Study on Numerical Simulation for Vibro-Acoustic Response of Spacecraft

Takashi TAKAHASHI, Keiichi MURAKAMI, Takashi Aoyama and Hideaki AISO

Institute of Space Technology and Aeronautics, Japan Aerospace Exploration Agency, Chofu, Tokyo

The limitations of the finite element method (FEM) mainly on numerical dispersion errors and on model dimension indicate that an alternative deterministic approach is necessary for the coupled vibro-acoustic analysis in the higher frequency range. Therefore, a novel approach called the wave base method (WBM) is discussed since it requires no meshes, and subsequently provides no dispersion errors and small model dimension. A 2-dimensional WBM code is then developed, and some examples are solved to show some advantageous features of the WBM. Moreover, a simple vibro-acoustic simulation on spacecraft is demonstrated, and local responses can be obtained due to its deterministic characteristic. It is also shown that the WBM has high potential for the vibro-acoustic analysis with the wide frequency range.

1. INTRODUCTION

Spacecraft are excited with mechanical vibrations via adapters between spacecraft and launch vehicles during the lift-off. It can be classified as sinusoidal vibrations with 5-100[Hz] and random vibrations with 20-2000[Hz] in ground tests. Spacecraft are also exposed to acoustic pressure transmitted through the air and payload fairings with the wide frequency range (typically 20-10000[Hz]). The sound pressure level (SPL) is generally maximum at the lift-off. Lightweight and large area structures, such as solar array panels and antenna dishes, respond to acoustic pressure. Some components with relatively high resonant frequencies, such as actuator and sensor units, are also sensitive to the acoustic environment. For large spacecraft (>1000 [kg]), acoustic pressure generates responses greater than those of mechanical random vibrations\(^1\). Therefore, it is quite important to predict the acoustic environment in order to develop reliable spacecraft. This paper focuses on vibro-acoustic analysis for spacecraft inside a payload fairing.

The statistical energy analysis (SEA)\(^2\) has been applied to predict vibro-acoustics of spacecraft and the International Space Station\(^3, 4\). In the high frequency range where individual vibration modes can no longer be distinguished, structural responses are generally quite sensitive to variations of material properties and dimensions. In such cases, probabilistic and statistical approaches, such as the SEA, are preferred to deterministic approaches. A simplified prediction equation for the changes in the SPL inside a fairing called ‘fill effect’ or ‘fill factor’ also has been given based on the SEA\(^3\). However, since the SEA deals with only spatially and frequency averaged quantities, it can not predict local responses such as resonance peaks. Furthermore, the lower the analysis frequency is, the worse the prediction accuracy becomes. On the other hand, the finite element method (FEM) is the almost only prediction method in the low frequency range typically less than 100[Hz] in spacecraft mechanical vibration analysis at the moment\(^9\). The FEM is a deterministic method which is capable of analyzing local
acoustic and structural responses. In spacecraft vibro-acoustic analysis, this method has also been applied to some simple structures\(^6\),\(^7\). However, the model size becomes larger, and numerical dispersion error due to the difference in wave numbers between the physical problem and numerically discretized problem becomes more dominant in the higher frequency range. It is known that there generally exists mid-frequency range where it is difficult to obtain accurate prediction results using both methods for the higher frequency range (e.g. SEA) and for lower frequency range (e.g. FEM).\(^9\) Note that the mid-frequency range may include resonance frequencies of spacecraft components which are quite critical for the spacecraft design as shown in Fig. 1.

The objective of this paper is to present the applicability of an alternative deterministic approach called the wave base method (WBM)\(^9\) to steady-state (frequency domain) vibro-acoustic analysis of spacecraft with the wide frequency range. Because of the novel approach, no practical numerical simulation codes have widely been spread yet. A 2-dimensional (2D) simulation code based on the WBM is thus developed, and some examples are used to verify the code. Moreover, a simple vibro-acoustic simulation on spacecraft is demonstrated.

2. WAVE BASED METHOD

2.1 Problem Definition

The WBM\(^9\) is a deterministic method for steady-state coupled vibro-acoustic analysis. It is based on an indirect Trefftz approach, and has overcome an ill-conditioned problem in a Trefftz formulation by defining a complete wave function set. The main feature of the method is that there are no numerical dispersion errors since the wave function set exactly satisfies the governing equations. Therefore, the method has high potential to predict vibro-acoustic responses with the wide frequency range.

This section introduces a theoretical description of the WBM for the 2D steady-state coupled interior vibro-acoustic problem. Fig. 2 shows a cavity filled with a fluid and surrounded with some boundary surfaces. First, the acoustic cavity must be decomposed into some convex subdomains due to a feature of the WBM described later. For simplicity, the acoustic cavity is composed of only two convex
subdomains $V_e$ ($e = 1, 2$). Each subdomain $V_e$ is assumed to have a circumscribing minimum rectangle with dimension $L_{x_e}$ and $L_{y_e}$. An acoustic boundary surface $\Omega_{ae}$ in $V_e$ may consist of six kinds of surfaces, i.e., $\Omega_{ae} = \Omega_{pe} \bigcup \Omega_{ve} \bigcup \Omega_{Ze} \bigcup \Omega_{ses} \bigcup \Omega_{cl2x} \bigcup \Omega_{cl2c}$. Surfaces $\Omega_{pe}$, $\Omega_{ve}$, and $\Omega_{Ze}$ are imposed pressure, normal velocity, and normal impedance boundary conditions (BCs), respectively. Surface $\Omega_{ses}$ consists of $n_{es}$ flat thin plates, and each surface $\Omega_{ses}$ ($s = 1, \ldots, n_{es}$) is assumed to be a plate (length $L_{as}$ and infinite wideness perpendicular to the paper) imposed some BCs at both edges. Surfaces $\Omega_{cl2}$ and $\Omega_{cl2c}$ are boundary surfaces between subdomain $V_1$ and $V_1$, and consist of $n_{12c}$ surfaces and $n_{12c}$ flat thin plates, respectively. Surface $\Omega_{cl2c}$ ($c = 1, \ldots, n_{12c}$) is imposed both pressure and normal velocity continuity conditions. For simplicity, surface $\Omega_{cl2c}$ ($s_e = 1, \ldots, n_{cl2c}$) is ignored in the derivation in this chapter, but is implemented in a numerical code used in the next chapter. Moreover, several external normal line forces $f_{esi}$ ($i = 1, \ldots, n_{esi}$) can be applied at local position $x'_{esi}$ on $\Omega_{ses}$. Furthermore, several external point sound sources $q_{es}$ ($i = 1, \ldots, n_{eq}$) can be placed at position $r_{eq}$ inside $V_e$. Both external excitations are assumed to be time-harmonic functions with angular frequency $\omega$.

![Fig. 2 2D steady-state coupled interior vibro-acoustic model.](image)

2.2 Governing Equations and Variable Expansions

The steady-state acoustic pressure $p_e$ at position $r_e$ in $V_e$ is governed by the Helmholtz equation:

$$\nabla^2 p_e(r_e) + k_e^2 p_e(r_e) = -j \rho_c \omega \sum_{i=1}^{n_{eq}} q_{es} \delta(r_e, r_{eq}), \quad r_e \in V_e,$$  

(1)

where $k_e$ is the acoustic wave number and $\delta$ is the Dirac delta function. In the WBM formulation, the acoustic pressure $p_e(r_e)$ is approximated by using an acoustic wave function set and a particular solution as follows.

$$p_e(r_e) \approx \hat{p}_e(r_e) = \sum_{i=1}^{n_{eq}} p_{em} \phi_{em}(r_e) + \hat{p}_{eq}(r_e),$$  

(2)

where $\phi_{em}(r_e)$ are the wave functions which exactly satisfy a homogeneous equation, and are normalized to relax the numerical poor condition. In order to guarantee the convergence, cavity domains
must be convex due to the feature of the wave functions, and the function must be truncated depending on the physical wave numbers and the dimensions of the bounding boxes. The function \( \hat{p}_{eq} (r_e) \) is the particular solution of the inhomogeneous equation (1).

On the other hand, steady-state plate normal displacement \( w_{es} \) at local position \( x_{es} \) on \( \Omega_{es} \) is governed by the plate bending equation, Kirchhoff equation:

\[
\frac{d^4 w_{es}(x_{es}')} {dx_{es}'^4} - k_{es}^2 w_{es}(x_{es}') = \frac{1}{D_{es}} \sum_{i=1}^{n_{e}} f_{esi} \delta(x_{es}', x_{esi}') + \frac{p_x(r_{esi})}{D_{es}},
\]

(3)

where \( k_{es} \) is the structural wave number, and \( D_{es} \) is bending stiffness of the plate. Then, the structural displacement \( w_{es}(x_{es}') \) is approximated in the WBM as follows:

\[
w_{es}(x_{es}') \approx \hat{w}_{es}(x_{es}') = \psi_{esi}^T(x_{es}') w_{esi} + \hat{w}_{esi}^T(x_{es}') p_x + \hat{\psi}_{esi} (x_{es}'),
\]

(4)

where the vector \( \psi_{esi}(x_{es}') \) consists of four structural wave functions \( \psi_{esi}(x_{es}') \) which exactly satisfy a homogeneous equation as follows:

\[
\psi_{esi}(x_{es}') = \exp(-j k_{esi} x_{es}')(i = 1, \ldots, 4).
\]

(5)

These functions are also normalized like acoustic ones. The function \( \hat{w}_{esi}(x_{es}') \) is a particular solution due to some external force terms in the inhomogeneous equation (5). The vector \( \hat{w}_{esi}(x_{es}') \) has an element \( \hat{\psi}_{esi}(x_{es}') \) which is also a particular solution due to an acoustic pressure term in equation (3) associated with the acoustic wave functions. The function \( \hat{w}_{esi}(x_{es}') \) is also a particular solution due to the acoustic pressure term in equation (3) related to the external sound sources.

From above definitions, we can see that all variable expansions (2) and (4) exactly satisfy the governing equations. This feature is quite important to predict vibro-acoustic responses accurately with the wide frequency range due to no numerical dispersion errors.

2.3. Weighted Residual Formulation and WBM model

In order to solve contribution coefficients \( p_{esi} \) and \( w_{esi} \) in expansions (2) and (4), the weighted residual method can be applied to acoustic BCs. That is, the BCs are satisfied approximately while the governing equations are satisfied exactly. As the Galerkin method in the FEM, a weighting function \( \tilde{p}_e \) can be expanded using the acoustic wave function set.

\[
\tilde{p}_e(r_e) = \sum_{i=1}^{n_{0}} \tilde{p}_{esi} \phi_{esi}(r_e) = \tilde{\phi}_{esi}^T(r_e) \tilde{p}_e = \tilde{\phi}_{esi}^T \phi_{esi}(r_e).
\]

(6)

Using the function \( \tilde{p}_e \), the weighted residual formulation can be derived as follows:

\[
\int_{\Omega_{e1}} \int_{\Omega_{e1}} \tilde{p}_e R_{e1} d\Omega + \int_{\Omega_{e1}} \tilde{p}_e R_{e2} d\Omega + \sum_{c=1}^{n_{esi}} \int_{\Omega_{esi}} \tilde{p}_e R_{esi} d\Omega + \sum_{s=1}^{n_{esi}} \int_{\Omega_{esi}} \tilde{p}_e R_{isi} d\Omega = 0
\]

(7)

\[
\int_{\Omega_{e2}} \int_{\Omega_{e2}} \tilde{p}_e R_{e2} d\Omega + \int_{\Omega_{e2}} \tilde{p}_e R_{e2} d\Omega + \sum_{c=1}^{n_{esi}} \int_{\Omega_{esi}} \tilde{p}_e R_{esi} d\Omega + \sum_{s=1}^{n_{esi}} \int_{\Omega_{esi}} \tilde{p}_e R_{isi} d\Omega = 0
\]

(8)
where $R_{e1}$, $R_{e2}$, $R_{e3}$, $R_{e4}$, and $R_{e5}$ are residual error functions of the BCs. The structural BCs on $\Omega_{\text{sat}}$ such as clamped, simply supported, free and symmetric BCs can also be derived using expansions (4).

Finally, the WBM model can be expressed as the matrix form:

$$
\begin{bmatrix}
A_{1a} & C_{1a} & O & O \\
C_{1a} & M_{1a} & O & N_{12} \\
O & O & A_{2a} & C_{2a} \\
O & N_{21} & C_{2a} & M_{2a}
\end{bmatrix}
\begin{bmatrix}
w_{1a} \\
p_{1a} \\
w_{2a} \\
p_{2a}
\end{bmatrix} =
\begin{bmatrix}
f_{1a} \\
f_{2a}
\end{bmatrix},
$$

(9)

where $C_{esa}$ and $C_{esa}$ explicitly present structural-acoustic coupling effects. The matrices $N_{12}$ and $N_{21}$ show subdomain coupling effects.

### 3. NUMERICAL EXAMPLES

Based on the theoretical description in the previous section, a 2D WBM code was implemented using MATLAB®. In this section, two examples are solved to verify this code, and to show some features of the WBM. Finally, a simple vibro-acoustic simulation on spacecraft is demonstrated.

#### 3.1 Convex domain and non-convex domain

As described in the previous section, non-convex acoustic domains must be divided into some convex subdomains in the WBM analysis. To show this effect in simulation results, a 2D coupled vibro-acoustic system as shown in the lower part of Fig.3 is considered.

![Structural and fluid displacements on the plate](image)

Fig. 3 Contour plots for the acoustic pressure fields in the lower part (a)-(c), and structural (line) and fluid (cross) normal displacements on the thin plate in the upper part.
An acoustic cavity (1.5[m] width and 1.0[m] height) filled with the air (density 1.225[kg/m³] and sound speed 340[m/s]) is surrounded by rigid walls except a thin aluminium plate (density 2790[kg/m³], thickness 0.002[m], Young modulus 70×10⁹[N/m²], Poisson ratio 0.3 and damping ratio 0.0) on the top, and has two rigid partitions with 0.5[m] height. The plate is clamped at both ends. The system is excited by a time-harmonic force with 1[N] amplitude and 200[Hz] frequency acted at local position x’ =0.75[m] on the plate. Three cases are considered by using (a) 1 non-convex subdomain, (b) 3 convex subdomains and (c) 4 convex subdomains. The case (b) is the same model as given in ref. 9. Figure 3 shows the structural and fluid normal displacements on the structural boundaries in the upper part, and contour plots for the acoustic pressure fields in the lower part. The number of wave functions (WFs) and quadrature points (QPs) used in the case (a) are 394 and 4320, respectively, but the BC is not still converged and it is clear that the acoustic pressure field is physically wrong. On the other hand, only 132 WFs and 1598 QPs are used in the case (b), and 154 WFs and 1899 QPs in the case (c) to satisfy the BCs. Furthermore, the same acoustic pressure fields are properly obtained in both cases (b) and (c). In addition, the results in the case (b) is exactly the same as those obtained in ref. 9, which means that the code is implemented correctly. Indeed, the case (b) is more efficient than the case (c), so the number of BCs and convex subdomains should be selected as small as possible.

3.2 Sound Transmission due to Structural Vibrations

A model shown in Fig. 4 is to demonstrate the sound transmission due to structural bendings. In this model, the acoustic cavity (2[m] width and 1[m] height) filled with the air (density 1.225[kg/m³] and sound speed 340[m/s]) has an aluminium thin plate (density 2790[kg/m³], thickness 0.002[m], Young modulus 70×10⁹[N/m²], Poisson ratio 0.3 and damping ratio 0.0) in the middle, the upper and lower walls are rigid, and an acoustic impedance BC is imposed on the right side. The flexible plate is clamped at both ends. The cavity is excited by a pressure BC expressed as a time-harmonic pressure with 100[Pa] amplitude and 1000[Hz] frequency on the left side. As can be seen, the model has the cavity-structure-cavity coupling. This is quite different from the cavity-structure coupling as shown in the previous example because the acoustic pressures and acoustic sources on both side of the structure can affect the structural responses, and the common structural displacement must be imposed as structural BCs in acoustic subdomains in both side of the structure. Figure 5 shows the simulation results using the model. A graph on the right side of the figure shows that the common structural BC is satisfied, so it indicates that the cavity-structure-cavity coupling can be solved using the WBM code.

![Fig. 4 Sound Transmission Model.](image)

Most texts on acoustics provide a simple prediction model on the sound transmission loss, called the mass law, which is generally used to evaluate the sound insulation effect. However, the structure is assumed to be infinitely large in the law even in the derivation on the coincidence effect by considering the plate bending. This assumption does not meet reality, so the simulation as given here should be done to obtain the accurate sound transmission loss.
Fig. 5  Sound transmission due to structural vibrations, showing contour plots for the acoustic pressure fields in two subdomains, and the structural (line) and fluid (cross for subdomain 1 and circle for subdomain 2) normal displacements.

3.3 Vibro-acoustic analysis for spacecraft

In order to examine structural-acoustic coupling effects in spacecraft vibro-acoustic analysis, a simple rigid spacecraft model (Case A) and a flexible spacecraft model (Case B) are built as shown in Fig. 6. In the Case B, the model is composed of 5 clamped thin flat plates. In this numerical example, the acoustic pressure based on the SPL given in ref. 11) is inputted on inside surfaces of a fairing as pressure BCs for simplicity. However, note that the SPL is actually the envelope level during launch and flight of the H-IIA (Japanese launch vehicle), and the acoustic pressure is assumed to be uniformly exposed to the spacecraft in the ground tests.

Fig. 6  Spacecraft model inside a fairing (the number is subdomain index).

Figures 7 and 8 illustrate some results of the steady-state vibro-acoustic analysis using the WBM code. Figure 7 shows variations of the structural acceleration PSD functions from 20[Hz] to 2000[Hz]. The PSD functions are generally the final outputs of the acoustic ground tests. Some resonant peaks can be observed even in higher frequency range because of the deterministic approach. Figure 8 illustrates the acoustic fields when the pressures on inside surfaces of a fairing are excited with (a) 20[Hz] and (b) 1000[Hz] frequencies as examples. Although the actual pressure field is the superimposition of each frequency response over the wide frequency range, the results in Fig. 8 can help to see that the local pressure fields are quite different due to structural vibrations, and therefore to understand the relationship between the acoustic field inside a fairing and spacecraft structural vibrations at each frequency.
Fig 7 Structural acceleration power spectral densities.

Fig 8 Acoustic pressure fields inside fairing (Case A and B).
4. CONCLUSIONS

In order to overcome the limitations of the FEM on numerical dispersion errors and on model dimension, the applicability of the WBM to steady-state vibro-acoustic analysis is investigated by showing some features of the method using the 2D code. It is demonstrated that local structural responses can be obtained due to its deterministic characteristic, and it is also shown that the coupled vibro-acoustic analysis can be performed easily and properly. All numerical results can be obtained by just setting all BC's without using any meshes. There is subsequently no dispersion error in the WBM formulation. Although the application of the WBM is limited to geometrically moderate complex systems, it can be stated that the WBM is a quite practical approach, and has high potential for the vibro-acoustic analysis with the wide frequency range. Further enhancements of the WBM such as a hybrid modelling between the WBM and the FEM to enlarge the application range remains as future work.

REFERENCES
6) J. Wijker, Mechanical vibrations in spacecraft design, (Springer, 2004).
10) http://www.mathworks.com/products/matlab/