Nonlinear Effect on the Plasma Blob Propagation in the Scrape-off Layer

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The 2D Hasegawa-Wakatani-type model equation is introduced to investigate the blob propagation in the scrape-off layer. The effect of the density nonlinearity on the blob propagation is investigated, focusing on the blob size effect. It is found that in the regime with the large blob size where the interchange instability is dominant, the theoretical estimation of the blob propagation velocity agrees well with the simulation results. However, in the regime with small blob size where the Kelvin-Helmholtz instability is dominant, it does not work well. This is because the blob breaks up into pieces in this regime. The center-of-mass velocity is estimated in this regime to understand the characteristic of the blob propagation.

1. INTRODUCTION

Plasma transport in the tokamak scrape-off layer (SOL) has attracted attention\(^1\),\(^2\). Recent experimental research indicates non-diffusive convective transport in the edge region, such as a 'plasma blob', which is a magnetic-field-aligned plasma filament that propagates in the radial direction\(^3\)\(^-\)\(^5\). It causes surface damage at the first wall, ejecting impurities into the core plasma\(^6\). Thus, research on blob transport in the SOL is crucial for the development of a thermonuclear fusion reactor such as ITER. The theoretical model for plasma blob propagation was proposed by S. I. Krasheninnikov\(^7\). The basic mechanism behind the rapid convection appears to be the following: any macroscopic clump of particles in a toroidal plasma tends to be polarized as a result of species-dependent drift due to \(\nabla \times B\) and curvature. In the core plasma with its closed flux surface, this charge separation is short circuited by free-streaming electrons. In the SOL, however, a finite potential can be supported on the open fields, due to sheath formation and resistivity at the end points. A poloidal electric field thus formed, coupled with the toroidal magnetic field, then leads to a rapid \(E \times B\) drift of the blob to the first wall. In the theoretical model, the blob is assumed to be a magnetic-field-aligned flux tube in the SOL, connected to the divertor plates at each end (Fig. 1). With this theoretical model, the single blob propagation is investigated by many researchers who use the 2D simulations\(^8\)\(^-\)\(^13\). However, there is less studies which take care of the blob propagation velocity. Larger radial velocity might cause more radial particle transport in the SOL, which contributes to the formation of 2nd SOL, so that it is important to investigate the blob propagation velocity. The simple picture of the single blob propagation velocity is investigated in the previous short paper\(^14\). In this paper, detailed characteristics during the radial propagation is discussed.

2. MODEL

The 2D Hasegawa-Wakatani-type model equation is introduced, based on the theoretical model in\(^15\). The vorticity equation and the continuity equation can be simplified by integrating along the magnetic field lines where all quantities except the current are assumed to be constant along \(B\). The current density at the sheath near the divertor plate is given by
\[ J_{\parallel \text{sh}} = e n \left( v \frac{\nu_{\text{th}, e}}{\sqrt{2\pi}} \exp \left[ -\frac{e \Phi}{T_e} \right] \right). \]  \tag{1}

where \( v = (T_e/m_i)^{1/2} \) is the ion acoustic speed and \( \nu_{\text{th}, e} = (T_e/m_i)^{1/2} \) is the electron thermal speed. \( n \) is the particle density and \( \Phi \) is the electrostatic potential. In this model, the quasi-neutrality condition is assumed, i.e., \( n_i = n_e = n \). Assuming that both ends of each field line terminate at grounded conducting plates, and taking the sheath boundary conditions into account, the vorticity equation and the continuity equation become

\[ \frac{\partial}{\partial t} \nabla^2 \Phi + \left[ \Phi, \nabla^2 \Phi \right] = \frac{1}{L_i} \frac{2eB^2}{m_i c_s} \left[ 1 - \nu \exp \left( -\frac{e \Phi}{T_e} \right) \right] + \frac{B}{m_i n_e} (\mathbf{b} \times \kappa) \cdot \nabla p, \] \tag{2}

\[ \frac{\partial n}{\partial t} + \left[ \Phi, n \right] = -n \frac{2c_s}{B} \left[ \frac{e \Phi}{T_e} \right] + n \frac{2}{B} \kappa \cdot (\nabla \Phi - \frac{T_e}{c} \nabla \ln n). \] \tag{3}

where \( \left[ f, g \right] = \mathbf{b} \cdot (\nabla \times f \times \nabla \times g) \) is the Poisson bracket, \( \mathbf{B} = B \mathbf{b} \) is the toroidal magnetic field and \( \kappa = \mathbf{b} \cdot \nabla \) is the magnetic curvature. \( L_i \) indicates the connection length of the magnetic field line in the SOL. \( \nu = (m_i/2\pi k_B)^{1/2} \) and \( p = n(T_e + T_i) \) is the total pressure. The normalizations are given as following: \( \Omega_i t \rightarrow t, \rho \rightarrow \rho, \kappa \rightarrow \kappa, c \Phi / T_e \rightarrow \Phi, n/n_i \rightarrow n, T_e/T_i \rightarrow T_e / T_i, \rho \rightarrow \rho / n_i, \) where \( \Omega_i = eB/m_i, \rho_i = \kappa / \Omega_i \) and \( n_i \) and \( T_i \) indicate the density and electron temperature at the separatrix, respectively. Since the plasma filaments are assumed to be uniform along the magnetic field line, a 2D slab geometry \( \{r, \phi \} \) is introduced, where \( r \) indicates the radial direction and \( \phi \) the poloidal direction. Expressing \( \Phi = \Phi_0 + \varphi \), where the Bohm potential \( \Phi_0 \) is defined by \( \Phi_0 = \Omega_i T_e / \ln \nu \), then we obtain

\[ \frac{\partial}{\partial t} \nabla^2 \varphi + \left[ \varphi, \nabla^2 \varphi \right] = \alpha \varphi - \frac{\beta \varphi}{n} \frac{\partial n}{\partial \nu} + \mu \nabla \varphi. \]  \tag{4}

\[ \frac{\partial n}{\partial t} + \left[ \varphi, n \right] = -\alpha \varphi + \beta \left( \frac{\partial \varphi}{\partial \nu} - \frac{\partial n}{\partial \nu} \right) + \nabla \varphi. \] \tag{5}

where the electron temperature is assumed to be constant for simplicity. The parameter \( \nu = 2R_0 / L_\parallel \) is a measure of the net parallel current into the divertor plates, and \( \beta = \sqrt{2} R / R \) is a measure of the strength of the curvature drift, where \( R \) is the major radius, \( L_\parallel = \omega R \) is the connection length of the magnetic field line in the SOL, and \( \omega \) is the safety factor. \( \mu \) and \( D \) are the ion viscosity and the diffusion coefficient, respectively.
The energy conservation relation in this system is written by

\[ \frac{\partial}{\partial t} \langle H \rangle = -\alpha \langle \varphi^2 \rangle - \alpha \langle \ln n \rangle - \mu \langle |\nabla \varphi|^2 \rangle - D \langle (1 - N) |\nabla N|^2 \rangle. \tag{6} \]

where \( H = |\nabla \varphi|^2 / 2 + N^2 / 2 \) with \( N = \ln n \) is the Hamiltonian, and the bracket implies the surface integral. This model conserves energy in the limit of \( \alpha \to 0, \mu \to 0 \) and \( D \to 0 \). The second term on the Right Hand Side (RHS) of Eq. (5) canceled with the second term on the RHS of Eq. (4), when the conserved quadratic energies are summed up. This energy conserving model is abbreviated as 'C-model'. We compare this model with two other models. In the second model, the density nonlinear terms are linearized and are approximated by \( \langle \beta n \rangle \partial m / \partial y \to \beta \partial m / \partial y \) in Eq. (4) and \( \beta n \partial \varphi / \partial y \to \beta \partial \varphi / \partial y \) in Eq. (5), then Eqs. (4) and (5) are modified by

\[ \frac{\partial}{\partial t} \nabla \varphi + \nabla \varphi = n \varphi - \beta \frac{\partial m}{\partial y} + \mu \nabla \varphi. \tag{7} \]

\[ \frac{\partial m}{\partial t} + |\varphi, n| = -\alpha m + \beta \left( \frac{\partial \varphi}{\partial y} - \frac{\partial m}{\partial y} \right) + D \nabla^2 n. \tag{8} \]

This model is abbreviated as 'L-model'. In this case, the Hamiltonian reduces to \( H = |\nabla \varphi|^2 / 2 + n^2 / 2 \). In the third model, the \( \beta \) term on the RHS of Eq. (5) is ignored, then Eqs. (4) and (5) are reduced to

\[ \frac{\partial}{\partial t} \nabla \varphi + \nabla \varphi = n \varphi - \beta \frac{\partial m}{\partial y} + \mu \nabla \varphi. \tag{9} \]

\[ \frac{\partial m}{\partial t} + |\varphi, n| = -\alpha m + D \nabla^2 n. \tag{10} \]

This simplified model is the same as that used by Aydemir\(^{10}\) and is abbreviated as 'S-model'. However, the energy conservation does not hold in this model. Eqs. (4) and (5) are more relevant model to compare with experimental observations.

The simulations are performed with the initial condition where the blob density distribution is assumed to be a 2D Gaussian:

\[ n_b \exp \left[ - (X/\delta_b)^2 - (Y/\delta_b)^2 \right] \]

where \( X = x - 2\delta_b \) and \( Y = L_y / 2 \). \( \delta_b \) indicates the initial blob size, and \( L_y \) is the system size in the \( y \) direction. The simulation parameters are set as \( \alpha = 3 \times 10^{-4}, \beta = 6 \times 10^{-4}, n_b/n_0 = 10 \) and \( \mu = D = 2 \times 10^{-3} \) where \( n_0 \) is the initial background density.

3. SIMULATION RESULTS

Figure 2 shows the time evolution of the peak radial position of a single blob, its velocity and the peak value of the poloidally-averaged density in the case with \( \delta_b = 5 \) for three models. The S-model agrees well with the C-model, while the L-model is quite different from the C-model. The density nonlinearity in Eq. (4) has a more dominant effect than the one in Eq. (5) on the phase of nonlinear evolution. Hence, we focus on the C-model and the S-model in the following. In the early stage of propagation, the peak velocity increases \( (0 < t \leq 100) \) for the C-model and the S-model and \( 0 < t \leq 200 \) for the L-model. In this phase, the peak position of the density shifts forward in the radial direction from the initial Gaussian shape, keeping the foot position. This phase is defined as 'initial phase'. In the initial phase, the density also increases, indicates the blob density concentrates at the peak position. After the initial phase, the blob spread out and the propagation velocity approaches to stationary \( (t \geq 200) \) for each model. This phase is defined as 'stationary phase'.

Figure 3 shows the density contour plots for a blob with \( \delta_b = 5 \) in the C-model and the S-model, respectively. The difference between the initial phase and the stationary phase is identified. As the initial condition, the blob density is given by the Gaussian shape \( (t = 0) \). In the initial phase, it concentrates to the front end of the blob \( (t \approx 600) \). Then, it is broken into pieces by the Kelvin-Helmholtz (K-H) instability \( (t \approx 19000) \). The difference between the C-model and the S-model is not remarkable in the contour plots,
Fig. 2  A time evolution of the peak location ($\nu$-coordinate), the peak velocity and the peak value of poloidally averaged density using three models. The length and time are normalized by $\rho_s$ and $\Omega_i$.

Fig. 3  Density contour plots of a blob with $\delta_b = 5$ propagation time evolution. The C-model is used in upper plots and the S-model in below plots. System size is $128 \times 128$. The length and time are normalized by $\rho_s$ and $\Omega_i$. Blobs break and spread with the Kelvin-Helmholtz instability.

however, the difference becomes noticeable in the time evolution of the electrostatic energy. Fig. 4 shows the time evolution of the electrostatic energy $|\nabla \psi|^2$ for a blob with $\delta_b = 5$ in the C-model and the S-model, respectively. In the linearly growing phase, both energy agree with each other. In the saturation phase, the difference becomes larger. The $\beta$ term in the Eq. (5) has less effect in the linearly growing phase. In the nonlinear phase, the fine structure appears so that $\partial \psi / \partial y \sim 1$ can be assumed. Thus, the $\beta$ term has the effect on the nonlinear evolution. In this phase, the symmetry in the $\nu$ direction is broken by the density nonlinearity.

In the following, the C-model is focused on. During the nonlinear evolution, the blobs are broken due to the instabilities. According to the reference\textsuperscript{10}, the K-H instability appears for a small size blob (due to the velocity shear accompanied by its rapid convection), on the other hand, the interchange instability appears for a large blob. The critical blob size, which categorizes the type of instability, is given as
Fig. 4  The time evolution of the electrostatic energy $|\nabla \varphi|^2$ for blob with $\delta_b = 5$ propagation in the C-model and the S-model.

Fig. 5  Density contour snap shots of blobs with $\delta_b = 10, 25, 40$ and 60 in the C-model. System sizes are $192 \times 192$, $256 \times 256$, $384 \times 384$ and $512 \times 512$ from the left picture. Blobs break and spread with the Kelvin-Helmholtz and interchange instability or propagate to the radial direction with keeping its coherent structure.

$\delta_{\text{crit}} = \rhoempt (L^2 / 2 \rho empt R)^{1/5}$. For our parameters, the dimensionless characteristic length is estimated as $\delta_{\text{crit}} \sim 15$ (in unit of $\rhoempt$). Figure 5 shows the density contour plots which indicates the characteristics of instabilities for various blob sizes. For the blob with $\delta_b = 10$, the K-H instability appears, and the evolution is similar as the one in Fig. 3. The larger blob with $\delta_b = 25$ propagates to the radial direction with keeping its coherent structure. For further larger blobs with $\delta_b = 40$ and 60, the Rayleigh-Taylor (R-T) fingers develop, which indicates the growth of the interchange instability. The number of the R-T fingers depends on the unstable wave number and the blob size. In the stationary phase, small blobs break up and spread due to K-H instability, as the results, the center-of-mass velocity becomes small. On the other hand, the peak velocity almost saturates in the stationary phase. Figure 6 shows the plot of the initial blob size and the peak velocity in the stationary phase. The solid line represents the results calculated by the theoretical blob velocity $V_s = (2 \beta / \alpha) / \delta_b$, which is derived assuming a traveling waveform\cite{10}. In the regime where the blob is subject to the K-H instability, the simulation results are quite different from the theoretical prediction. Blobs spread out a broader area, and the assumption of the traveling waveform is violated, then the velocity saturates at a value lower than the predicted one. On the other hand, the theoretical estimation of blob propagation velocity in the stationary phase agrees well with the simulation results, since the spreading is weak in the interchange regime. It is captured that propagation velocity is proportional to $\delta_b^{-2}$ in this regime. The critical size $\delta_{\text{crit}}$ depends on simulation conditions such as curvature and sheath resistivity and so on.
Fig. 6  Blob peak velocity in their stationary phase vs. initial blob size using the C-model. A log scale is used on the vertical and horizontal axes.

Fig. 7  Time evolutions of the propagation velocity of blobs with $\delta_b = 5, 25$ and $40$ estimated by center-of-mass.

So far, the peak location is used to estimate the blob velocity. The peak velocity is suitable to capture the blob characteristics in the interchange regime. However, it does not contain the information of the blob behavior as a whole. To take it into account, the center-of-mass motion is introduced. The coordinate of the center-of-mass is defined as $v_{CM}(t) = \int x \cdot n_{blob}(x) \, dx / \int n_{blob}(x) \, dx$ and the radial velocity of the center of mass is defined as $v_{CM} = \partial v_{CM} / \partial t$ where $n_{blob}(x)$ is the poloidally averaged blob radial density profile which is obtained from the difference between the system total density profile and the background density profile. Figure 7 shows the time evolution of $v_{CM}$ of the various blob sizes. It is found that the blob with $\delta_b = 5$ changes the propagation velocity dynamically. The velocity increases initially, in this phase, the blob density concentrates to the front end. The local minimum of the curve corresponds to the vortices generation and the upslope is related to the detachment of the vortices (see fig. 3). In the evolution of K-H instability, the blob breaks and spreads over. Hence, the velocity gradually decrease with the time evolution. The blob with $\delta_b = 25$ is less affected by instabilities, and the velocity increases gradually. For the blob with $\delta_b = 40$, the velocity is almost stationary till the growth of the interchange instability takes place. Once it appears, the velocity grows up with R-T fingers. From Fig. 7, it is seen that the decay time is roughly
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Fig. 8 Similar as Fig. 6, a blob with $\delta_b = 5$ peak velocity and the theoretical prediction are plotted. Red cross indicate the time trace of $v_{CM}$ versus $\sqrt{2\sigma}$.

Estimated as $t \sim 1500$ for the blob with $\delta_b = 5$ which is affected by K-H instability and as $t \sim 200$ for the blob with $\delta_b = 40$ which is affected by the interchange instability. During the decay time, the small blob propagates, up to the half scale of system size. On the other hand, the large blob is mainly dominated due to the interchange instability and remains at the local position.

Next, variance is introduced to estimate the spread of the blob quantitatively. The variance is defined as $\sigma^2(t) = \int (x - x_{CM})^2 n_{blob}(x) dx / \int n_{blob}(x) dx$ using the center-of-mass coordinate $x_{CM}$. Comparing the variance definition for the Gaussian distribution function and the blob initial density profile, it is shown that $\sqrt{2\sigma}$ corresponds to the effective blob size $r_b(t)$, where $r_b(t = 0) = \delta_b$. Figure 8 shows the time trace of $v_{CM}$ on the variance $\sqrt{2\sigma}$ for the blob with initial size $\delta_b = 5$. The peak velocity $v_{peak}$ for $\delta_b = 5$ and the theoretical value are also plotted. It is seen that according to the temporal evolution, the variance is getting large and the velocity approaches to the theoretical line although the assumption of the traveling waveform is not valid in this phase.

According to the paper, the blob size ensemble average is given by

$$\langle Q \rangle = \frac{\int_{r_b}^{r_{max}} d\delta_b f(\delta_b) Q}{\int_{r_b}^{r_{max}} d\delta_b f(\delta_b)}.$$  \hspace{1cm} (11)

where $f(\delta_b)$ is a size distribution of blobs and the lower bound of integration is cut off at the gyro-radius $\delta_b = 1$. The upper cutoff is determined by the requirement that the blob move fast enough before decaying to contribute to transport, i.e., $\delta_{max} = \left(\beta / \alpha^2\right)^{1/3}$. Then, the outward particle flux $\Gamma$ is estimated as

$$\Gamma(t) = \left(\frac{N(\delta_b)}{S(r_b(\delta_b, t))} v_{CM}(\delta_b, t)\right),$$ \hspace{1cm} (12)

where $N(\delta_b)$ is the total number of particles in the blob and $S(r_b(\delta_b, t))$ is the effective area of the spreading blob. The quantity $N$ is calculated from the initial blob density profile as $N(\delta_b) = \int \int n_{blob} \exp \left[-(X/\delta_b)^2 - (Y/\delta_b)^2 \right] dY dX$ and the effective area $S$ is approximated as $S(r_b) \approx \pi (2r_b)^2$. Fig. 8 shows $v_{CM}$ asymptotically approaches to the theoretical value. Using the asymptotic relation $r_b \sim (2\beta / \alpha) / v_{CM}$, we obtain

$$\Gamma \sim \frac{\alpha}{4\pi \beta} \left\langle N(\delta_b) v_{CM}^2(\delta_b) \right\rangle.$$ \hspace{1cm} (13)
This formula might be applicable to estimate blob size ensemble averaged flux.

SUMMARY

In summary, we investigated the motion of a single blob by solving the Hasagawa-Wakatani-type model equation. The model which conserves the energy in the dissipationless limit and that by Aydemir were compared. It was found that the density nonlinearity in the vorticity equation has a dominant effect on the blob propagation and that in the continuity equation causes the asymmetry in the poloidal direction. The difference between the two models was clearly observed in the energy evolution. The dependence of the blob size on the propagation velocity was also investigated. When the initial blob size is large, the propagation velocity scales as $\delta^{-2}$. In contrast, when the initial blob size is small, the blob is broken into the pieces due to the Kelvin-Helmholtz instability, which limits the blob velocity. Finally, we introduced the center-of-mass velocity and the variance to represent the time evolution of the blob size and velocity. Taking the blob size evolution into account, the center-of-mass velocity is comparable to the propagation velocity given by the traveling waveform assumption.

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