Punching Shear Capacity of RC Slab by Applying Limit State Design Method

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In many cases where a reinforced concrete (RC) slab directly supports the wheel load of large-sized vehicles, the static strength of the slab is generally evaluated as punching shear capacity. Given the situation, this study proposes equations to be used to determine the punching shear capacity for the slab fabricated by applying the limit state design method, which were derived from the relationship between the experimental maximum load-carrying capacity and the reinforcement strain in the static loading experiment conducted using two types of the double reinforcement slab specimens. This is an attempt to perform a theoretical evaluation of such strength with a view to making some contributions to the establishment of a reasonable design method of RC slabs.

1. INTRODUCTION

Reinforced concrete (RC) slabs directly support the wheel loads of large vehicles. The static load-carrying capacity of such RC slabs is usually evaluated as punching shear capacity. Studies of punching shear capacity for RC slabs began with the experiments of Talbot in 1913, and many studies have been conducted since that time. These studies include reports of the results of punching shear capacity tests for slabs, studies of influencing factors, consideration of fracture mechanisms, proposed equations for calculating load-carrying capacity and so on. Many tests have been conducted by researchers in Japan as well, and fracture mechanisms and equations for calculating punching shear capacity have been proposed. Equations for calculating the punching shear capacity of RC slabs include equations proposed by Matsui et al., Kakuta, and the Japan Society of Civil Engineers. Previously, the authors focused on the Matsui equation for the RC slabs of highway bridges, and they proposed a punching shear dynamic model and a revised theoretical equation and verified conformance with test values. As a result, the authors applied the shear strength equation that they had proposed for the concrete compressive strength range of 20-80 N/mm² to the RC slab punching shear capacity equations of Matsui et al., and approximated the test values and the theoretical values.

In this study, the authors conducted static loading tests using double-reinforced for two types of RC slab test specimens in order to evaluate, from a theoretical perspective, the punching shear capacity of RC slabs to which the limit state design method had been applied, based on the relationship between maximum load-carrying capacity and reinforcement strain in the test. In addition, they proposed a punching shear capacity and a punching shear dynamic model for punching shear in the elastic region and plastic region and validated conformance with the test values, as a contribution to the effort to establish a rational design method for RC slabs.

2. MATERIALS USED IN TEST SPECIMENS, TEST SPECIMEN SIZE AND TEST METHOD

2.1 Materials used

The concrete used in the test specimens was ordinary Portland cement with coarse aggregate of max. 20 mm. The reinforcements were D10 and D13 of SD295A had been used for Type I. D10 of SD295 was used for Type II. Three

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>Concrete compressive strength (N/mm²)</th>
<th>Yield strength (N/mm²)</th>
<th>Tensile strength (N/mm²)</th>
<th>Young's modulus (kN/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-S-1, 2</td>
<td>30.0</td>
<td>365</td>
<td>510</td>
<td>200</td>
</tr>
<tr>
<td>I-S-3(D13)</td>
<td>27.0</td>
<td>371</td>
<td>506</td>
<td>200</td>
</tr>
<tr>
<td>II-S-1, 2</td>
<td>32.0</td>
<td>370</td>
<td>511</td>
<td>200</td>
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<tr>
<td>II-S-3, 4</td>
<td>35.0</td>
<td>368</td>
<td>516</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 1 Characteristics of materials.
for Type I, D10 of SD295 was used for Type II. Three specimens were for Type I: I-S-1, I-S-2; and I-S-3(D13) and four specimen for Type II: II-S-1, II-S-2, II-S-3 and II-S-4. Table 1 shows the mechanical properties of materials used in the test.

2.2 Test specimen size and arrangement
For the size and arrangement of the test specimens, in accordance with the provisions of the Specifications for Highway Bridges and Accompanying Commentary, the planned traffic volume of heavy vehicles per day in one direction was presumed to be 500 or less for the Type I specimens and 2000 or more for the Type II specimens, and a model half that size was used.

Isotropic test specimens of Type I with a total length of 1470 mm and span of 1200 mm were used, and double reinforcement was used. On the tension side, D10 reinforcements were placed at 100 mm intervals in the axial direction and at 120 mm intervals in the direction perpendicular to the axial direction, and an effective depth of 90 mm and 80 mm, respectively, was used. On the compression side, half the number of reinforcements on the tension side were provided. Moreover, specimen I-S-3(D13) had the same reinforcement arrangement as arrangement I-S-1 and I-S-2. Figure 1 (1) shows the size and placement of the reinforcements.

Furthermore, the Type II specimens had the same reinforcement arrangement as the Type I specimens, and double reinforcements were used. On the tension side, the D10 reinforcements were placed at 100 mm intervals in the axial direction and the direction perpendicular to the axial direction at effective depths of 105 mm and 95 mm, respectively. The compression side had half the number of reinforcements that were on the tension side. The RC slab test specimen was made a square plate with 4-sided simple support. Figure 1 (2) shows the size and arrangement of the reinforcements.

![Fig. 1 Dimensions of test specimens (unit in mm).](image)

2.3 Punching shear test method
As shown in Fig. 1, the punching shear tests conducted for the RC slabs involved placing a wheel load (250 × 40 mm) in the center of the slab span to conduct a static loading test. The load in the static loading test was a stage loading in which the load was increased by increments of 5.0 kN. Measurements were made of tension reinforcement strain and load.

3. EXPERIMENTAL PUNCHING SHEAR CAPACITY AND STRAIN

3.1 Punching shear capacity test value
Table 2 shows the test value for the load-carrying capacity of the RC slab used in the test. The mean values for the maximum load-carrying capacity in the static loading test were 174.3 kN for test specimens I-S-1 and I-S-2, and 196.0 kN for test specimen I-S-3(D13). This means that the load-carrying capacity of reinforcement D13 had increased by a factor of 1.15. The failure mode was punching shear failure for all of the Type I specimens. The mean values for maximum

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>Maximum load-carrying capacity (kN)</th>
<th>Average load-carrying capacity (kN)</th>
<th>Failure Modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-S-1(D10)</td>
<td>171.3</td>
<td>174.3</td>
<td>Punching shear failure</td>
</tr>
<tr>
<td>I-S-2(D10)</td>
<td>177.2</td>
<td></td>
<td>Punching shear failure</td>
</tr>
<tr>
<td>I-S-3(D13)</td>
<td>196.3</td>
<td>196.0</td>
<td>Punching shear failure</td>
</tr>
<tr>
<td>II-S-1(D10)</td>
<td>221.3</td>
<td>223.4</td>
<td>Punching shear failure</td>
</tr>
<tr>
<td>II-S-2(D10)</td>
<td>225.4</td>
<td></td>
<td>Punching shear failure</td>
</tr>
<tr>
<td>II-S-3(D10)</td>
<td>235.2</td>
<td>237.7</td>
<td>Punching shear failure</td>
</tr>
<tr>
<td>II-S-4(D10)</td>
<td>240.2</td>
<td></td>
<td>Punching shear failure</td>
</tr>
</tbody>
</table>
load-carrying capacity in the static loading test were 223.4 kN for test specimens II-S-1 and II-S-2, and 237.7 kN for test specimens II-S-3 and II-S-4. The difference in load-carrying capacity was due to the difference in the compressive strength of the concrete.

The fracture status of the RC slab in the static loading test was as follows: fractures spread at a 45° angle from the wheel load contact surface (25 cm × 4 cm) in the direction of the slab bottom, and stripping of the bottom concrete occurred. The fracture mode in the static loading test was punching shear fracture.

3.2 Relationship between load and strain

Figure 2 (1) and Figure 2 (2) show the strain for the main reinforcements in the center of the slab during the tests for the Type I and Type II specimens, respectively. Figure 2 (1) shows the tension reinforcements in the direction perpendicular to the axial direction. Figure 2 (2) shows the strain of the distribution reinforcements, i.e., the tension reinforcements in the axial direction. Figure 2 (3) shows the mean strain in the axial direction and the direction perpendicular to the axial direction. As shown by the material property values in Table 1, the yield strains of the tension reinforcements for the Type I specimens were $1825 \times 10^6$ for test specimens I-S-1 and I-S-2 and $1855 \times 10^6$ for test specimen I-S-3(D13). The yield strains of the tension reinforcements for the Type II specimens were $1850 \times 10^6$ for test specimens II-S-1 and II-S-2 and $1840 \times 10^6$ for test specimens II-S-3 and II-S-4. From the relationship between yield strain and load, a punching shear dynamic model for the ultimate limit state and a load-carrying capacity equation were proposed. Moreover, the strains of tensile strength on the reinforcements for the Type I specimens were $2550 \times 10^6$ for specimens I-S-1 and I-S-2 and $2530 \times 10^6$ for specimens I-S-3(D13). For the Type II specimens, the strain of tensile strength on the tension reinforcements was $2560 \times 10^6$ for test specimens II-S-1 and II-S-2 and $2580 \times 10^6$ for test specimens II-S-3 and II-S-4. From this strain relationship, a load-carrying capacity equation and a punching shear dynamic model for cases in which the tension strength is reached were proposed.

1) Direction perpendicular to the axial direction

2) Axial direction

3) Mean strain in axial direction and direction perpendicular to axial direction

Fig. 2 Strain in tensile reinforcement
(1) Direction perpendicular to the axial direction (main reinforcements)

Regarding the strain in the direction perpendicular to the axial direction, as Fig. 2 (1) shows, for the Type I specimens, the maximum values for the load at which the tension reinforcements yielded were 115 kN for the test specimen I-S-1 and 110 kN for test specimen I-S-2. The value was 130 kN for test specimen I-S-3(D13). Moreover, for the Type II, the maximum values for the load at which the tension reinforcements yield were 160 kN for the test specimen II-S-1 and 150 kN for test specimen II-S-2. The value was 160 kN for test specimens II-S-3 and II-S-4. The maximum load near the tensile strength was 205 kN and 200 kN for test specimens S-1 and S-2, respectively, and the value was 210 kN for test specimens S-3 and S4.

(2) Axial direction (distribution reinforcements)

Regarding the strain in the axial direction for the Type I specimens, as Fig. 2 (2) shows, the maximum values for the load which the tension reinforcement yielded were 110 kN and 115 kN for test specimens I-S-1 and I-S-2, respectively, and the value was 125 kN for test specimen I-S-3(D13). The maximum load near the tensile strength was 125 kN for test specimens I-S-1 and I-S-2 and 150 kN for test specimen I-S-3(D13). For the Type II specimens, the maximum values in the ultimate limit state were 145 kN for test specimen II-S-1 and 140 kN for test specimen II-S-2. The values were 155 kN and 150 kN for test specimen II-S-3 and II-S-4, respectively. The maximum loads near the tensile strength were 160 kN and 165 kN for test specimens II-S-1 and II-S-2, respectively, and 175 kN and 180 kN for test specimens II-S-3 and II-S-4, respectively. The tensile strain of the reinforcements was greater for the reinforcements placed in the axial direction.

(3) Mean strain in axial direction and direction perpendicular to axial direction

For theoretical strength equations using the limit state design method, the limit state should be established based on the strain in the axial direction and the direction perpendicular to the axial direction. However, since the analysis is unmanageable, the mean values for strain in the axial direction and the direction perpendicular to the axial direction are used to evaluate the theoretical strength by assuming the ultimate limit state for the limit state design method, the value near the tensile strength, and the fracture load. Accordingly, for the mean values for strain in the axial direction and the direction perpendicular to the axial direction, as shown in Fig. 2(3), the maximum values for the Type I specimen yield load were 115 kN for test specimens I-S-1 and I-S-2, and 125 kN for test specimen I-S-3(D13). The maximum loads near tensile strength were 135 kN for test specimens I-S-1 and I-S-2, and 160 kN for test specimen I-S-3(D13). For the Type II specimens, the maximum values for the yield load were 150 kN and 145 kN for test specimens II-S-1 and II-S-2, respectively, and 155 kN for test specimens II-S-3 and II-S-4. The maximum loads near tensile strength were 175 kN for test specimens II-S-1 and II-S-2, 180 kN for test specimen II-S-3, and 185 kN for test specimens II-S-4. The theoretical strength was studied using the relationship between these strain and load values to determine the conformance between the test values and the theoretical values. Table 3 shows the maximum load and the mean strain of the tension reinforcements in the axial direction and the direction perpendicular to the axial direction at the ultimate limit state and near the tensile strength.

4. REVISED THEORETICAL EQUATION FOR PUNCHING SHEAR CAPACITY

The punching shear dynamic models proposed by Matsui et al. consider the punching shear capacity provided by shear strength from the upper surface of the concrete to the neutral axis position, as well as the impact of the dowel effect from the tensile reinforcement position to the bottom surface, resulting from the fact that concrete stripping fracture extends over a wide area on the bottom surface of the RC slab. Figure 3 shows the punching shear dynamic model when a static load is applied. The punching shear capacity equation is provided as Eq. (1). The authors conducted single shear tests using square column test specimens in the concrete compressive strength range of 20-80 N/mm² and proposed an equation for estimating shear strength. Accordingly, the authors' proposed equation (1.a) was applied to the concrete shear strength equation in Eq(1). In addition, the Okamura equation (1.b)’ was applied to the concrete tensile strength equation.

\[
V_o = f_{uo} \left[ 2(a+2X_o)X_o +2(b+2X_o)X_o \right] \quad + \left[ 2(4C_o + 2d_o + b)C_o +2(a+2d_o)C_o \right] \quad \text{--- (1) } \\
\begin{align*}
  f_{uo} &= 0.688f_{c,28d}^{0.90}
  \quad \Rightarrow f_{uo} = 80 \text{ N/mm}^2 \\
  f_c &= 0.269f_{c,28d}^{0.63}
  \quad \Rightarrow f_c = 80 \text{ N/mm}^2
\end{align*}
\]

where

- \( V_o \): Punching shear capacity (N), \( a, b \): Length of sides of loaded slab in main reinforcement and distribution reinforcement direction (mm), \( X_o \): Position of neutral axis in main reinforcement and distribution reinforcement direction (mm), \( d_o, d_c \):
Effective depth of main tension reinforcements and distribution reinforcements (mm), \(C_m, C_e\): Covering of main tension, \(f_{vc}\): Shear strength of concrete (N/mm\(^2\)), \(f_t\): Tensile reinforcements and distribution reinforcements (mm), strength of concrete (N/mm\(^2\)), \(f':\) Compressive strength of concrete (N/mm\(^2\)).

A comparison by the authors of the test punching shear capacity using RC slabs and the theoretical strength determined using the Matsui equation reveals that, while the test value was slightly higher, it generally approximated the theoretical value.

5. PUNCHING SHEAR CAPACITY USING THE LIMIT STATE DESIGN METHOD

In order to identify the theoretical punching shear capacity, this study evaluated, in three stages as shown below, the punching shear capacity when the limit state design method is applied. First, an evaluation was conducted for the case in which the yield strength of the reinforcements was applied-in, other words, the maximum punching shear capacity in the elastic region. Since this is the maximum load-carrying capacity within the scope of Hooke's law, the value was evaluated as the punching shear capacity in ultimate limit state using the limit state design method. Next, the punching shear capacity in the plastic region was evaluated with regard to the maximum punching shear capacity near the fracture load in static load tests and the punching shear capacity when the main reinforcements reach the tensile strength. In addition, it was necessary to evaluate the maximum punching shear capacity near the fracture load in order to evaluate the fatigue durability and to identify the strength with respect to the bottom side reinforcement of the RC slab in terms of the load-sharing capacity of the RC slab section and the reinforcement section.

5.1 Equation for determining punching shear capacity in the ultimate limit state

To calculate the punching shear capacity in the ultimate limit state, first a study was conducted for the punching shear dynamic model near the maximum load in the ultimate limit state, from the relationship between load and strain in the tension reinforcements in the static loading test.

As shown in Table 3, the mean value for yield load when a static load was applied in the test was 147.5 kN. The mean value for maximum strength in the test was 223.4 kN. The ratio of these two values is 0.65, so the yield load was 65% of the maximum strength in the test. Accordingly, the punching shear dynamic model for the ultimate limit state was proposed considering approximately 65% of the punching shear dynamic model proposed by Matsui et al.

(1) Punching shear dynamic model showing the effect of concrete shear strength and load-carrying capacity equation

The punching shear dynamic model proposed by Matsui et al. (Fig. 3) assumes that the fracture on the surface receiving the concrete shear strength will be distributed at a 45° from the side length in the main reinforcement direction and distribution reinforcement direction of the loaded slab. In other words, the surface to which the shear strength \((f_{vc})\) is exerted in Eq. (1) was assessed as \(2(a + 2x)X + 2(b + 2x)X\). Accordingly, the surface receiving the concrete shear strength \((f_{vc})\) was studied as the area projecting area from the concrete surface to the neutral axis position, in the main reinforcement direction \((X)\) and the distribution reinforcement direction \((x)\). In addition, the scope in which the shear strength \((f_{vc})\) in Eq. (1) is exerted was calculated as being from the position of the loaded slab to the neutral axis. However, since in this study the limit state design method was used, the sizes of the stress equivalent block in the main reinforcement direction \((a)\) and the stress equivalent block in the distribution reinforcement direction \((a)\) were determined, and the mean value \((a=(a+a)/2\) was used. In calculating the sizes of the stress equivalent blocks in the main reinforcement direction and distribution reinforcement direction \((a, a)\), the yield strength of the reinforcements \((f_t)\) was used, and calculations were performed for each 100 cm section width.

Considering the above, Fig. 4 shows the punching shear model resulting from the effect of shear strength, and the punching shear capacity \(V_{pc}\) resulting from shear strength is provided as Eq. (2).

\[
V_{pc} = f_{vc} \left\{ 2(B+2a_x) \times 2(A \times x) \right\}
\]

\[
f_{vc} = 0.688 \sigma'_{c} \leq f'_{c} = 80 \text{N/mm}^2
\]

where

\(V_{pc}\): punching shear capacity (N), \(A, B\): side length in main reinforcement and distribution reinforcement direction of loaded slab (mm), \(a\): Mean value for size of stress equivalent blocks in main reinforcement direction \((a)\) and distribution reinforcement direction \((a)\) \((a=(a+a)/2)\), \(f_{vc}\): Shear strength of concrete (N/mm\(^2\))
1) Size of the stress equivalent block in the ultimate limit state: \( f \)

Figure 5 shows the size of the stress equivalent block of a double-reinforced rectangular section using the limit state design method. In addition, the stress equivalent blocks in the ultimate limit state \( (a_{u} = (a_{b} + a_{d})/2) \) is the mean value of the main reinforcement direction \( (a_{b}) \) and distribution reinforcement direction \( (a_{d}) \).

I: When compression reinforcements yield

\[
a = (A_{c} f_{c} - A_{d} f_{d})/d(0.85 f_{c} a_{b} b d)
\]  

(3)

II: When compression reinforcements do not yield

\[
a/d = m/2[p'(e_{c} - E/d) + \sqrt{(p' - p')(e_{c} - E/d)^2} + p'R/m'd'd'e_{c} - E'/f_{d}]
\]

\[
m = f_{c}/0.85f_{y}, \ p = A_{c}/b, \ p' = A_{d}/b d
\]

(4)

where

\( f_{c} \): Design compressive strength of concrete (in this study, as a result of approximation of the test strength, the concrete compressive strength \( f_{c} \) is used) (N/mm²), \( f_{c} \): Yield strength of reinforcements (N/mm²), \( f_{d} \): Yield strength of compression reinforcements (N/mm²), \( A_{c} \): Quantity of reinforcements on tension side (per slab span), \( A_{d} \): Quantity of reinforcements on compression side (per slab span), \( d \): Effective depth (= \( d_{e} \)), \( d' \): Distance from compression edge to compression reinforcement central, \( b \): Member width (slab span)

(2) Punching shear dynamic model considering the dowel effect and punching shear capacity equation

In the punching shear dynamic model proposed by Matsui et al, the scope in which the dowel effect is exerted is double the tension reinforcement covering \( (C_{t}) \) and distribution reinforcement covering \( (C_{d}) \) \( (2C_{t}, 2C_{d}) \). In this study, 1/2 of the surface \( (C_{s}, C_{t}) \) was considered. In addition, the main reinforcement covering \( (d_{e}) \) and the distribution reinforcement covering \( d' \) were calculated, and the mean value \( (C_{s} = (d_{e} + d')/2) \) was multiplied by the ratio of tension reinforcement yield strength and tensile strength \( (f_{d}/f_{y}) \) as a coefficient, and the result was used as the size effect for which the dowel effect is exerted. Accordingly, the size effect for which the dowel effect is exerted \( C_{s} \) is \( C_{s} = C_{s} 	imes f_{d}/f_{y} \) \( (C_{s} = (d_{e} + d')/2) \). Figure 6 shows the relationship between the tension reinforcement covering \( d_{e} \) and the distribution reinforcement covering. The punching shear dynamic model when this is considered is included in Fig. 4.

The equation used to calculate punching shear capacity that is affected by the concrete tensile strength because of the dowel effect is provided as Eq. (5).

\[
V_{o}= f_{c} \{2(2C_{s} + 2d_{e} + B)C_{s} + 2(A + 2d_{s})C_{d}\}
\]

\[
f_{c} = 0.269 f_{d}^{2/3} \leq f'_{c} = 80 N/mm^{2}
\]

(5)

where

\( A, B \): Side length in main reinforcement and distribution reinforcement direction of loaded slab (mm), \( C_{s} \): Size effect showing impact of dowel effect (= \( C_{s} = C_{s} \times f_{d}/f_{y} \)), \( C_{d} \): Mean value of main reinforcement covering \( (d_{e}) \) and covering in distribution reinforcement direction \( (d_{e} + d')/2 \), \( d_{e} \): Mean value of effective depth of main reinforcements \( (d_{e}) \) and effective depth in distribution reinforcement direction \( (d_{e} = (H-C_{s})) \), \( H \): Slab thickness (mm), \( f_{c} \): Compressive strength of concrete (N/mm²)

(3) Maximum punching shear capacity in ultimate limit state

The maximum punching shear capacity in the ultimate limit state \( V_{o} \) was derived by adding the punching shear capacity resulting from the effect of shear capacity \( (V_{o}) \) and the punching shear capacity resulting from the effect of the tensile strength of the concrete \( (V_{o}) \). The equation used to calculate the punching shear capacity using the ultimate limit state design method is provided as Eq. (6).

\[
V_{o} = V_{o} + V_{o} = f_{o} \{2(2h + 2a_{b})a_{b} + 2(A_{o}a_{b})\} + f_{o} \{2(2C_{s} + 2d_{e} + B)C_{s} + 2(A + 2d_{s})C_{d}\}
\]

\[
f_{o} = 0.688 f_{n}^{0.5} \leq f'_{o} = 80 N/mm^{2}
\]

\[
f_{c} = 0.269 f_{d}^{2/3} \leq f'_{c} = 80 N/mm^{2}
\]

(6)
Table 3 shows the ultimate punching shear capacity in the ultimate limit state calculated using Eq. (6).

5.2 punching shear capacity in the plastic region

With regard to the punching shear capacity in the plastic region, a punching shear dynamic model for the situation in which the main reinforcements are near reaching tensile strength and near fracture load was proposed, in addition to an equation for calculating load-carrying capacity.

1) Punching shear capacity near the tensile strength

The punching shear capacity near the tensile strength is provided as the punching shear dynamic model and load-carrying capacity near the point at which the strain of the tension reinforcements reaches the tensile strength.

1) Punching shear dynamic model from the effect of concrete shear strength and punching shear capacity equation

The punching shear dynamic model and load-carrying capacity equation from the effect of concrete shear strength were the same as for the ultimate state, in other words, Fig. 4 and Eq. (2). However, the size of the stress equivalent block was different. The tensile strength of the reinforcements $f_{t}$ was used as the mean value for the size of the stress equivalent block near the point at which tensile strength is reached. Figure 7 shows the punching shear dynamic model near the point at which the tension reinforcements reach the tensile strength. The punching shear capacity is provided as Eq. (7)

$$V_{pl} = f_{w0} \left\{ 2(B + 2a)\alpha_{w} + 2(A\times\alpha_{w}) \right\}$$

$$f_{w0} = 0.688f'_{w}^{	ext{6.50}} \leq 80N/mm^2$$

Size of stress equivalent block using the ultimate limit state design method: $a$

I : When compression reinforcements yield

$$a = (A_{v} - A_{h})d/(0.85f_{w0}b'd)$$

II : When compression reinforcements do not yield

$$a/d = m/2 \{p-p'(e_{w}E_{d}) + \sqrt{(p-p'(e_{w}E_{d})^2 + p'4\beta/m'd'd)} \}$$

$$m = f_{s}/0.85f_{w0}, p = A_{v}/b'd, p' = A_{h}/b'd$$

where,

$f_{t}$: Tensile strength of reinforcements ($N/mm^2$), $b$: Member width ($b = 100 f_{w0}a$)

2) Punching shear dynamic model and punching shear capacity equation that consider the dowel effect

For the distribution surface of the dowel effect (in other words, the punching shear capacity resulting from concrete tensile strength), the main tensile reinforcement covering ($C_{v}$) and the distribution reinforcement covering ($C_{h}$) were doubled ($2C_{v}$, $2C_{h}$), the same as in the punching shear dynamic model proposed by Matsui et al. Accordingly, the dynamic model proposed in this study doubles the size effect $C_{v}$ indicating the covering of the reinforcements in the axial direction and the direction perpendicular to the axial direction ($2C_{h}$). Moreover, since the size effect that indicates the dowel effect is near the tensile strength ($f_{d} + f_{t}/2f_{s}$) and that value was used as a coefficient. Accordingly, the size effect indicating the dowel effect is provided by $C_{v} = C_{v}(f_{d} + f_{t}/2f_{s})$. The punching shear dynamic model determined from the above is shown in Fig. 7. The punching shear capacity equation is provided as Eq. (10)

$$V_{pl} = f_{w0} \left\{ 2(4C_{v} + 2d_1 + B)C_{h} + 2(A + 2d_2)C_{v} \right\}$$

$$f_{w0} = 0.269f'_{w}^{2/3} \leq 80N/mm^2$$

where,

$A$, $B$: Length of sides of loaded slab in main reinforcement and distribution reinforcement direction (mm), $C_{v}$: Size effect indicating dowel effect ($=C_{v} = C_{v}(f_{d} + f_{t}/2f_{s})$, $C_{h}$: Mean value of main reinforcement covering ($d_{1i}$) and covering in the distribution reinforcement direction ($d_{2i}$), $d_{i}$: Mean value of effective depth of main reinforcements ($d_{i}$) and effective depth in distribution reinforcement direction ($d_{i}$) ($d_{i} = (H-C_{i})f_{w0}$), $C_{v}$: Compressive strength of concrete ($N/mm^2$),
3) Punching shear capacity near tensile strength

From the punching shear dynamic model shown in Fig. 7, the punching shear capacity near the tensile strength in the plastic region is provided by Eq. (11).

\[
V_o = V_{ao} + V_{ko}
\]

\[
f_o = f_{o} = \frac{2(2B+2a_0)k_o+2(4x_0)}{f_t} \leq f_t = 80 \text{N/mm}^2
\]

(11)

(11.a)

(11.b)

The ultimate punching shear capacity near the fracture load as calculated using Eq. (11) is shown in Table 3.

(2) Punching shear capacity near the fracture load

For the punching shear capacity near the fracture load, the punching shear dynamic model near the tensile strength as shown in Fig. 7 and the punching shear capacity equation shown in Eq. (11) were applied.

1) Punching shear dynamic model and punching shear capacity equation for punching shear resulting from the effect of concrete shear strength

The punching shear capacity near the fracture load was calculated using Eq. (11). In this case, as cracking will occur, the size of the stress equivalent block was calculated with the effective width \( b \), used when calculating the size of the stress equivalent block for the main reinforcements \( (a_r) \) and the distribution reinforcements \( (a_d) \), as the width \( b \) that takes into consideration the cracking width as a load distribution by multiplying width \( b \) by the ratio of the yield strength and tensile strength of the tension reinforcements \( (f_{o}/f_{t}) \) as a coefficient \( (b = 100 \times f_{o}/f_{t}) \). With regard to the quantity of reinforcements, the quantity per meter was used. The punching shear dynamic model was the same as the one shown in Fig. 7. Moreover, for the size of the stress equivalent block, Eq. (9) was used, as the compression reinforcement strain did not reach the yield strength near the fracture load. Accordingly, Eq. (7) was used for the punching shear dynamic model and punching shear capacity for the punching shear resulting from the effect of the shear strength of the concrete.

2) Punching shear dynamic model and punching shear capacity equation that consider the dowel effect

As the impact of the dowel effect is near the fracture load, the punching shear dynamic model proposed by Matsui et al. was used. Accordingly, the punching shear capacity was calculated using Eq. (10).

3) Punching shear capacity at fracture

The punching shear dynamic model near fracture load is shown in Fig. 7, and the punching shear capacity is provided by Eq. (11). From the above, Eq. (11) was used for the punching shear dynamic model and assessment equation for the maximum punching shear capacity near the fracture load.

6. THEORETICAL LOAD-CARRYING CAPACITY RESULTS

6.1 Ultimate punching shear capacity in the ultimate limit state

From Table 3, it can be seen that, from the relationship between load and mean strain for the main reinforcements provided in the axial direction and the direction perpendicular to the axial direction in the test specimens used in these tests, the maximum load in the ultimate limit state was 150 kN and 145 kN for test specimens S-1 and S-2, respectively, and 155 kN for test specimens S-3 and S-4. In contrast, the results calculated using the punching shear dynamic model (Fig. 5) and punching shear capacity equation (11) for the punching shear in the ultimate limit state were 146.4 kN for test specimens S-1 and S-2 and 150.6 kN for test specimens S-3 and S-4. This demonstrates that the values were generally approximated. Accordingly, the design can be made safer by incorporating material coefficients and member coefficients into the design punching shear capacity.

6.2 Punching shear capacity in the plastic region

(1) Punching shear capacity near tensile strength

From Table 3, it can be seen that the maximum load-carrying capacity near the point at which the tension reinforcements reach the tensile strength was 175 kN for test specimens S-1 and S-2, and 180 kN and 185 kN for test specimens S-3 and S-4, respectively. In contrast, the results calculated using the punching shear dynamic model (Fig. 7) and punching shear capacity equation (6) for the punching shear in the ultimate limit state were 173.0 kN for test specimens S-1 and S-2 and 179.4 kN for test specimens S-3 and S-4. If the mean strain and yield load of the tension reinforcements are compared, it can be seen that the values were generally approximated.

(2) Maximum load-carrying capacity near the fracture load

The punching shear capacity near the fracture load was 221.3 kN and 225.4 kN for test specimens S-1 and S-2, respectively, and 235.2 kN and 240.2 kN for test specimens S-3 and S-4, respectively. In contrast, the results calculated using the punching shear dynamic model (Fig. 7) and punching shear capacity equation (11) for the punching shear near the fracture load were 208.8 kN for test specimens S-1 and S-2 and 216.1 kN for test specimens S-3 and S-4, demonstrating that the values were generally approximated.
Table 3  Experimental maximum load-carrying capacity and theoretical load-carrying capacity

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Ultimate punching shear strength in the ultimate limit state</th>
<th>Punching shear strength in the plastic region</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yield load (kN)</td>
<td>Strain (x10⁶)</td>
</tr>
<tr>
<td>I-S-1(D10)</td>
<td>115</td>
<td>1824</td>
</tr>
<tr>
<td>I-S-2(D10)</td>
<td>115</td>
<td>1813</td>
</tr>
<tr>
<td>I-S-3(D10)</td>
<td>125</td>
<td>1787</td>
</tr>
<tr>
<td>II-S-1(D10)</td>
<td>150</td>
<td>1812</td>
</tr>
<tr>
<td>II-S-2(D10)</td>
<td>145</td>
<td>1826</td>
</tr>
<tr>
<td>II-S-3(D10)</td>
<td>155</td>
<td>1835</td>
</tr>
<tr>
<td>II-S-4(D10)</td>
<td>155</td>
<td>1827</td>
</tr>
</tbody>
</table>

7. CONCLUSIONS

An analysis was conducted for (a) the punching shear capacity of reinforced concrete (RC) slabs in static load tests and (b) the punching shear dynamic model for punching shear capacity and theoretical punching shear capacity equation in the elastic and plastic regions and near the fracture load when the limit state design method was used. The results of the analysis were as follows.

(1) The combined fracture mode in the static loading test was punching shear fracture spreading at a 45° angle from the wheel load loading surface.

(2) The punching shear capacity in the ultimate limit state when a static load was applied using the limit state design method approximated the load-carrying capacity in the test and theoretical punching shear capacity equation in the elastic and plastic regions for the two Type specimens modeled from the RC slab of Highway Bridge. The value calculated using the proposed punching shear dynamic model and the theoretical punching shear capacity equation to be clearly assessed.

(3) With regard to the punching shear capacity near the tensile strength as well, the load-carrying capacity in the test was approximated with the value calculated using the proposed punching shear dynamic model and the punching shear capacity equation.

(4) With regard to the punching shear capacity near the fracture load, a punching shear capacity equation was proposed with a revised value for the size effect used in the punching shear dynamic model and punching shear capacity equation proposed by Matsui et al. As the results, the test values of two type specimens and theoretical values were approximated. The limit state design method was suitable for the proposed punching shear dynamic model then the theoretical model that had been valued.

(5) In calculating the design punching shear capacity, the safety of the design can be increased by incorporating the material coefficients (for concrete and steel) and the member coefficients.

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