown in Fig. 15 for the spherical particle model. The evolution of the maximum values of the particle stress and the interfacial stress can be divided into three stages. The particle stress is equal to the interfacial average stress during an elastic deformation. The interfacial stress is smaller than the particle stress at the start of plastic deformation. For the T7 composite, the specimen was fractured under a smaller plastic deformation. The particle stress is greater than the interfacial average stress in this state. In T7+TC composite, the interfacial stress becomes greater than the particle stress under the larger plastic deformation, which is because the stress concentration on
the interface between the particle and the matrix is more obvious. The ability of transferring the stress from the matrix to the particle decreases with an increase in the strain in the neighborhood of the stress concentration region\(^{31,33}\). Therefore for the thermally cycled composite, interfacial debonding will occur easier than particle fracture.

For the T7 treated composite, the maximum stresses in the particle is larger than the interfacial average stress as shown in Fig. 10 and 11. Although the strength of the particle and the interface are difficult to evaluate experimentally, the particle fracture is preferable in this case due to high particle stress. For the composite with thermal cycling, the maximum particle stresses are less than the interfacial average stress as shown in Fig. 12 and 14, which means that interfacial debonding tends to occur before particle fracture. As reported in the previous paper, the results based on SEM observation accord with the FEM prediction\(^{23,34}\).

The distributions of the direction and the magnitude of the principle stress under larger global strain are shown in Fig. 16. The direction and the magnitude of the principle stress on the interface between the particle and the matrix, around \(\theta = 45\) degrees, are different from the elastic one. As interfacial damage or locally concentrated plastic strain becomes significant, the stress coming from the particle avoids the damaged region on the interface as shown in Fig. 16. Instead, these stresses pass through the neighboring region which have the ability to afford a greater load. This might explain the stress concentrations in this region, Fig. 12.

6. CONCLUSION

The microscopic deformation of a particulate reinforced metal matrix composite was modeled by a unit cell model with Mises and Gurson yield conditions based on metal plasticity. The numerical predictions were compared and calibrated with the results of monotonic experiments of a cast aluminum alloy discontinuously reinforced with SiC particulates with and without thermal cycling. The responses of monotonic work hardening and the microscopic fracture mechanisms were investigated numerically. The micromechanical considerations of monotonic deformation in the particulate reinforced metal matrix composite were examined. The results obtained are as follows; (1) Whether there is a plastic deformation band penetrating a matrix or not, is the controlling factor of the magnitude of a work hardening rate for particulate reinforced composites, with or without thermal cycling. (2) For T7 treated composites, the maximum stress in the particle was larger than that of the interfacial average stress. For the composite with thermal cycling, the maximum particle stress was less than that of the interfacial average stress.

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