The Parametrization of All Stabilizing Modified Repetitive Controllers for Time-Delay Systems with the Specified Input-Output Frequency Characteristics

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In this paper, we examine the parametrization of all stabilizing modified repetitive controllers for single-input/single-output time-delay systems with the specified input-output frequency characteristics. The parametrization of all stabilizing modified repetitive controllers for time-delay systems was considered by Yamada et al. However, using the parametrization of all stabilizing modified repetitive controllers for time-delay systems by Yamada et al., the input-output frequency characteristics cannot be settled easily. From a practical point of view, the input-output frequency characteristics of the control systems are required to be easily settled. In this paper, we expand the result by Yamada et al. and propose the parametrization of all stabilizing modified repetitive controllers for time-delay systems with the specified input-output frequency characteristics. Finally, a numerical example is shown to illustrate the effectiveness of the proposed parametrization.

1 Introduction

In this paper we investigate the parametrization of all stabilizing modified repetitive controllers for time-delay systems with the specified input-output frequency characteristics. The repetitive control system is a type of servomechanism for periodic reference input. That is, the repetitive control system follows the periodic reference input without steady state error, even if a periodic disturbance or uncertainty exists in the plant\textsuperscript{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Because the repetitive control system that follows any periodic reference input without steady state error is a neutral type of time-delay control system, it is difficult to design stabilizing controllers for the plant\textsuperscript{8). In order to design the repetitive control system that follows any periodic reference input without steady state error, the plant needs to be biproper\textsuperscript{3, 4, 5, 6, 7, 8). Since almost all plants are strictly proper, many modifications of repetitive control systems that can apply to the strictly proper plants are considered\textsuperscript{3, 4, 5, 6, 7, 8). These studies are divided into two types. One uses a low-pass filter\textsuperscript{3, 4, 5, 6, 7} and the other uses an attenuator\textsuperscript{8). The latter is difficult to design because it uses a state variable time-delay in the repetitive controller\textsuperscript{8). The former has a simple structure and is easily designed. Therefore, the former type of repetitive control system is called the modified repetitive control system and applied to many applications\textsuperscript{1, 2, 7).\n
On the other hand, there exists important control problem to find all stabilizing controllers named the parametrization problem\textsuperscript{11, 12). Initially, the parametrization of all stabilizing repetitive controller is studied by Hara and Yamamoto\textsuperscript{4). In\textsuperscript{4), since the stability sufficient condition of repetitive control system is decided as $H_{\infty}$ norm problem, the parametrization of all stabilizing modified repetitive controller is given by resolving into the interpolation problem of Nevanlinna-Pick. Katoh and Funahashi give the parametrization of all stabilizing modified repetitive controllers for minimum phase plants by solving exactly Bezout equation\textsuperscript{10). In\textsuperscript{10), since the parametrization is not given based on the stability sufficient condition, this result is important in the sense that the class of stabilizing modified repetitive controllers in\textsuperscript{10} is extensive than the class of stabilizing modified repetitive controllers given in\textsuperscript{4). But, in\textsuperscript{10), the plant is assumed to be an asymptotically stable or be stabilized by local feedback control. This implies that the reference in\textsuperscript{10} give a parametrization of all causal repetitive controllers for an asymptotically stable and minimum phase plant. That is, the reference in\textsuperscript{10} does not give the complete....}
parametrization of all stabilizing modified repetitive controllers for minimum phase systems. In addition, in \cite{10}, it is assumed that the relative order of low-pass filter in modified repetitive controller is equal to that of the plant. This implies that the reference in \cite{10} give the parametrization for special case of repetitive control systems. Yamada and Okuyama overcome this problem and give the complete parametrization of all stabilizing modified repetitive controllers for minimum phase systems using the fusion of the parallel compensation technique and the solution of Bezout equation \cite{13}. Yamada, Satoh and Okuyama expand the result in \cite{13} and give the parametrization of all stabilizing modified repetitive controllers for the certain class of non-minimum phase systems using the fusion of the parallel compensation technique and solution of Bezout equation \cite{14}. Yamada, Satoh, Iida and Okuyama give the complete parametrization of all stabilizing modified repetitive controllers for non-minimum phase systems \cite{15}. Using the result in \cite{15}, Yamada, Satoh and Arakawa give the parametrization of all stabilizing modified repetitive controllers for time-delay systems \cite{16}. Thus the parametrization of all stabilizing modified repetitive controllers for strictly proper plant has been considered.

When we design a stabilizing modified repetitive controller for time-delay systems using the parametrization in \cite{16}, the input-output frequency characteristics of the control system cannot be settled so easily. From a practical point of view, the input-output frequency characteristics of the control systems are required to be easily settled. This problem is solved by obtaining the parametrization of all stabilizing modified repetitive controllers for timedelay systems with the specified input-output frequency characteristics. However, no paper considers the problem to obtain the parametrization of all stabilizing modified repetitive controllers for time-delay systems with the specified input-output frequency characteristics.

In this paper, we give the parametrization of all stabilizing modified repetitive controllers for time-delay systems with the specified input-output frequency characteristics. The basic idea to obtain the parametrization of all stabilizing modified repetitive controllers for time-delay systems with the specified input-output frequency characteristics is very simple. If a modified repetitive controller that specifies the input-output frequency characteristics of the repetitive control system stabilizes the plant, then the modified repetitive controller must be included in the class of all stabilizing controllers for the plant. The parametrization of all stabilizing controllers for the plant is obtained by Youla et al. \cite{11} and Vidyasagar \cite{19}. The parametrization of all stabilizing controllers includes a free parameter, which is stable. That is, stabilizing modified repetitive controllers for time-delay systems with the specified input-output frequency characteristics can be designed using the free parameter in the parametrization of Youla et al. \cite{11} and Vidyasagar \cite{19}. Using this idea, we obtain the parametrization of all stabilizing modified repetitive controllers for time-delay systems with the specified input-output frequency characteristics. Finally, a numerical example is shown to illustrate the effectiveness of the proposed parametrization.

**Notation**

- \( R \) the set of real numbers.
- \( R(s) \) the set of real rational functions with \( s \).
- \( R H_{\infty} \) the set of stable proper real coefficient rational functions.
- \( H_{\infty} \) the set of stable causal functions.

**2 Problem formulation**

Consider the unity feedback control system in

\[
\begin{cases}
  y = G(s)e^{-sL}u \\ 
  u = C(s)(r - y)
\end{cases}
\]

where \( G(s)e^{-sL} \) is the plant, \( G(s) \in R(s) \) is assumed to be strictly proper, \( L > 0 \) is the time-delay, \( C(s) \) is the controller, \( y \in R \) is the output and \( r \in R \) is the periodic reference input with period \( T > 0 \) satisfying

\[
r(t + T) = r(t) \quad \forall t \geq 0.
\]

According to \cite{5}, in order for the output \( y \) to follow the periodic reference input \( r \) in (2) with small steady state error, the controller \( C(s) \) must have the following structure

\[
C(s) = C_1(s) + C_2(s)C_T(s),
\]

where \( C_1(s) \in R(s) \) and \( C_2(s) \neq 0 \in R(s) \) are proper. \( C_T(s) \) is an internal model for the periodic reference input with period \( T \) and written by

\[
C_T(s) = \frac{q(s)e^{-sT}}{1 - q(s)e^{-sT}},
\]
where \( q(s) \) is a proper low-pass filter satisfying \( q(0) = 1 \). From \(^5\), if the low-pass filter \( q(s) \) satisfy

\[
1 - q(j\omega) \simeq 0 \quad \forall \omega = \frac{2\pi i}{T} \quad (i = 0, \ldots, N),
\]

then the output \( y \) in (1) follows frequency components in frequency \( \omega_i(i = 0, \ldots, N) \) of reference input \( r \) with small steady error. Therefore in order to design modified repetitive control systems, it is important problem to settle \( q(s) \) in (4) in wide frequency range. In order for \( q(s) \) to satisfy (5) in wide frequency range, \( q(s) \) must be stable and of minimum phase. In the following, we call settling \( q(s) \) in (4) by specifying the input-output frequency characteristics of the repetitive control system.

The parametrization of all stabilizing modified repetitive controllers in (3) for \( G(s)e^{-sL} \) is given in \(^6\). When we design a stabilizing modified repetitive controller using the parametrization in \(^6\), the input-output frequency characteristics of the control system cannot be settled so easily. That is, using the parametrization in \(^6\), it is difficult to settle \( q(s) \) in (4) so easily. From a practical point of view, the input-output frequency characteristics of the repetitive control systems are required to be easily settled. If \( q(s) \) is settled beforehand, then the input-output frequency characteristics of the repetitive control system are easily specified. From the importance of the parametrization problem, it is important problem to obtain the parametrization of all stabilizing modified repetitive controllers for time-delay systems such that \( q(s) \) in (5) is settled beforehand.

The problem considered in this paper is to give the parametrization of all stabilizing modified repetitive controllers for time-delay systems with the specified input-output frequency characteristics. That is, when the low-pass filter \( q(s) \) in (5) is settled beforehand to be stable and of minimum phase, we obtain the parametrization of all stabilizing modified repetitive controllers \( C(s) \) written by (3) for the time-delay system in (1).

3 The parametrization of all stabilizing modified repetitive controllers for time-delay systems with the specified input-output frequency characteristics

In this section, we give the parametrization of all stabilizing modified repetitive controllers for time-delay systems with the specified input-output frequency characteristics.

The low-pass filter \( q(s) \) in (4) is assumed to be settled beforehand and to be stable and of minimum phase. The parametrization of all modified repetitive controllers with the specified input-output frequency characteristics written by the form in (3) is given by following theorem.

**Theorem 1** The control system in (1) is internally stable and the controllers \( C(s) \) are written by the form in (3) if and only if \( C(s) \) is written by

\[
C(s) = \frac{\tilde{X}(s) + D(s)Q(s)}{\tilde{Y}(s) - N(s)Q(s)}e^{sL},
\]

where \( N(s) \in RH_\infty, D(s) \in RH_\infty, \tilde{N}(s) \in RH_\infty \) and \( \tilde{D}(s) \in RH_\infty \) are coprime factors of \( G(s) \) on \( RH_\infty \) satisfying

\[
G(s) = N(s)D^{-1}(s) = \tilde{D}^{-1}(s)\tilde{N}(s).
\]

\( \tilde{X}(s) \in RH_\infty \) and \( \tilde{Y}(s) \in RH_\infty \) satisfy

\[
\begin{bmatrix}
  Y(s) & X(s) \\
  -\tilde{N}(s) & \tilde{D}(s)
\end{bmatrix}
\begin{bmatrix}
  D(s) & -\tilde{X}(s) \\
  N(s) & \tilde{Y}(s)
\end{bmatrix} = I,
\]

where \( X(s) \in RH_\infty \) and \( Y(s) \in RH_\infty \). \( Q(s) \in H_\infty \) is the free parameter written by

\[
Q(s) = \frac{Q_{n1}(s) + Q_{n2}(s)e^{-sL} + Q_{n3}(s)q(s)e^{-sT} + Q_{n4}(s)q(s)e^{-s(T+L)}}{Q_{d1}(s) + Q_{d2}(s)e^{-sL} + Q_{d3}(s)q(s)e^{-sT} + Q_{d4}(s)q(s)e^{-s(T+L)}},
\]

where \( Q_{ni}(s) \in RH_\infty(i = 1, \ldots, 4), Q_{di}(s) \in RH_\infty(i = 1, \ldots, 4) \) are any functions satisfying

\[
\frac{\tilde{Y}(s)Q_{d1}(s) - N(s)Q_{n3}(s)}{\tilde{Y}(s)Q_{d1}(s) - N(s)Q_{n1}(s)} = -1,
\]

\[
\tilde{X}(s)Q_{d1}(s) + D(s)Q_{n1}(s) = 0, \quad \tilde{Y}(s)Q_{d2}(s) - N(s)Q_{n2}(s) = 0.
\]
\[ \dot{X}(s)Q_{41}(s) + D(s)Q_{43}(s) = 0, \quad \dot{Y}(s)Q_{44}(s) - N(s)Q_{n4}(s) = 0 \]  
\[ \text{and} \]

\[ \frac{\dot{X}(s)(Q_{21}(s) + Q_{44}(s)) + D(s)(Q_{22}(s) + Q_{n4}(s))}{\dot{Y}(s)Q_{21}(s) - N(s)Q_{n1}(s)} \neq 0. \]  

Proof of this theorem requires following lemma.

**Lemma 1**  
Unity feedback control system in (1) is internally stable if and only if \( C(s) \) is written by

\[ C(s) = \frac{\dot{X}(s) + D(s)Q(s)}{\dot{Y}(s) - N(s)Q(s)}, \]  

where \( N(s) \in RH_{\infty}, D(s) \in RH_{\infty}, \dot{N}(s) \in RH_{\infty} \) and \( \dot{D}(s) \in RH_{\infty} \) are coprime factors of \( G(s) \) on \( RH_{\infty} \) satisfying (7). \( \dot{X}(s) \in RH_{\infty} \) and \( \dot{Y}(s) \in RH_{\infty} \) satisfy (8) and \( Q(s) \in H_{\infty} \) is the free parameter \(^{12}\).

**Lemma 2**  
Assume that \( C_1(s), C_2(s) \) in (3) and \( q(s) \) in (4) are rational functions. The modified repetitive controller \( C(s) \) written by (3) stabilizes the plant \( G(s)e^{-sT} \) if and only if the controller \( C(s) = C(s)e^{-sT} \) stabilizes \( G(s) \).

(Proof)  
Proof is obvious, because \( C_1(s), C_2(s) \) in (3) are rational functions and \( G(s) \) and \( C(s) \) in (1) are single-input/single-output system.

Using Lemma 1 and Lemma 2, we show the proof of Theorem 1.

(Proof)  
First, the necessity is shown. That is, we show that if \( C(s) \), written as (3), stabilizes \( G(s)e^{-sT} \), then the controller \( C(s) \) and the free parameter \( Q(s) \) are written by (6) and (9), respectively. From Lemma 2, the parametrization of all stabilizing controllers \( C(s) \) for the plant \( G(s)e^{-sT} \) is equivalent to that for all stabilizing controllers \( \hat{C}(s) = C(s)e^{-sT} \) for \( G(s) \).

From Lemma 1, the parametrization of all stabilizing controllers \( \hat{C}(s) \) for \( G(s) \) is written by

\[ \hat{C}(s) = C(s)e^{-sT} = \frac{\dot{X}(s) + D(s)Q(s)}{\dot{Y}(s) - N(s)Q(s)}, \]  

This equation is equivalent to (6). Next we show that if the controller \( \hat{C}(s) = C(s)e^{-sT} \) stabilizes \( G(s) \), then the free parameter \( Q(s) \) in (6) is written by (9). Substituting \( C(s) \) in (3) into (6), we have

\[ Q(s) = \frac{Q_{n1}(s) + Q_{n2}(s)e^{-sT} + Q_{n3}(s)q(s)e^{-sT} + Q_{n4}(s)q(s)e^{-s(T+L)}}{Q_{d1}(s) + Q_{d2}(s)e^{-sT} + Q_{d3}(s)q(s)e^{-sT} + Q_{d4}(s)q(s)e^{-s(T+L)}}, \]  

where \( Q_{n1}(s) \in RH_{\infty}(i = 1, \ldots, 4) \) and \( Q_{d1}(s) \in RH_{\infty}(i = 1, \ldots, 4) \) are

\[ Q_{n1}(s) = -C_{1d}(s)C_{2d}(s)\dot{X}(s) \in RH_{\infty}, \quad Q_{n2}(s) = C_{1n}(s)C_{2d}(s)\dot{Y}(s) \in RH_{\infty}, \]
\[ Q_{n3}(s) = C_{1d}(s)C_{2d}(s)\dot{X}(s) \in RH_{\infty}, \quad Q_{n4}(s) = -C_{1n}(s)C_{2d}(s) + C_{1d}(s)C_{2n}(s)\dot{Y}(s) \in RH_{\infty}, \]
\[ Q_{d1}(s) = C_{1d}(s)C_{2d}(s)D(s) \in RH_{\infty}, \quad Q_{d2}(s) = C_{1n}(s)C_{2d}(s)N(s) \in RH_{\infty}, \]
\[ Q_{d3}(s) = -C_{1d}(s)C_{2d}(s)D(s) \in RH_{\infty}, \quad Q_{d4}(s) = -C_{1n}(s)C_{2d}(s) + C_{1d}(s)C_{2n}(s)N(s) \in RH_{\infty}. \]

Here \( C_{1n}(s) \in RH_{\infty}, C_{1d}(s) \in RH_{\infty}, C_{2n}(s) \in RH_{\infty} \) and \( C_{2d}(s) \in RH_{\infty} \) are coprime factors of \( C_1(s) \) and \( C_2(s) \) on \( RH_{\infty} \) satisfying

\[ C_1(s) = C_{1n}(s) C_{1d}(s), \quad C_2(s) = C_{2n}(s) C_{2d}(s). \]  

From (16), if the controller written by (3) stabilizes the control system in (1), then \( Q(s) \) is written by (9). Substituting (17)\sim(20) into (10)\sim(13), we can confirm that (10)\sim(13) hold true. Thus, the necessity has been proved.

Next, the sufficiency is shown. That is, under the assumption of (10)\sim(13), we show that if \( Q(s) \) in the controller written by (6) is given by (9), then the controller is written by (3). Substituting (9) into (6), we have

\[ C(s) = C_1(s) + C_2(s) \frac{q(s)e^{-sT}}{1 - q(s)e^{-sT}}, \]
where $C_1(s)$ and $C_2(s)$ are written by

$$C_1(s) = \frac{\dot{X}(s)Q_{d2}(s) + D(s)Q_{n2}(s)}{Y(s)Q_{d1}(s) - N(s)Q_{n1}(s)}$$

(23)

and

$$C_2(s) = \frac{\dot{X}(s)(Q_{d2}(s) + Q_{d4}(s)) + D(s)(Q_{n2}(s) + Q_{n4}(s))}{Y(s)Q_{d1}(s) - N(s)Q_{n1}(s)}$$

(24)

respectively. From the assumption of (13), $C_2(s) \neq 0$ holds true. This implies that $C(s)$ in (6) works as a modified repetitive controller. Thus, sufficiency has been proved.

From above discussion, we have proved Theorem 1.

\section{Numerical example}

In this section, a numerical example is shown to illustrate the effectiveness of the proposed parametrization.

Let us consider to obtain the parametrization of all stabilizing modified repetitive controllers with the specified input-output frequency characteristics for the plant $G(s)e^{-sT}$ written by

$$G(s)e^{-sT} = \frac{s + 7}{s^2 + 9s + 20}e^{-2s}.$$  

(25)

The period $T$ for the periodic reference input $r$ is $T = 1[sec]$. The low-pass filter $q(s)$ in (4) are settled by

$$q(s) = \frac{1}{0.0001s + 1}.$$  

(26)

Since $G(s)$ is stable, $D(s)$, $N(s)$, $\dot{X}(s)$ and $\dot{Y}(s)$ in (6) satisfying (7) and (8) are given by

$$D(s) = \dot{D}(s) = 1, \ N(s) = \dot{N}(s) = G(s), \ \dot{X}(s) = X(s) = 0, \ \dot{Y}(s) = Y(s) = 1.$$  

(27)

According to Theorem 1, the parametrization of all stabilizing modified repetitive controllers with the specified input-output frequency characteristics is written by (6). Here the free parameter $Q(s) \in H_{\infty}$ in (6) is written by (9).

In order for $Q(s)$ to satisfy $Q(s) \in H_{\infty}$ and (10) to (13), $Q_{n1}(s) \in RH_{\infty}(i = 1, \ldots, 4)$ and $Q_{d1}(s) \in RH_{\infty}(i = 1, \ldots, 4)$ in (9) are selected as follows: From (10), (11), (12) and (27), we have

$$Q_{n1}(s) = 0, \ Q_{d2}(s) = G(s)Q_{d2}(s), \ Q_{d3}(s) = 0, \ Q_{d4}(s) = -Q_{d1}(s), \ Q_{d4}(s) = G(s)Q_{d4}(s).$$  

(28)

$Q_{d1}(s)$, $Q_{d2}(s)$ and $Q_{d4}(s)$ are used to make $Q(s) \in H_{\infty}$ and to specify control characteristic. From (28), since the transfer function from the periodic reference input $r$ to the error $e = r - y$ is written by

$$e \begin{pmatrix} r \end{pmatrix} = \frac{1 - q(s)e^{-sT}}{(1 - q(s)e^{-sT}) + G(s)\left(\frac{Q_{d2}(s)}{Q_{d1}(s)} + \frac{Q_{d4}(s)}{Q_{d1}(s)}q(s)e^{-sT}\right)},$$  

(29)

when the gain of $Q_{n2}(s)/Q_{d1}(s)$ and that of $Q_{n4}(s)/Q_{d1}(s)$ are made bigger, the error $e = r - y$ have a tendency to be small. Therefore $Q_{d1}(s)$, $Q_{d2}(s)$ and $Q_{d4}(s)$ are settled to make the gain of $Q_{n2}(s)/Q_{d1}(s)$ and that of $Q_{n4}(s)/Q_{d1}(s)$ bigger as far as possible and to satisfy $Q(s) \in H_{\infty}$. From the Nyquist theorem, $Q_{n1}(s) \in RH_{\infty}(i = 1, \ldots, 4)$ and $Q_{d1}(s) \in RH_{\infty}(i = 1, \ldots, 4)$, if the nyquist plot of $Q_{d1}(s) + Q_{d2}(s)e^{-sT} + Q_{d3}(s)e^{-sT} + Q_{d4}(s)e^{-s(T + L)}$ does not encircle the origin, then $Q(s) \in H_{\infty}$ holds true. That is, $Q_{d1}(s)$, $Q_{d2}(s)$ and $Q_{d4}(s)$ are settled to make the gain of $Q_{n2}(s)/Q_{d1}(s)$ and that of $Q_{n4}(s)/Q_{d1}(s)$ bigger as far as possible and not to make the nyquist plot of $Q_{d1}(s) + Q_{d2}(s)e^{-sL} + Q_{d3}(s)e^{-sT} + Q_{d4}(s)e^{-s(T + L)}$ encircle the origin. From above discussion, $Q_{n1}(s)(i = 2, 4)$ and $Q_{d1}(s)(i = 1, \ldots, 4)$ in (9) are selected by

$$Q_{n2}(s) = \frac{35(s + 2)(s + 4)(s + 5)}{(s + 0.5)(s + 12)(s + 50)}, \ Q_{n4}(s) = \frac{3(s + 4)(s + 5)}{(s + 7)(s + 15)}.$$  

(30)

$$Q_{d1}(s) = \frac{3(s + 2)}{s}, \ Q_{d2}(s) = \frac{35(s + 2)(s + 7)}{(s + 0.5)(s + 12)(s + 50)}, \ Q_{d3}(s) = \frac{-3(s + 2)}{s + 0.5}, \ Q_{d4}(s) = \frac{3}{s + 15}.$$  

(31)
Using $Q_m(s)(i = 1, \ldots, 4)$ and $Q_{di}(s)(i = 1, \ldots, 4)$ in (30) and (31), the nyquist plot of $Q_{d1}(s) + Q_{d2}(s)e^{-sL} + Q_{d3}(s)e^{-sT} + Q_{d4}(s)e^{-s(T+L)}$ is shown in Fig. 1. From Fig. 1, since the nyquist plot of $Q_{d1}(s) + Q_{d2}(s)e^{-sL} + Q_{d3}(s)e^{-sT} + Q_{d4}(s)e^{-s(T+L)}$ does not encircle the origin, $Q_{d1}(s) + Q_{d2}(s)e^{-sL} + Q_{d3}(s)e^{-sT} + Q_{d4}(s)e^{-s(T+L)}$ has no zero in the closed right half plane. Thus, we find that $Q(s)$ in (9) is included in $H_\infty$.

From (23) and (24), $C_1(s)$ and $C_2(s)$ in (3) are written by

$$C_1(s) = \frac{35s^2 + 350s + 945}{3s^2 + 192s + 1800}$$

and

$$C_2(s) = \frac{35(s + 2)(s + 4)(s + 5)(s + 7)(s + 15) + 3(s + 0.5)(s + 4)(s + 5)(s + 12)(s + 50)}{3(s + 0.5)(s + 7)(s + 12)(s + 15)(s + 50)(s + 2)}$$

respectively. Using (26), (33) and (34), the stabilizing modified repetitive controller is given by (3).

Using the obtained modified repetitive controller $C(s)$ for the plant $G(s)e^{-sL}$ written by (25), the response of the output $y(t)$ in (1) for the reference input $r(t) = \sin(2\pi t)$ is shown in Fig. 2. Here the solid line shows the response of the output $y(t)$ and the dotted line shows that of the reference input $r(t) = \sin(2\pi t)$. Fig. 2 shows that the output $y(t)$ follows the reference input $r(t)$ with very small steady state error.

In this way, we can design stabilizing modified repetitive controllers for time-delay plant easily.
5 Conclusion

In this paper, we proposed the parametrization of all stabilizing modified repetitive controllers for time-delay systems with the specified input-output frequency characteristics. That is, when the low-pass filter $q(s)$ in (5) is settled beforehand to be stable and of minimum phase, we found out the parametrization of all stabilizing modified repetitive controllers $C(s)$ written by (3) for the time-delay system in (1). A numerical example was shown to illustrate the effectiveness of the proposed parametrization.

References


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