An Analytical Model of Electric Field by Plasma Actuator

Hiro YOSHIDA*, Takehiko SEGAWA*, Yoshinobu HOSHI* and Kwing So CHOI**

* National Institute of Advanced Industrial Science and Technology (AIST), Tsukuba, Ibaraki
** University of Nottingham, Nottinghamshire, United Kingdom

In order to understand the performance of the dielectric barrier discharge plasma actuator (DBD PA), analytical solutions of the electrostatic field by DBD PA were derived using the Laplace equation and the electric image method. Various electrodes shapes can be composed by the solutions. Main results are: 1) Electric field by a line electrode was composed using the solutions for the point electrodes. 2) Under a certain parameter condition, streamwise component of the electric fields seemed to be related qualitatively to the behavior of the streamwise jet velocity experimentally observed. 3) The relative permittivity of dielectric layer was supposed to have appreciable influence on the induced jet velocity suggesting that, under elevated temperatures, the jet induced by DBD PA may behave differently from that at room temperature.

1. INTRODUCTION

There are many passive and active flow control methods. Fin type solid vortex generator (SVG) is a typical passive device installed in various facilities.1) Since the SVG protrudes into the main stream, it is prone to cause extra drags.2) Moreover, the SVG sometimes lacks in the adaptability to flow changes. Thus a novel jet array actuator has been developed.3) It is called the functional jet vortex generator (FJVG) and has the equivalent effect to the SVG without protrusion. In the case of surface jets, however, a few apertures for issuing jets must be machined on the surface. The machining process and piping arrangement in the body are usually not easy task. Moreover, those apertures often generate resonant vibration of air in the apertures to emit noises.

Recently, dielectric barrier discharge plasma actuator (DBD PA) is drawing people’s attention greatly.4,5,6) The typical DBD PA consists of a thin dielectric layer sandwiched between two electrodes and high voltage AC power supply (Fig.1). As shown in Fig.1 the DBD PA has a very simple structure without any moving parts. It can be flash mounted on the surface of the body to control the flow around it. Thus it is very attractive to examine availability of DBD PA for various thermo-fluid systems. For this reason, there have been numerous reports on experimental4-8) and numerical9-11) studies on DBD PA. Cheng and Davidson gave enlightening discussion on the acceleration mechanism.12,13) Irrespective of those investigations, there are two main questions

![Figure 1 Schematic diagram of DBD PA (side view) and coordinate system.](image-url)
remain to be solved: 1. What is the principal mechanism of the plasma jet acceleration? 2. Are there any limitations to the induced jet velocity? In many cases, the jet velocities observed so far seem to be less than 10 m/sec. 13 In order to further understand the relationship between the electric and flow fields, it is helpful for us if we can easily estimate the electric fields for various electric conditions. Moreover, it is important to do the parametric survey for the influences of the electrode and dielectric layer configurations and its electric properties on the resultant electric fields. For such examination, the analytical solution of the electric fields of DBD PA is quite useful for giving us some insight into the phenomena.

In the most DBD PAs, AC voltage has been applied. However, the positive or the negative polarity states in AC application are almost the same as the corresponding polarity states in DC application. Moreover, it is known that the acceleration mechanism is basically the same in AC and DC application. 12, 13 Thus in this paper only the electrostatic field was considered. The exact solutions of the electrostatic fields generated by DBD PA were derived and, based on them, the influence of the geometrical condition of the electrodes and dielectric layer on the electric fields was discussed. In the actual phenomena in DBD PA, there must exist various charged particles near the electrodes and are very complicated. Even if they are very complicated, those are originated from and are supposed to be influenced in some extent by the initial configuration of the electrodes and the dielectric layer. In this sense the present model is the most fundamental one to be examined at first. The precise comparison between the simplest mode and the actual phenomena is not discussed here, but it should be done in future.

2. ACCELERATION OF IONS

As mentioned in the previous section, the available jet velocity of DBD PA does not exceed 10 m/sec. It has been not clarified whether or not the upper limit of the jet velocity exists. If there is a limit, what does bound the maximum jet velocity? In what follows, we roughly estimate the jet velocity. According to Loeb, 14 velocity of ions, \( v_i \), in an electric field, \( E \), is given as:

\[
  v_i = kE,
\]

where \( k \) ([cm/sec]/[volt/cm]) is the mobility of ion. Let us apply the momentum conservation law to the acceleration process of the fluid. First some part of the neutral gas is ionized by the electric field. The ionized ions are accelerated by the electric field and then repeat collisions with neutral particles to reach an equilibrium state with an averaged velocity, \( \bar{v} \). The equilibrium velocity, \( \bar{v} \), is obtained as follows:

\[
  \bar{v} = (n_i m_i v_i + n_m m_n v_n) / (n_i m_i + n_m m_n),
\]

where \( n_i \), \( m_i \), and \( v_i \) are the number density, the mass, and the velocity, respectively. Suffixes \( i \) and \( n \) denote the quantities related to the ion and the neutral particle, respectively. Introducing the degree of ionization, \( \alpha \), as eq.(3), \( \bar{v} \) is derived by using eqs.(1), (2) and (3) as eq.(4).

\[
  \alpha = n_i / (n_i + n_m) = n_i / N
\]

\[
  \bar{v} = \alpha v_i + (1 - \alpha) v_n = \alpha kE
\]

where we assume that

\[
  v_i \approx 0 \quad \text{and} \quad m_i \approx m_n
\]

If we take, as an example, the air temperature of 11,000 K, the degree of ionization is \( \alpha \approx 0.02 \). 15 As a whole, the acceleration takes place under room temperature. Thus, the mobility is given as \( k \approx 1.3 - 2.1 \) (cm/sec)/(volt/cm). 16 From those values, the averaged velocity is roughly estimated as \( \bar{v} \approx 1.3 - 2.1 \) m/sec. The estimation is different from that by Roth & Sherman’s, 25 m/sec. 17 But it is close to the experimental observations so far reported 5,6,7

Based on the experimental results by Forte et al., 7 the acceleration mechanism is supposed as following: As the applied voltage and its frequency increase, the degree of ionization increases to generate many ions. Those ions fiercely collide with gaseous particles. The collisions finally increase the induced velocity of the ambient air.

3. MODEL OF ELECTRIC FIELD

Figure 2 shows the coordinate system of the analytical model of the electrostatic field expressed by
two point charges (point electrodes). The z-axis directs from the back to the front of the page. A dielectric layer is sandwiched between the top, placed at \((x, y, z) = (0, a, 0)\), and the bottom, at \((b, -a, d, 0)\), point charges. In Fig.2, region 1 \((y \geq 0)\) and region 3 \((-a-d \geq y)\) are the air layers while region 2 \((0 > y > -a-d)\) is the dielectric layer. Electrodes with designated geometries are composed as a combination of the point electrodes.

3.1 Governing equations

The electric potential is given as a solution of the following Laplace equation:

\[
\nabla^2 \Phi_i = 0, \quad (6)
\]

where \(\Phi_i = \phi_i + \psi_i\). \(\phi_i\) and \(\psi_i\) are, respectively, the electric field by the positive and negative point charges. The suffix \(i\) represents quantities related to region \(i\), that is, \(i=1\) and \(2\) mean the air layer regions and \(i=3\) means the dielectric layer in Fig.2.

3.2 Boundary conditions

The boundary conditions are given as follows:

At \(y = 0\), \(\phi_i = \phi_1\) and \(\psi_i = \psi_2\)

\[
\varepsilon_1 \frac{\partial \phi_1}{\partial y} = \varepsilon_2 \frac{\partial \phi_2}{\partial y} \quad \text{and} \quad \varepsilon_1 \frac{\partial \psi_1}{\partial y} = \varepsilon_2 \frac{\partial \psi_2}{\partial y}
\]

At \(y = -d\), \(\phi_2 = \phi_3\) and \(\psi_2 = \psi_3\)

\[
\varepsilon_3 \frac{\partial \phi_2}{\partial y} = \varepsilon_1 \frac{\partial \phi_3}{\partial y} \quad \text{and} \quad \varepsilon_3 \frac{\partial \psi_2}{\partial y} = \varepsilon_1 \frac{\partial \psi_3}{\partial y}
\]

where \(\varepsilon_1\) and \(\varepsilon_2\) are, respectively, the relative permittivity of the air and that of the dielectric material.

3.3 Analytical solution

Based on the electric image method, the solutions of eqs.(6)-(10) are given as:

\[
\phi_1 = \frac{q}{4\pi \varepsilon_1} \left\{ \frac{1}{\sqrt{x^2 + (y + a)^2 + z^2}} + \frac{(\varepsilon_1 - \varepsilon_2)}{(\varepsilon_1 + \varepsilon_2)} \frac{1}{\sqrt{x^2 + (y - a)^2 + z^2}} \right\}
\]

\[
\phi_2 = \frac{q}{2\pi \varepsilon_1 \varepsilon_2} \frac{1}{\varepsilon_1} \frac{1}{\sqrt{x^2 + (y + a)^2 + z^2}}
\]

\[
\phi_3 = \frac{q}{4\pi \varepsilon_1} \left\{ \frac{1}{\sqrt{x^2 + (y + a)^2 + z^2}} + \frac{(\varepsilon_1 - \varepsilon_2)}{(\varepsilon_1 + \varepsilon_2)} \frac{1}{\sqrt{x^2 + (y + 2d - a)^2 + z^2}} \right\}
\]

\[
\psi_1 = -\frac{q}{4\pi \varepsilon_1} \left\{ \frac{1}{\sqrt{x^2 + (y + d + a)^2 + z^2}} + \frac{(\varepsilon_1 - \varepsilon_2)}{(\varepsilon_1 + \varepsilon_2)} \frac{1}{\sqrt{x^2 + (y - d - a)^2 + z^2}} \right\}
\]

\[
\psi_2 = \frac{q}{2\pi \varepsilon_1 \varepsilon_2} \frac{1}{\varepsilon_1} \frac{1}{\sqrt{x^2 + (y + d + a)^2 + z^2}}
\]

\[
\psi_3 = -\frac{q}{4\pi \varepsilon_1} \left\{ \frac{1}{\sqrt{x^2 + (y + d + a)^2 + z^2}} + \frac{(\varepsilon_1 - \varepsilon_2)}{(\varepsilon_1 + \varepsilon_2)} \frac{1}{\sqrt{x^2 + (y - d + a)^2 + z^2}} \right\}
\]
\[ \psi_s = \frac{-q}{4\pi \varepsilon_0} \left\{ \frac{1}{\sqrt{x^2 + (y + d + a)^2 + z^2}} \right\} + \frac{(\varepsilon_1 - \varepsilon_2)}{(\varepsilon_1 + \varepsilon_2) \sqrt{x^2 + (y + d - a)^2 + z^2}} \right\} \] (16)

where \( q \) is the electric charge. Those solutions reduce to the case of electric dipole when \( d = 0 \). The electric field vector \( E_i \) is defined:

\[ E_i = -\nabla \Phi_i \] (17)

In this study, we focused our attention on the patterns of the electric fields rather than their magnitude. Thus the left hand sides of eqs.(11)-(16) were normalized by \( (4\pi \varepsilon_0/q) \). As far as those solutions are concerned, the pattern of the electric fields with same electrode geometry, i.e. \( a \) and \( b \), and dielectric layer properties, i.e. \( d, \varepsilon_1 \) and \( \varepsilon_2 \), are similar irrespective of the magnitude of \( q \). However, in the more advanced analysis including the special electric charges, the solutions (11)-(16) are no more valid. Such case is beyond the range of the present study and should be examined in future.

4. RESULTS AND DISCUSSION

4.1 Approximation of line electrode

In order to compare qualitatively the analytical results with those of the other studies, we need to match the electrode geometry. If we refer to Forte et al.'s experimental results, the rectangular electrode must be represented by using the point electrodes. For simplicity, we consider a line electrode instead of the rectangular one. The general structures of the electric fields for the both electrodes are supposed to be similar to each other.

The solutions eqs.(11)-(16) are for the electric fields generated by a pair of point charges (electrodes) as shown in Fig.2. If the pairs of point charges are aligned along z-axis with a spacing \( dz \), those series of charges are assumed to describe the electric field by the line electrode (Fig.3). The x- and y-components of the electric field vectors along y-axis at \( x = 10 \text{ mm} \) and \( z = 0 \text{ mm} \) under 1- to 79-pairs of point charges are compared in Figs.4 and 5. The geometrical conditions are: The spacing between pairs of point charges is \( dz = 0.4 \text{ mm} \); the distance between the positive and negative point charges (electrode gap) is \( a = 8 \text{ mm} \); the stand off distance of the point charges from the dielectric layer surface is \( b = 0 \); and the thickness of the dielectric layer is \( d = 2 \text{ mm} \). The relative permittivity is set \( \varepsilon_2/\varepsilon_1 = 3 \), which is equivalent to that of PMMA. As can be seen in Figs.4 and 5, for the number of pairs above 50, both the x- and y-components seem to converge to the final values, which are supposed to be attained by the line electrodes. It is found that the peak positions in the electric fields are not changed irrespective of the number of pairs. In what follows, we adopt 79-pairs of point charges to compose the line electrode. The \( E_z \) is supposed to be closely related to the streamwise jet velocity, \( U_z \), induced by DBD PA.

4.2 Electric field near wall

In this section, the relative permittivity and the thickness of the dielectric layer are the same as in the previous section, i.e. \( \varepsilon_2/\varepsilon_1 = 3 \) and \( d = 2 \text{ mm} \). First we calculated the electric field near wall for various electrode gaps, \( a \). Figure 6 shows the electric field \( E_x \) and \( E_z \), observed at \( x = 10 \text{ mm} \) and \( z = 0 \). We can see that the electrode gap has significant influence on both components for the limited region \( 0 < y < 6 \text{ mm} \). That is, the cases \( a = 0 \) and \( 9 \text{ mm} \) bring, respectively, large negative and positive amplitude of \( E_z \). Moreover, the case \( a = 9 \text{ mm} \) results in striking oscillation in \( E_y \). Although the gap and the dielectric material are not clearly stated, Forte et al made precise measurement of the velocity field induced by DBD PA (see Fig.4 in Ref.7). Their experimental results indicate also that the jet acceleration is
remarkable within $0 < y < 6$ mm.

4.3 Influence of electrode gap on electric field

Figure 7 shows the maximum amplitudes of $E_x$ and $E_y$ at $x = 10$ mm and $z = 0$ as a function of the gap, $a$. It is seen that $E_x$ and $E_y$ have peaks at $a = 9$ mm and 10 mm, respectively. This suggests that there is an optimum electrode gap of DBD PA for the effective acceleration of the jet. Actually, in Fig. 6 of Ref 7, the jet velocity induced by DBD PA shows a peak at about $a = 5$ mm. Although the gap values are not the same in the present analysis and the experiment, both results indicate the existence of the optimum electrode gap for the acceleration.

4.4 Influence of permittivity on electric field

Generally, the relative permittivity of the dielectric material changes as the ambient temperature changes. For example, the permittivities of the silicon nitride (SN252, Kyocera) and the alumina (SSA-T, Nikkato) are roughly 10 and 9.3 at room temperature and 510 and 66 at 1,000°C, respectively. The

Figure 4 Electric field composed by pairs of point charges observed at $x = 10$ mm and $z = 0$ mm. Top: x-component $E_x$, bottom: y-component $E_y$. The spacing of the point charges along $z$-axis $dz = 0.4$ mm, the electrode gap $a$ = 8 mm, the stand off distance from the dielectric surface $b = 0$ mm, the thickness of the dielectric layer $d = 2$ mm, $\varepsilon_2/\varepsilon_1 = 3$ (PMMA).
permittivity was measured at 1 kHz frequency using the test circuit. Based on those permittivities measured, we chose the values of 3, 10, 30, and 60 to see the trend of the influence of the permittivity on the electric field. At the same time, we can see the influence of the ambient temperature through the permittivity. This is important because the authors also intend to develop DBD PA being used under the elevated temperatures aiming at the applications to gas turbines and heat exchangers. 6)

In Fig.8, we see that the dependency of $E_x$ on the gap drastically changes for $\varepsilon_2/\varepsilon_1 \geq 30$, that is, for $a > 10$ mm, the induced jet direction for the cases $\varepsilon_2/\varepsilon_1 = 3$ and 10 and the direction for the cases 30 and 60 may be opposite. While for $E_y$, except the case $\varepsilon_2/\varepsilon_1 = 10$, the dependency of $E_y$ on $a$ is similar to the cases $\varepsilon_2/\varepsilon_1 = 3, 30$ and 60. Those results imply that the ambient temperature may have appreciable effect on the induced jet.

![Convergence in $x$-comp](image1)

![Convergence in $y$-comp](image2)

Figure 5 Convergency of $x$- and $y$- components of the electric field by the point electrode pairs to those by the line electrodes as a function of number of the pairs. Top: $x$-component normalized by the positive largest value $E_x$ at $(x,y,z) = (10\text{ mm}, 2.2\text{ mm}, 0)$, bottom: $y$-component normalized by the negative largest value $-E_y$ at $(x,y,z) = (10\text{ mm}, 0.2\text{ mm}, 0)$. Other parameters are the same as in Fig.4.

5. CONCLUSIONS

In order to simulate the dielectric barrier discharge plasma actuator (DBD PA), a theoretical work has been done. The main results are summarized as follows:

1. The analytical solutions for DBD PA were derived for point charges (electrodes).
2. The electric field of line electrode was composed successfully using the solutions for the point
3. Within the present configuration of electrodes and dielectric layer, the streamwise component of the electric fields \( E_x \) seemed to be related qualitatively to the behavior of the streamwise jet velocity experimentally observed.

4. The relative permittivity of the dielectric layer was supposed to have appreciable influence on the induced jet velocity. This suggests that, under elevated temperatures, the jet induced by DBD PA may behave differently from that at room temperature.

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![Graphs showing electric field components](image)

\( E_x \)

\( E_y \)

Figure 6 Electric field for various electrode gaps observed at \( x = 10 \text{ mm} \) and \( z = 0 \). Top: \( x \)-component, bottom: \( y \)-component. \( \varepsilon_y/\varepsilon_1 = 3 \) (PMMA) and \( d = 2 \text{ mm} \).
Figure 7 Magnitude of $E_x$ (top) and $E_y$ (bottom) at $x = 10$ mm and $z = 0$ as a function of the electrode gap, $a$. $\varepsilon_2/\varepsilon_1 = 3$ (PMMA) and $d = 2$ mm.

Figure 8 Influence of the permittivity on $E_x$ (top) and $E_y$ (bottom) at $x = 10$ mm and $z = 0$ as a function of the electrode gap, $a$. The key $\varepsilon_2$ means $\varepsilon_2/\varepsilon_1$ and $d = 2$ mm.

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