Heat-Flux Interface Transport on One-Dimensional Turbulence Model for Stratified Flow of Thermal Effluent

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A condenser cooling sea water discharged from coastal nuclear power plant has been diffused into the sea area at an outlet temperature-rise of the order of 7 °C in thermally-stratified turbulent shear flow. The purpose of this research is that the vertical thermal diffusion phenomenon and heat-flux interface stability mechanism in the thermal effluent is analyzed based on one-dimensional heat-flux turbulence models. An important applied hydrodynamic formula that is the relation between a local gradient Richardson number $R_i$ and a dimensionless buoyancy parameter $B_p$ is formulated, and it is seen to keep the buoyancy parameter $B_p \to \infty$ in the case of flow region of $R_i \geq 0.612$. It is principal to perform the protection of environment.

1. INTRODUCTION

A waste heat quantity in the sea area where the cooling sea water is discharged from a condenser of a turbine building in a coastal nuclear power plant (commercial use) can be calculated using the following equation by a thermal efficiency \( \eta_t \) of generating-end output.

\[
W_h (\text{kcal/kWh}) = \frac{860}{\eta_t} (1 - 0.05) - 860
\]

(1)

where \( W_h \) denotes the waste heat quantity, and the heat quantity of generated output is 1kWh=860kcal, and 0.05 is the heat loss of a power plant. The water temperature rise \( \Delta T_0 \) (°C, about 7 °C) at the thermal effluent outlet is given in accordance with the cooling sea water discharge \( Q_0 \) (m³/s) by the following equation.

\[
\Delta T_0 = \frac{GW_h}{3600\rho C_p Q_0}
\]

(2)

where \( G \) is the generating-end electric output power (kW), and \( \rho \) & \( C_p \) are the sea water density (kg/m³) and sea water specific heat (kcal/kg °C) respectively. As for the thermal effluent problems, it is strongly required to avoid an injurious action to a self-governing community and persons engaged for coastal fishery and local inhabitants respectively. Because there is a danger of the injured fishes and shellfishes attributable to thermal effluent, the turbulent thermal diffusion phenomena in the sea-water-depth direction must be solved about the thermally-stratified turbulent shear flow of the surface discharge system. It is a definition of turbulence models to derive from such transport equations as would close Reynolds equations (RANS equation) based on N-S equations, in which the \( k-\varepsilon \) model and the stress model are typical ones of turbulence models\(^2\). The turbulence model of heat-flux interface in the thermal effluent is analyzed based on an assumption that is incompressible, non-isothermal and universal equilibrium range (the theory of local isotropy, A.N. Kolmogoroff (1941))\(^3\). **Fig. 1** is a schematic diagram of the one-dimensional turbulence model for the thermal effluent. As shown in **Fig. 1**, a current of the thermal effluent diffuses on the ocean current \( U_h, T_h \), and \( u, w \) are the turbulent
fluctuating velocities in the X*, Z-axes directions, and $\downarrow \uparrow$ are the mixing length, and $\delta$ are the eddies motions of the equilibrium range. Fig. 1 shows whether turbulence can be developed or not by the buoyancy effect due to gravity.

$$Z_{+}, \overline{W}=0, \overline{w}=w$$

- Free surface $P=P_o$ (Atmospheric pressure)
- $\delta \downarrow \uparrow \pi w \overline{u}(z)=\overline{U}(z)+u, \overline{T}(z)=\overline{T}(z)+\theta$
- Thermal effluent $-g$
- Heat-flux interface $X$
- $Y, \overline{V}=0, \overline{v}=v$
- $\overline{U}_{+} \overline{T}_{+} > 0, \overline{\theta}_{+} > 0, U_o, T_o$

**Fig. 1 Schematic diagram of the one-dimensional turbulence model.**

### 2. ANALYSES OF RICHARDSON NUMBER $R_i$ AND $B_p$ IN THERMAL EFFLUENT

The law of dynamic similarity which appears in the interface stability of stratified flow in the horizontal turbulent thermal diffusion for the thermal effluent should be described by the local gradient Richardson number $R_i$. $R_i$ is the important dimensionless parameter to determine the buoyancy-affected turbulence, and it is given by

$$R_i = -\frac{g}{\rho_0 \overline{U}} \overline{\nabla} \overline{\nabla} \overline{U} = \frac{\alpha}{T} \overline{g} \overline{\nabla} \overline{\nabla} \overline{U}$$

(3)

where $\alpha$ is the dimensionless coefficient of volume expansion, $\overline{T}$ is the time-averaged water temperatures, $g$ is the acceleration due to gravity, $\overline{U}$ is the time-averaged velocity in the flow direction, $z$ is the position coordinate in the upward vertical direction, $\rho$ is the fluid density and $\rho_0$ is a basic fluid density (see Fig. 1). A dimensionless buoyancy parameter $B_p$ is defined as

$$B_p = \frac{\alpha}{T} \overline{g} \left( \frac{k}{\varepsilon} \frac{\partial \overline{T}}{\partial z} \right)$$

(4)

where, $k=\overline{U}^2/2$ is the turbulent kinetic energy and $\varepsilon$ is the dissipation rate of turbulent kinetic energy. For the transport equations of the Reynolds stress, there are the shear generation term $P$ and the buoyant forces generation term $G$ for the generation terms of turbulent energy, and a relational expression of $G+P=\varepsilon$ is applicable to high Reynolds number shear flow in the local equilibrium state. The following equations have been introduced by Mizushima who obtained from one-dimensional turbulence model.

$$G = -\frac{\alpha}{T} u_i \overline{\theta} g_i = \frac{\alpha}{T} \overline{w} \overline{\theta}$$

(5)

$$P = -u_i \mu_k \frac{\partial \overline{U}}{\partial x_k} = -\overline{uv} \frac{\partial \overline{U}}{\partial z}$$

(6)

$$\overline{w} \overline{\theta} = \left( \frac{0.1639}{1 + 0.4584 B_p} \right) k^2 \frac{\partial \overline{T}}{\partial z}$$

(7)

$$\overline{uv} = \left( \frac{0.1213 + 0.0294 B_p}{1 + 0.0772 B_p} \right) \left( 1 + 0.4584 B_p \right) \frac{k^2}{\varepsilon} \frac{\partial \overline{U}}{\partial z}$$

(8)

$$\varepsilon = G + P = -\frac{\alpha}{T} \overline{g} \left( \frac{0.1639}{1 + 0.4584 B_p} \right) k^2 \frac{\partial \overline{T}}{\partial z} + \left( \frac{0.1213 + 0.0294 B_p}{1 + 0.0772 B_p} \frac{k^2}{\varepsilon} \frac{\partial \overline{U}}{\partial z} \right)^2$$

(9)
where $\theta$ is the turbulent fluctuating temperature and $\overline{\theta \theta}$ is the heat-flux correlation. In accordance with Eq. (9) described above, an important applied mechanical (hydrodynamic) formula in which the heat-flux interface phenomenon in turbulent thermal diffusion of thermal effluent is analyzed and given by

$$R_i = \frac{B_p \left( 0.1213 + 0.0294B_p \right)}{\left( 1 + 0.0772B_p \right) \left( 1 + 0.6223B_p \right)}$$

has been proposed. Fig. 2 shows the results calculated using Equation (10). The dimensionless buoyancy parameter $B_p$ increases rapidly at $R_i$ number above $R_i = 0.5$, and $R_i = 0.612$ is approximately equal to an asymptotic line, as indicated in Fig. 2. Since the value of $B_p$ takes $B_p \to \infty$ in the flow region of $R_i$ number larger than 0.612, the non-diffusion effects on the interface should be held by the buoyant effective stabilization. Fig. 3 shows the experimental results of $R_i$ number distributions in the dimensionless stratified depth direction, where the measured values at $x/\delta = 45$ are ones of the outlet sea area, and the measured values at $x/\delta = 63$ are ones of the faraway sea area, $x$ is the flow-distance between the outlet and the measured point, $\delta$ is the stratified flow depth at the measured points, and $H_o$, is the outlet depth. The distributions of $R_i$ number in Fig. 3 are changed considerably within inside of stratified depth, which are remarkably different between the outlet and the faraway sea area. After $R_i$ number reaches a minimum value of $R_i = 0.04$–0.4 in the vicinity of $1/2$ stratified depth, $R_i$ number gradually increases to $R_i = 7.0$–2.0 towards the interface. Therefore, the stability on the interface is clearly noticed,
because $B_p \to \infty$ on the interface is derived from the above mentioned.

3. TURBULENCE MODEL OF HEAT-FLUX CORRELATIONS

The turbulent thermal diffusion equation can be expressed as follows:

$$
\frac{\partial \bar{T}}{\partial t} + U_i \frac{\partial \bar{T}}{\partial x_i} = \frac{\partial}{\partial x_j} \left( \lambda \frac{\partial \bar{T}}{\partial x_j} - u_i \theta \right) + F_r \tag{11}
$$

where $u_i$ is the turbulent fluctuating velocity of $i$ coordinate axis component, $t$ is time, $\lambda$ is the molecular thermal diffusivity, and $F_r$ is the term of heat-exchange between the sea surface and the atmosphere. In Eq. (11), $u_i \theta$ that are defined as the heat-flux correlations can be expressed by means of the following equations.

$$
\begin{pmatrix}
-u_i \bar{\theta} \\
-w \bar{\theta} \\
-k \bar{\theta}
\end{pmatrix} =
\begin{pmatrix}
K_{xx} & K_{xy} & K_{xz} \\
K_{xy} & K_{yy} & K_{yz} \\
k_{xz} & k_{yz} & k_{zz}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial \bar{T}}{\partial x} \\
\frac{\partial \bar{T}}{\partial y} \\
\frac{\partial \bar{T}}{\partial z}
\end{pmatrix} \tag{12}
$$

where $K_{xx}$ is a $x$ axis component of the eddy thermal diffusion coefficient tensor. $\bar{u} \bar{\theta}$ and $K_{xx}$ are given in the following equations that are analyzed based on the one-dimensional turbulence model$^{31}$.

$$
\bar{u} \bar{\theta} = \frac{0.0703 + 0.0119 B_p}{\left[ 1 - 0.772 B_p \left( 1 + 0.4584 B_p \right) \right]} \left( \frac{k}{\varepsilon} \right)^2 \frac{\partial \bar{U}}{\partial \bar{z}} \frac{\partial \bar{T}}{\partial \bar{z}} \tag{13}
$$

$$
K_{zz} = \frac{0.1639}{\left( 1 + 0.4584 B_p \right)} \left( \frac{k}{\varepsilon} \right)^2 \tag{14}
$$

As a result of one-dimensional turbulence model, the heat-flux correlation in the $Y$ direction becomes $v \bar{\theta} = 0$. $\bar{w} \bar{\theta}$ in the $Z$ direction is given by Eq.(7). A basic equation required for an analysis of the value for $k/\varepsilon$ that can be expressed by rearranging Eq. (4) as follows:

$$
\frac{k}{\varepsilon} = \left[ \frac{B_p}{\alpha/T \left( \frac{\partial \bar{T}}{\partial \bar{z}} \right)} \right]^{1/2} \tag{15}
$$

where $k/\varepsilon$ values can be obtained by the measured values of $\partial \bar{T} / \partial \bar{z}$. Then, $\bar{\theta}^2$ equation that is

![Figure 4](image.png)

**Fig. 4** $\bar{u} \bar{\theta}$ (cm $^\circ$C/s) distributions in the dimensionless stratified depth direction.

obtained on the basis of the transport equation for $\bar{\theta}^2$ can be expressed for one-dimensional turbulence model by the following equation$^{31}$.
Heat-Flux Interface Transport Flow of Thermal Effluent

\[
\bar{\theta} = -C_1 \frac{k}{\varepsilon} u_\varepsilon \frac{\partial \bar{T}}{\partial \bar{z}} = -C_1 \frac{k}{\varepsilon} \frac{w}{\varepsilon} \frac{\partial \bar{T}}{\partial \bar{z}} = \left( \frac{0.2622}{1 + 0.4584 B_p} \right) \left( \frac{k}{\varepsilon} \right)^2 \left( \frac{\partial \bar{T}}{\partial \bar{z}} \right)^2
\]  

(16)

Hence, the values of \( k \) are given by using Eq. (16) as follows:

\[
k = \frac{\bar{\theta}^2}{\left( \frac{\partial \bar{T}}{\partial \bar{z}} \right)^2} \left( 1 + 0.4584 B_p \right) \left( \frac{0.2622}{k / \varepsilon} \right)^2
\]  

(17)

where the values of \( k \) are calculated by using both \( k / \varepsilon \) values obtained from Eq. (15) and the experimental values of \( \bar{\theta} \left( \frac{\partial \bar{T}}{\partial \bar{z}} \right)^2 \). The horizontal heat-flux correlation \( \bar{w} \bar{\theta} \) (cm°C/s) distributions that are calculated by Eq. (13) are shown in Fig. 4 for the dimensionless depth direction. The vertical heat-flux correlation \( -\bar{w} \bar{\theta} \) (cm°C/s) distributions that are calculated by Eq. (7) are shown in Fig. 5 for the dimensionless depth direction.

![Graph showing vertical heat-flux distributions.](image)

**Fig. 5** – \( -\bar{w} \bar{\theta} \) (cm°C/s) distributions in the dimensionless stratified depth direction.

4. CONCLUDING REMARKS

The law of applied dynamical similarity in thermal stratified flow is maintained by Richardson number \( R_t \) that determines whether turbulence can be developed or not. In the flow region of \( R_t \geq 0.612 \), it is found that the dimensionless buoyancy parameter \( B_p \) takes value of \( B_p \rightarrow \infty \) that is applicable to the non-diffusion on the interface owing to the buoyancy performance effect. Though \( R_t \) number on the interface is increasing to such a large value of \( R_t \geq 2.0 \sim 7.0 \) that the stability on the heat-flux interface is clearly noticed, it is recognized that the heat-flux \( -\bar{w} \bar{\theta} \) on the interface is not zero because of the turbulence produced by the ocean current. Although the increase depth that is diffused to the faraway sea area from the outlet for the heat-flux interface is about 1.5 Ho dimensionless depth, it is considered that the additional thermal diffusion on the interface in the faraway sea area is harmless to the fishes and shellfishes bodies on account of a decreasing within 1°C in the interface water temperature rise. For turbulent thermal diffusion phenomena in the sea-water-depth direction, the stability mechanism of the heat-flux interface transport in stratified flow was elucidated.

REFERENCES