Two-Dimensional Computation for Aerodynamic Vibrations of Two Close Circular Cylinders in Side-by-Side Arrangement

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We present numerical results for aerodynamic vibrations of two elastically supported circular cylinders, which are placed at a certain distance between two cylinders, by two-dimensional computation. The computation is achieved by means of the finite element method and the arbitrary Lagrangian-Eulerian method, and numerical stabilization of solutions of the Navier-Stokes equations is performed by a third-order upwind scheme. In our computation we show clearly aerodynamic characteristics of two vibrating circular cylinders in side-by-side arrangement.

1. Introduction

It has been well-known from experimental data and numerical results that aerodynamic characteristics of two circular cylinders in various arrangements vary violently with variation of spacing between the cylinders. Classification of flow regions around the cylinders in side-by-side and tandem arrangements has been clearly shown by Zdravkovich and the numerical results are in good agreement with the experimental data.

On the other hand, when two circular cylinders that are elastically supported by spring-damper systems are close to each other, aerodynamic vibrations of the cylinders are very complicated. Kondo showed two-dimensional numerical results for vibrations of two circular cylinders in tandem arrangement by using the finite element method and the arbitrary Lagrangian-Eulerian (ALE) method. It was shown that vortex shedding from a stationary windward cylinder or a vibrating one has a great influence on vibration of a leeward cylinder.

The purpose of this paper is to clearly show both flow patterns around two vibrating circular cylinders in side-by-side arrangement and features of vibration of the cylinders at various reduced velocities. The circular cylinders are placed in a uniform flow. In addition, each cylinder is treated as a system of two-degrees of freedom which moves to both in-line and transverse directions. A numerical method which is used in this paper is a two-dimensional numerical scheme. In this case, it has been known that there are slight differences in a comparison between the fluid forces obtained by two- and three-dimensional computations. Thus, numerical results shown in this paper are qualitatively discussed. In order to make computations of the vibrating circular cylinders, the arbitrary Lagrangian-Eulerian method is applied to the third-order upwind finite element scheme. We, in this paper, focus to aerodynamic vibrations of two circular cylinders at the Reynolds number 1000.

2. Governing equations for a coupled problem of fluid flow and two circular cylinders

Aerodynamic characteristics for free vibrations of two circular cylinders can be obtained by numerical computation of the equations of motion of both a fluid flow and the cylinders. We here consider a computational model of two elastically supported circular cylinders which are arranged in side-by-side at spacing T between the centers of the cylinders, which are placed in a uniform flow $U_0$, as shown in Figure 1.
Fig. 1 Model of two circular cylinders in side-by-side arrangement

The Navier-Stokes equations and the continuity equation, in two-dimensional domain $\Omega$, are written as

$$\frac{\partial u_i}{\partial t} + u_j u_{i,j} = \sigma_{ij} + f_i, \quad (1)$$

$$u_{i,i} = 0, \quad (2)$$

where $u_i$ is the dimensionless velocity, $f_i$ is the dimensionless body force, $X_i$ is the dimensionless coordinate system of a fluid domain, $t$ is the dimensionless time, and $( )_{,i}$ denotes the derivative with respect to $X_i$. These variables are non-dimensionalized on the basis of the diameter $D$ and the flow velocity $U_0$. The total stress $\sigma_{ij}$ is given by

$$\sigma_{ij} = -p \delta_{ij} + \tau_{ij}, \quad (3)$$

where $p$ is the dimensionless pressure and $\tau_{ij}$ denotes the viscous stress given by

$$\tau_{ij} = \frac{1}{Re} (u_{i,j} + u_{j,i}), \quad (4)$$

where $Re$ denotes the Reynolds number.

The pressure Poisson equation for making computations of the pressure field $p$ in the fluid domain is given as follows, by taking the divergence of the Navier-Stokes equations (1):

$$p_{,ii} = -\hat{u}_{i,i}, \quad (5)$$

where $\hat{u}_i$ in the right-hand side of the above equation is defined by

$$\hat{u}_i = \frac{\partial u_i}{\partial t} + u_j u_{i,j} - \tau_{ij,j} - f_i. \quad (6)$$

Because we consider the free vibrations of each cylinder with two-degrees of freedom as shown in Figure 1, the equations of motion of the circular cylinders for in-line and transverse directions are given by

$$\frac{d^2 x_i}{dt^2} + \frac{4 \pi h}{U_r} \frac{dx_i}{dt} + \left( \frac{2 \pi}{U_r} \right)^2 x_i = \frac{1}{2 \rho_c \pi} C_{D,i}, \quad (7)$$

$$\frac{d^2 y_i}{dt^2} + \frac{4 \pi h}{U_r} \frac{dy_i}{dt} + \left( \frac{2 \pi}{U_r} \right)^2 y_i = \frac{1}{2 \rho_c \pi} C_{L,i}, \quad (8)$$

for $i = 1, 2$, where $h$ is the cylinder height, $x_i$ and $y_i$ are the in-line and transverse coordinates, $\rho_c$ is the cylinder density, and $C_{D,i}$ and $C_{L,i}$ are the drag and lift coefficients, respectively.
where \( i = 1 \) and \( 2 \) of the subscript \( i \) in equations (7) and (8) denote symbols of the cylinders I and II, respectively. \( x_i (\equiv x_i/D) \) and \( y_i (\equiv y_i/D) \) are the dimensionless displacements of the in-line and transverse directions of two circular cylinders, which are non-dimensionalized by the diameter \( D \) of the cylinders, \( h \) is the structural damping ratio, \( U_r \) is the reduced velocity ( \( U_r = U_0/Df_n \)), \( f_n \) is natural frequency of the cylinder, \( \rho \) is the density of fluid, \( \rho_c \) is the density of the cylinder, and \( C_{Di} \) and \( C_{Li} \) denote the drag and lift coefficients.

3. Third-order upwind finite element procedure

Spatial discretizations for equations (1) \( \sim \) (6) are performed by means of the finite element method, and numerical stabilization\(^1\) of solutions of the Navier-Stokes equations is achieved by the use of a third-order upwind scheme based on the Petrov-Galerkin formulation. In addition, we consider a computational space \((\xi_i, \tau)\) instead of a physical space \((x_i, t)\) in order to accurately make a third-order upwind scheme by means of a finite element procedure. The relationship between the computational space and the physical space is given by

\[
\xi_i = \xi_i(X_1, X_2, t), \quad \tau = t. \tag{9}
\]

By using equation (9), a modified weighting function which is applied to equation (1) is given as

\[
\tilde{w}_i = w_i + \tilde{w}_i, \tag{10}
\]

where \( w_i \) is a standard weighting function which leads to the Galerkin formulation. \( \tilde{w}_i \) plays an important role in making the numerical dissipation and is defined by

\[
\tilde{w}_i = -\frac{1}{3} \beta \Delta \xi_i^2 w_i_{ij(j)} - \frac{1}{12} \alpha \Delta \xi_i^2 sgn(U_{ij}) w_i_{ijjj}, \tag{11}
\]

where \( \Delta \xi_i \) is a length of an element in the computational space, \( \beta \) and \( \alpha \) are the parameters, \( sgn(x) \) denotes the signum function, and \((.)_{ij} \) is the derivative with respect to \( \xi_j \). We do not employ the summation convention to the index \((j)\) with parentheses, and we define that the index \((j)\) is equal to the repeated index \( j \). Because we use the ALE method in the numerical algorithm, \( U_i \) in equation (11) denotes the contravariant velocity written as

\[
U_i = \xi_{,i}(u_j - \frac{\partial X_j}{\partial \tau}). \tag{12}
\]

By using equations (9) \( \sim \) (11), we can obtain the finite element equations of the governing equations in a flow field, and see Reference 1) for the detail methodology.

We now briefly show the finite element equations of the governing equations of fluid, which are developed in the computational space, below:

1. Navier-Stokes equations

\[
M \frac{\partial \mathbf{V}}{\partial \tau} + (\mathbf{N}^* + \mathbf{K}) \mathbf{V} - \mathbf{C} \mathbf{P} = \mathbf{F}, \tag{13}
\]

2. Continuity equation

\[
\mathbf{C}' \mathbf{V} = 0. \tag{14}
\]

3. Pressure Poisson equation

\[
\mathbf{S} \mathbf{P} = \mathbf{C}' \mathbf{\hat{V}} + \mathbf{G} \tag{15}
\]

\[
M \mathbf{\tilde{V}} = M \frac{\partial \mathbf{V}}{\partial \tau} + (\mathbf{N}^* + \mathbf{K}) \mathbf{V} - \mathbf{F}_r, \tag{16}
\]

where \( \mathbf{M} \) is the mass matrix, \( \mathbf{N}^* \) is the convection matrix, \( \mathbf{K} \) is the viscosity matrix, \( \mathbf{C} \) is the pressure gradient, \( \mathbf{F} \) is the force vector, \( \mathbf{S} \) is the matrix for the Laplacian operator and \( \mathbf{G} \) is the vector for the pressure on a boundary, \( \mathbf{F}_r \) denotes the force vector which is made from \( f_i \) and \( \tau_{ij} \).
Because two circular cylinders are freely vibrating under the fluid forces, the continuity condition of velocity and the pressure condition, on the surfaces of the cylinders, are given as follows:

\[ u_1 = \frac{dx_1}{dt}, \quad u_2 = \frac{dy_1}{dt} \]

(17)

\[ p_n = -n_1 \frac{\partial u_1}{\partial \tau} \]

(18)

The above equations can be simulated by the use of any numerical time integration method.

4. Numerical results

4.1 Computational model of two circular cylinders

We here describe a two-dimensional model of two circular cylinders in side-by-side arrangement as shown in Figure 1. The cylinders, which are put in two-dimensional domain, are elastically supported by spring-damper systems in the in-line and transverse directions, and then we present two-dimensional numerical results of two circular cylinders which vibrate under fluid forces.

The spacing ratio, \( T/D \), between the centers of two cylinders is set as \( T/D = 1.5 \) in our computations because of occurrence of strong interference for the fluid flow between the cylinders. The Reynolds number, \( Re \), is given by \( Re = 1000 \).

And also we give the density ratio \( \rho_c/\rho = 40.0 \) and the structure damping ratio \( h = 5\% \) for the vibrating circular cylinders, in order that we may obtain large amplitudes of the vibrating cylinders. Therefore, the Scruton number \( S_c \) is \( S_c = 2\rho_c/\rho \times 2\pi h \simeq 25.12 \). The reduced velocity \( Ur \) is set as \( Ur = 2 \sim 8 \).

Figure 2 shows the finite element mesh, and the total number of elements is 36310. The Crank-Nicolson method is used for numerical time integration of the finite element equations of fluid and the linear acceleration method is applied to the equations (7) and (8).

4.2 Aerodynamic characteristics of two vibrating circular cylinders

Figure 3 shows the time histories of the drag and lift coefficients, \( C_D \) and \( C_L \), of two stationary circular cylinders and two vibrating circular cylinders at \( Ur = 7 \), obtained by our two-dimensional computation. Because the fluid motion is simulated by computation in two-dimensional domain, the amplitudes of \( C_D \) and \( C_L \) are relatively larger\(^{27}\). Figure 4 shows time-averaged pressure distributions \( C_p \) on the surfaces of two stationary circular cylinders and two vibrating circular cylinders. When two circular cylinders are stationary states and are closely placed, difference between two pressures on the surfaces of the cylinders is clearly seen as shown in Figure 4 – 1) as well as experimental data\(^{9,9}\). However, when two circular cylinders are freely vibrating under fluid forces, the difference between two pressures is very small as shown in Figure 4 – 2).

Figure 5 shows vorticity contours around the vibrating circular cylinders in a range of approximately one period of the transverse displacements, \( y_1 \) and \( y_2 \), at \( Ur = 7 \). In the side-by-side

2-1) Mesh near two cylinders 2-2) Overall mesh in a computational domain

Fig.2 Finite element mesh.
3-1) Two stationary circular cylinders

3-2) Two vibrating circular cylinders at \( U_r = 7 \).

Fig. 3. Time histories of \( C_D \) and \( C_L \) of two stationary and two vibrating circular cylinders at \( T/D = 1.5 \) and \( Re = 1000 \).

4-1) Two stationary circular cylinders

4-2) Two vibrating circular cylinders at \( U_r = 7 \)

Fig. 4 Time-averaged pressure, \( C_p \), on two circular cylinders in side-by-side arrangement at \( T/D = 1.5 \) and \( Re = 1000 \).
arrangement of two stationary circular cylinders, Zdravkovich presented that the narrow and wide wakes are formed at the range of spacing of $1.2 < T/D < 2.2$. Such flow is called the biased gap flow. It is seen from Figure 5 that the biased gap flow appears behind two vibrating circular cylinders at any time of simulation as well as the experimental data by Zdravkovich. In addition, it is found that the variation of vortex shedding from two vibrating cylinders is dependent on spacing which varies between the cylinders.

Figure 6 shows time histories of the in-line and transverse displacements, $x_1, x_2, y_1$ and $y_2$, at $U_r = 4, 5$ and 7, respectively. When the value of $U_r$ is small, the displacements or the amplitudes of vibration are quite small. However, when the value of $U_r$ becomes gradually large, the displacements or the amplitudes become large with $U_r$. The profiles of vibrations of two cylinders are considerably complicated in all cases of $U_r$ because the variations of $C_D$ and $C_L$ of two circular cylinders due to the biased gap flow are very irregular as shown in Figure 3. In addition, the time histories of the in-line and transverse displacements from non-dimensional time $t = 280$ up to $t = 400$ at $U_r = 7$ are shown in Figure 7. We can find that the same or different profile of the period and the amplitude of $y_1$ and $y_2$ occurs alternately because of influence of the strong biased gap flow.

Fig.5 Variations of flow patterns around two vibrating circular cylinders in side-by-side arrangement at $T/D=1.5$, $Re=1000$ and $Ur=7$. 

5-1) Symbols

5-2) Flow patterns: Vorticity contours
Figure 8 shows power spectra of the transverse displacements, $y_1$ and $y_2$, in the range of $U_r = 2 \sim 7$. In the case of $U_r = 2$, there are four peaks, $fD/U_0$ ($f$ is frequency) $\simeq 0.11, 0.17, 0.27$ and 0.47, of the power spectra. The first, second and third $fD/U_0$ of peaks are near the Strouhal numbers $S_t$ of two stationary circular cylinders in side-by-side arrangement\(^a\), and the fourth

Fig. 6: Time histories of displacements, $x_1$, $x_2$, $y_1$ and $y_2$, of two circular cylinders in side-by-side arrangement at $T/D=1.5$ and $Re=1000$. 
7-1) Time histories of $x_1$ and $x_2$

7-2) Time histories of $y_1$ and $y_2$

Fig. 7 Time histories of displacements $x_1$, $x_2$, $y_1$ and $y_2$ at $T/D=1.5$, $Re=1000$ and $Ur=7$.

8-1) $Ur=2$

8-2) $Ur=3$

8-3) $Ur=4$

8-4) $Ur=5$

8-5) $Ur=6$

8-6) $Ur=7$

Fig. 8 Power spectra of displacements, $y_1$ and $y_2$.

9-1) In-line displacements, $x_1$ and $x_2$.

9-2) Transverse displacements, $y_1$ and $y_2$.

Fig. 9 Displacements of in-line and transverse directions, $x_1$, $x_2$, $y_1$ and $y_2$, at $T/D=1.5$ and $Re=1000$. 
$fD/U_0$ is approximately equal to $1/U_r$. However, when $U_r \geq 3$, the first, second and third peaks disappear from power spectra, and each circular cylinder is vibrating by the non-dimensional frequency given as approximately $fD/U_0 \approx 1/U_r$.

Figure 9 shows displacements of the in-line and transverse directions in the range of $U_r = 2 \sim 7$. In Figure 9, for example, the symbols $(x_1)_{max}$, $(y_1)_{mean}$, $(y_1)_{max}$ and $(y_1)_{rms}$ denote the maximum $x_1$, the mean $x_1$, the maximum amplitude of $y_1$ and the R.M.S of $y_1$, respectively. It is seen that all the displacements increase gradually with increment of $U_r$. In the case of a single circular cylinder, it is well-known from experimental data that the vortex-induced vibration appears when $U_r$ is nearly equal to $1/S_t$. However, in the case of two close circular cylinders in side-by-side arrangement, such vortex-induced vibration does not appear in the range of $U_r = 2 \sim 7$ because of strong interference by the biased gap flow.

On the other hand, there is any possibility of a collision of two circular cylinders when the cylinders are closely placed by side-by-side arrangement. Figure 10 shows the flow profile around two circular cylinders and the time histories of displacements of the cylinders which moved until just before the collision when $U_r = 8$. In this case of $U_r = 8$, two circular cylinders are vibrating by the same periods and the like amplitudes until just before the collision. The flow behind the cylinders is the like twin vortices.

10-1) Flow pattern around two cylinders which moved until just before a collision.

10-2) Time histories of transverse displacements, $y_1$ and $y_2$.

Fig.10 Collision of two circular cylinders, $T/D=1.5$, $Re=1000$ and $Ur=8$. 
5. Conclusions

We have presented numerical results for aerodynamic vibrations of two circular cylinders which are arranged in side-by-side of $T/D = 1.5$ between the centers of cylinders, by using both the third-order upwind finite element method and the ALE method for fluid computation. The basic equations we used in this paper are the incompressible Navier-Stokes equations and the equations of motion of two circular cylinders which are elastically supported by spring-damper systems to both in-line and transverse directions.

Each vibration to in-line and transverse directions of two circular cylinders denotes a very complicated profile because of occurrence of the biased gap flow. However time-averaged displacements or time-averaged amplitudes of two cylinders quite coincide at all reduced velocities. On the other hand, in the case of two close circular cylinders in side-by-side arrangement, the vortex-induced vibration does not appear in the range of $U_r = 2 \sim 7$ and the displacements of the cylinders show increment with the reduced velocity. In addition, in the case of side-by-side arrangement, we could show any possibility of a collision of the close circular cylinders at relatively large $U_r$.

By our numerical computation, we could show fully the results of aerodynamic vibrations of two elastically supported circular cylinders, which were located in a uniform flow, for some reduced velocities and the spacing ratio $T/D = 1.5$ between the centers of two cylinders.

References