On a Singular Solution in Higgs Field (II)—The Structure of SM Higgs Boson

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The structure of SM Higgs boson is discussed by the set of top quark decay processes in electroweak and quark sectors with newly enlarged equation of motion (Non-Linear Klein-Gordon eq.). Their asymptotic behaviors are mathematically described on an infinitesimal hyperboloid of one sheet in multi dimensional space. Then mass value of top quark is calculated as 171.26(6) GeV/c² which is consistent with CDF/D0’s result by minimizing the theoretical top quark mass. And from the difference between the value by assuming that SM Higgs boson is a virtual bound state of top quark-pair ((t̄t)*) itself with the mass formula obtained by requirement of minimal mass production as 171.26(6)/√2 = 121.10(3) GeV/c², and the recently obtained theoretical mass value of SM Higgs boson (120.611 GeV/c²), it is expected that SM Higgs boson is to be a composite scalar meson after emitting one photon from the (t̄t)* which will be a vector meson through radiative decay. Finally, a structure of SM Higgs boson mass which is composed of all spin 0 mesons’ masses, is proposed. Where the truncated-Octahedron mass structure is recursively (doubly) seen.

1. INTRODUCTION

The mass of top quark and the structure of SM Higgs boson are still open problems which have not been fully clarified theoretically yet, although they must be related each other from the viewpoint of that they have (or seem to have) both huge masses. So until now several efforts have been done. In preceding paper 2), the author developed the Mass triangle method by which the mass formula of SM Higgs boson and its mass value were calculated, and also it was expected that SM Higgs particle would be a composite boson. Therefore we shall hereafter try to solve the next problems above by applying the technique of our method.

2. FORMULATION AND THE RESULT

2.1 LAGLANDIAN DENSITY OF GAUGE FIELD AND EOM OF HIGGS FIELD

Since we later treat top quark decay processes in electroweak and quark sectors to which weak boson and quark are produced, we start with the Lagrangian 3) of SU(2) × U(1) invariant gauge field after spontaneous symmetry breaking and with unitary condition to make a gauge invariant formulation of the theory: -

\[ \mathcal{L} = \psi_L^\mu \left[ \partial_\mu - ig(W^+_\mu S_+ + W^-_\mu S_- + W^3_\mu S_3) \right] \psi_L - \frac{1}{4} \left[ \bar{\psi}_L \phi M_D D_R + \bar{\psi}_L \bar{\phi} M_U U_R + h.c. \right] + \mathcal{L}_{\text{Higgs}}, \]

(1)

where, \( \psi_{L,R} = \frac{1}{2} \left( 1 \pm \gamma_5 \right) \left( \begin{array}{c} U \\ D \end{array} \right) \), \( \gamma_5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3 \), \( \gamma^\mu, \mu = 0 - 3 : \text{Dirac's } \gamma \text{ matrix} \)
\[ U = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad D = \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad S_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad S_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad S_3 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \]

\[ \frac{1}{2} Y_L = \begin{pmatrix} 1/6 & 0 \\ 0 & 1/6 \end{pmatrix}, \quad \frac{1}{2} Y_R = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix}; \quad \text{where} \quad 0 = 3 \times 3 \text{ zero matrix}, \quad 1 = 3 \times 3 \text{ unit matrix} \]

\[ \mathcal{L}_{\text{Higgs}} = (D^\mu \phi) \Gamma (D_\mu \phi) - \lambda \left( \frac{\phi^\dagger \phi}{2 \lambda} \right)^2 \frac{1}{4} \left( \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + \frac{1}{4} \epsilon^{abc} W^b_\mu W^c_\nu \right)^2 - \frac{1}{4} \left( \partial_\mu B_\nu - \partial_\nu B_\mu \right)^2, \quad (2) \]

where, \( D^\mu \equiv \partial^\mu - ig W^a_\mu T^a - ig' B_\mu Y/2; \mu = 0 \sim 3. \)

\( W^a_\mu, B_\mu: \) gauge fields which belong to \( SU(2), U(1) \) respectively

\( T^a = (1/2) \tau^a: a = 1 \sim 3, \tau^a: 2 \times 2 \) Hermite matrices which have same form of Pauli matrices

\( g, g': \) gauge coupling constants of \( SU(2), U(1) \) respectively

\( Y = 1, \) for complex scalar field \( \phi \)

\( f^{abc}: \) structure constant of the Lie group \( SU(2) \)

\[ v = \sqrt{\mu^2/2 \lambda}, \quad \lambda: \text{self-coupling constant of } \phi \]

\[ W^z_\mu = \frac{1}{\sqrt{2}} (W^1_\mu \pm i W^2_\mu), \quad \phi^2 = 2 \lambda v^2, \quad G = \sqrt{g^2 + g'^2} \]

Then mass term in eq.(1) of 3rd term is written as

\[ \mathcal{L}_{\text{mass}} = -\left(1 + \frac{\phi}{v}\right) \left[ D_L M_D D_R + U_M U_R + h.c. \right] = -\left(1 + \frac{\phi}{v}\right) \left[ D_L \Lambda D_R + U_M U_R + h.c. \right] \]

\[ = -\left(1 + \frac{\phi}{v}\right) \left\{ \begin{pmatrix} m_d & 0 \\ 0 & m_i \end{pmatrix}, \quad \Lambda = \begin{pmatrix} m_s & 0 \\ m_s & m_i \end{pmatrix} \right\}. \]

Then after we apply Euler-Lagrange equation onto eq.(1):

\[ \frac{d}{dx_\mu} \left( \frac{\partial L}{\partial (\partial \phi/\partial x_\mu)} \right) - \frac{\partial L}{\partial \phi} = 0, \]

we get the EOM of the field as a nonlinear Klein-Gordon equation regarding \( \phi : \)

\[ \lambda \phi^3 + 3 \lambda \phi \phi^2 + \left( \frac{\partial^2}{\partial t^2} - \nabla^2 \right) + \left\{ \phi_m^2 - \frac{1}{2} g^2 W^+_{\mu} W^{-\mu} - \frac{1}{4} G^2 \left( Z_\mu \right)^2 \right\} \phi^2 \]

\[ - \left\{ g M_W W^+_{\mu} W^{-\mu} + \frac{1}{2} GM_Z \left( Z_\mu \right)^2 \right\} + \frac{1}{v} \left\{ m_{b_i} b_i + m_{\tau_i} \tau_i \right\} + \frac{1}{v} \left\{ m_d \bar{d} d + m_u \bar{u} u + m_s \bar{s} s \right\} = 0. \]

2.2 DERIVATION OF TOP QUARK MASS FORMULA

2.2.1 HIGGS MASS FORMULA IN ELECTROWEAK AND QUARK SECTORS

According to eq.(3), we also take an asymptotic solution via its Lorentz invariant form near singular solution of the field at "vacuum expectation value" (vev: \( \phi = 0 \)) where Higgs scalar field has been extended, to extract an information about SM Higgs boson mass from eq.(5) (NLKGB) so that

\[ \phi - \varepsilon^2 v, \quad (\varepsilon \to 0). \]
By inserting eq (6) into eq (5) and using Higgs mass definition, etc. of eq (2), we are able to have a Higgs mass formula in these sectors:

$$m^2 = 2 \left( \frac{\left( W_\mu \cdot W^\mu \right)}{(2 \varepsilon_\lambda / g)^2} - m_{q(i(bad))}^2 \frac{\left( \bar{q}_b q_b \right)}{(2 \varepsilon_\lambda / \sqrt{2} G_F)^2} \right)$$

$$+ \left( \frac{G_F}{2 \varepsilon_\lambda / G} \right)^2 - 2 m_{q(j)}^2 \frac{\left( \bar{q}_b q_b \right)}{(2 \varepsilon_\lambda / \sqrt{2} G_F)^2} \right) - 2 m_{t_j}^2 \left( \frac{\sqrt{u_i}}{(2 \varepsilon_\lambda / \sqrt{2} G_F)^2} \right)^2,$$

where $G_F = 1 / (\sqrt{2} v^2)$: Fermi constant. Here we introduced an infinitesimal Grassmann number $\varepsilon_\lambda$ as $\varepsilon_\lambda = \varepsilon, \varepsilon_\lambda^2 = 0$ because of its nilpotent property. And, $\bar{q}_b q_b = \bar{b}b + \bar{s}s + \bar{d}d, \bar{q}_c q_c = \bar{c}c + \bar{u}u$. Therefore eq (6) is elegantly rewritten as

$$\varphi(\varepsilon_\lambda) = 0.$$ (8)

After all, we now could have an asymptotic behavior of eq (7) for eq (8).

2.2.2 TOP QUARK MASS FORMULA

It will be understood that first and second terms in right-hand side of eq (7) are both related to decay of top quark to weak boson and to other quark. While third term corresponds to decay to another sector of top quark. Then we shall hereafter focus on former decay in which its behavior is described on an infinitesimal hyperboloid of one sheet in multi dimensional space with $(\sqrt{w^* \cdot w^*}, \sqrt{\bar{b}b}, \sqrt{\bar{s}s}, \sqrt{\bar{d}d}, \sqrt{Z^- \cdot Z^-}, \sqrt{\bar{c}c}, \sqrt{\bar{u}u})$ coordinates as shown in Fig. A1 of APPENDIX-A. Since dimensions of two terms in right-side of eq (7) are both square of mass, we can put as

$$c_\mu \cdot \left( W_\mu \cdot W^\mu \right)_0^2 \left( 2 \varepsilon_\lambda / G \right)^2 = m^2, \quad c_i \cdot m_{b_i} \left( \sqrt{\bar{b}b} \right)_0^2 \left( 2 \varepsilon_\lambda / \sqrt{2} G_F \right)^2 = m^2,$$

$$c_s \cdot m_s \left( \sqrt{\bar{s}s} \right)_0^2 \left( 2 \varepsilon_\lambda / \sqrt{2} G_F \right)^2 = m^2, \quad c_d \cdot m_d \left( \sqrt{\bar{d}d} \right)_0^2 \left( 2 \varepsilon_\lambda / \sqrt{2} G_F \right)^2 = m^2,$$

$$c_Z \cdot \left( Z^- \cdot Z^- \right)_0^2 \left( 2 \varepsilon_\lambda / G \right)^2 = m^2, \quad c_c \cdot m_c \left( \sqrt{\bar{c}c} \right)_0^2 \left( 2 \varepsilon_\lambda / \sqrt{2} G_F \right)^2 = m^2,$$

$$c_u \cdot m_u \left( \sqrt{\bar{u}u} \right)_0^2 \left( 2 \varepsilon_\lambda / \sqrt{2} G_F \right)^2 = m^2$$

at certain point (designated by 0) in respective field. So eq (9) are understood as it fixes a coordinate of certain point respectively on the hyperboloid curve in the first quadrant in $(\sqrt{w^* \cdot w^*}, \sqrt{\bar{b}b})$- plane, etc. Then let us write a top quark mass formula with $r_0 \ (0 < r_0 < 1)$ as decay-mode probability parameter for $m_t$:

$$m_t = n C_t r_0^{k_1} (1 - r_0)^{k_2} m_t \equiv n C_t r_0^{k_1} (1 - r_0)^{k_2} m_{t(bad)} + n C_t r_0^{k_1} (1 - r_0)^{k_2} m_{t(z(t))}$$

$$= \frac{m_{t(bad)}}{(1 - r_0)} m_{t(z(t))},$$

where we assumed that the probability of $m_t$-decay process obeys binomial distribution of being k-times in n-trials (-particles), producing masses of particles in virtual states with $m_{t(bad)}$ and $m_{t(z(t))}$:

$$m_{t(bad)} = \sqrt{c_{\mu} \left( 2 m_{t(bad)}^2 - 2 m_{t(bad)} m_{t(z(t))} + 2 m_{t(z(t))} \right) / c_{\mu}}$$

$$m_{t(z(t))} = \sqrt{c_{\mu} \left( 2 m_{t(z(t))}^2 - 2 m_{t(z(t))} m_{t(bad)} + 2 m_{t(bad)} \right) / c_{\mu}}$$

from eqs. (7), (9), and we let,$$ c_{\mu} / c_t \equiv \kappa_{t(bad)} (c_{t(bad)} / c_t) \equiv 1 / k_{t(bad)}$$

$$c_z / c_t \equiv \kappa_{t(z(t))} (c_{t(z(t))} / c_t) \equiv 1 / k_{t(z(t))}.$$ (12)
It is important that we should group each mass of weak boson together with its relevant quarks’ masses in closed form, since we here assumed the decaying virtual states such as \((\alpha \cdot m_{W(bad)} + \beta \cdot m_{Z(uu)})\) during decay processes of \(t \to bW^\prime\), etc. and \(t \to cZ\), etc. Furthermore we may write as \(^6\)

\[m_{W(bad)} = M_{W(bad)} y_{W(bad)}, \quad m_{Z(uu)} = M_{Z(uu)} y_{Z(uu)}\]

where \(c\): velocity of light; \(u_{W(bad)}, u_{Z(uu)}\): velocity of produced-mass

\[M_{W(bad)} = M_{Z(uu)} = \text{rest produced-mass from top quark.}\]

By inserting eqs.(11),(12) and (13) to eq.(10), and to maintain Lorentz invariance, we are able to write as

\[m_t = \left\{ M_{W(bad)} (y_{W(bad)} / r_0 + M_{Z(uu)} / (1-r_0)) / \{ M_{W(bad)} / r_0 + M_{Z(uu)} / (1-r_0) \} \right\} y_t\]

\(= M_t y_t, \quad \text{where} \ y_{W(bad)} = y_{Z(uu)} = y_t, \ \text{and} \)

\[M_{W(bad)} = \sqrt{k_{bad}^2 \{ 2m_{W}^2 - 2\kappa_{bad} \kappa_{uu} \kappa_{uu} \} (m_{b}^2 + m_{u}^2 + m_{d}^2)}, \quad M_{Z(uu)} = \sqrt{k_{uu}^2 \{ m_{b}^2 - 2\kappa_{uu} \kappa_{uu} \kappa_{uu} \} (m_{c}^2 + m_{u}^2)}.\]

Then we will get values of coefficients in eq.(15) by comparing the Mass triangles shown in Figs.B1(a), (b) and Figs.B2(a), (b) of APPENDIX-B as,

\[k_{bad} = 1/(2\sqrt{2}), \quad \kappa_{bad} = 1; \quad k_{uu} = 1/2, \quad \kappa_{uu} = 1/\sqrt{2}.\]

Therefore we finally obtain top quark mass value by minimizing the theoretical mass value through differentiation regarding decay-mode parameter \(r_0\) on eq.(14).

\[\frac{dM_t}{dr_0} = 0, \quad \therefore r_0 = -\frac{\Delta m^2}{\sqrt{\Delta m^2 - (\Delta m^2)^2}} = \frac{1}{\sqrt{\frac{M_1^2 - M_2^2}{2} - \frac{M_3^2 - M_4^2}{2}}} = 171.26(6) \text{ [GeV/c}^2\text{]} \]

and inserting eq.(17) to eq.(14) with eqs.(15) and (16) to get stationary mass formula;

\[M_t = (1/2) \left\{ \left( \frac{m_{b}^2 - m_{d}^2}{2} \right)^{1/4} + \left( \frac{m_{c}^2 - m_{u}^2}{2} \right)^{1/4} \right\}^2 = 171.26(6) \text{ [GeV/c}^2\text{]} \]

which is consistent with CDF/D0’s experimental result \(171.2\pm2.1 \text{ GeV/c}^2\). Where we used the value of 4.68 \([\text{GeV/c}^2]\) for b-quark mass.\(^7\) It is noteworthy that eq.(10) is symmetrical between \(m_{W(bad)}\) and \(m_{Z(uu)}\) regarding \(r_0\) because two terms in \(\{\}\) of eq.(18) have an equal power form \(1/4\).

2.3 THE STRUCTURE OF SM HIGGS BOSON

Let us look again at eq.(7) of which the first and second terms in right-hand side both appear also to describe processes of producing top quark with different point of view. So we shall write eq.(7) as

\[m_{\nu}^2 \equiv \left( c_1 m_t, \sqrt{m_t^2 + m_r^2} \right) \left( 2\varepsilon_{1a} / \sqrt{2\varepsilon_{1b} G_F} \right)^2.\]

Then if we let;

\[\left( c_1 m_t, \frac{\sqrt{m_t^2 + m_r^2}}{\sqrt{2\varepsilon_{1a} / \sqrt{2\varepsilon_{1b} G_F}}} \right)^2 = m_r^2 = m_{\nu}^2, \quad \left( c_2 m_t, \frac{\sqrt{m_t^2 + m_r^2}}{\sqrt{2\varepsilon_{1a} / \sqrt{2\varepsilon_{1b} G_F}}} \right)^2 = m_r^2,\]

we can rewrite eq.(19),

\[m_{\nu}^2 = \left( \frac{2m_r^2}{c_1} + \frac{2m_r^2}{c_2} \right) = \sqrt{2k^2 \left( 1 + \kappa^2 \right)} \cdot m_r, \quad \text{where} \ c_1 = \kappa^2 c_2 = 1/k^2.\]

Thus we are able to write from Lorentz invariance as well as Sec.2.2,

\[M_{\nu} \equiv M_{\nu} = \sqrt{2k^2 \left( 1 + \kappa^2 \right)} \cdot M_r.\]

By comparing Mass triangles shown in Fig.1(a) and (b) below we get,

\[M_{\nu} = \eta_t \cdot M_t \cos \theta + \eta_t \cdot M_t \sin \theta, \quad \text{where} \ \eta_t + \eta_t = 1\]
and \( \eta_t = k' k^2, \eta_t = k' \sqrt{2} \); \( k' = \sqrt{1/(\sqrt{2}(1+\kappa))} \). (24)

Then by inserting eq.(24) to eq.(22), \( M_{\eta^*}(\kappa) = \frac{\sqrt{1+\kappa^2}}{1+\kappa} M_t \). (25)

And by differentiating eq.(25) regarding \( \kappa \) to compute stationary value, we finally get
\[
\frac{dM_{\eta^*}(\kappa)}{d\kappa} = 0, \quad \kappa = 1 \quad \Rightarrow \quad M_{\eta^*} = M_t/\sqrt{2}.
\] (26)

So we will have the mass of a virtual bound top quark-pair \( (\bar{t}t)^* \) by assuming that the virtual bound top quark-pair is to be SM Higgs boson. Actually, we can see that right-hand side of eq.(21) such as \( '1(1+1^2)' \) in square root describes a binding process of \( \text{two} \) top quarks, namely, one top quark-pair when \( \kappa = 1 \). From eqs.(18) and (26),
\[
M_{(\bar{t}t)^*} = M_{\eta^*} = M_t/\sqrt{2} = 121.10(3) \text{[GeV/c}^2\text{]}.
\] (27)

Then there is a mass difference because theoretical SM Higgs mass value \( ^2 \) will be 120.611 GeV/c\(^2\);
\[
\therefore \Delta M = M_{(\bar{t}t)^*} - M_{\eta^*} = 0.49(2) \text{[GeV/c}^2\text{]}.
\] (28)

This value is a little smaller than masses of \( K^{\pm,0} \) mesons, and is smaller than mass of \( \eta_0 \) meson. Therefore it is expected that SM Higgs boson is to be a composite scalar meson after emitting one photon from the virtual bound top quark-pair: \( (\bar{t}t)^* \) which will be a vector meson and has zero charge. Namely,
\[
\therefore \ (\bar{t}t)^* \rightarrow \gamma H^0.
\] (29)

As shown in Fig.2, our theoretical mass value of bound top quark-pair looks to be on the line of experimental mass values of other bound quark-pairs of spin 1 (while \( \pi \) meson's spin is 0). In addition from eq.(27), the mass of \( (\bar{t}t)^* \) is ametry regarding \( M_w \) and \( M_z \) while the mass of \( H^0 \) is symmetry.\(^2\) And a short discussion for the physical interpretation of eqs.(27),(28),(29) is given in APPENDIX-C.

![Top Quark Mass Triangles](image1)

![Bq̄q̄ - Meson Mass](image2)

3. FOR THE BASIC STRUCTURE OF SM HIGGS BOSON MASS

The spin of SM Higgs boson is, as known, 0 (zero). So let us consider that SM Higgs boson mass is constructed basically by heavy mesons' masses of all spin 0, such as

\[
\begin{pmatrix} B_s^+ B_s^- \end{pmatrix}, \begin{pmatrix} B_c^+ B_c^- \end{pmatrix}, \begin{pmatrix} D_s^+ D_s^- \end{pmatrix}.
\] (30)

Then it is expected that they will form a polyhedron which is composed of planes of hexagon, in space. And, 'effective' (non-overlapped) number of planes of hexagon should be
\[ n_{\text{eff.}} \equiv \text{Int} \left\{ \frac{M_{\text{H}^0(\text{theory})}}{M \left[ B_s^0 \bar{B}_s^0 \left( B_c^+ B_c^- \right) \left( D_s^+ D_s^- \right) \right]} \right\} = 4. \] (31)

We will see that eq.(31) is satisfied, in all polyhedron, only by truncated-Octahedron\(^9\),\(^10\) (tr-O) shown in Fig.3, on which two hexagons and one square gather at each twenty-four vertexes.

Fig.3 truncated-Octahedron (tr-O)

And we think that the residue of mass from eq.(31) will form, inside the tr-O, meson masses at central position and of surrounding cloud. Actually by comparing SM Higgs boson mass values between experimental\(^7\) and theoretical\(^2\) ones along with this model, we have two phases of SM Higgs boson mass which appear relevant according to the heavy meson (\(\eta_c\) or three-\(\eta_c\)'s) at the central position of tr-O to be able to interact with W\(^\pm\) or Z gauge particle before acquiring own mass as

\[ M_{\text{H}^0(\eta_c, \text{exp. val})} \equiv \text{Int} \left\{ \sum_{M_1} \left[ 3\eta_c, 18\pi^+ + 4 \left( B_s^0 \bar{B}_s^0 \right) \left( B_c^+ B_c^- \right) \left( D_s^+ D_s^- \right) \right] \right\} \]

= 120.610 GeV/c\(^2\),

\[ M_{\text{H}^0(3\eta_c, \text{exp. val})} \equiv \text{Int} \left\{ \sum_{M_1} \left[ 3\eta_c, 10\pi^+ \pi^- + 4 \left( B_s^0 \bar{B}_s^0 \right) \left( B_c^+ B_c^- \right) \left( D_s^+ D_s^- \right) \right] \right\} \]

= 120.612 GeV/c\(^2\),

where, \( M_{\text{H}^0(\text{theor. with } M_w, M_z)} = 120.611 \text{ GeV/c}\(^2\). \) (from ref.2) (34)

The developed outer shell ('basic structure') is shown in Fig.4.

Fig.4 Developed outer shell of tr-O
Moreover we will see that the central mesons are both surrounded by cloud of \( \pi \) mesons\(^{11}\) and by shell of heavy mesons\(^{12}\), and similar mass structure (tr-O) is seen again in the \( \eta \) and the three-\( \eta \)'s themselves. Because their masses are described with same manner of eqs.\((32),(33),(34)\), in accordance between experimental\(^{7}\) and our theoretical values as,

\[
M_{\eta_0(\text{theor.)}} = \sum_{M_s} \left[ 2\eta_0, 15\pi^0 + 2 \left\{ \left( K^- K^+ \right) \left( \pi^+ \pi^- \right) \left( \pi^- \pi^+ \right) \right\} + \left( K^0 \bar{K}^0 \right) \left( \pi^+ \pi^- \right) \left( \pi^- \pi^+ \right) \right\} = 9300.3 \text{ MeV/c}^2,
\]

where, \( M_{\eta_0(\text{theor.)}} = 9300 \pm 20 \pm 20 \text{ MeV/c}^2 \) (35)

\[
M_{3\eta_0(\text{theor.)}} = \sum_{M_s} \left[ 3\eta_0, 4\pi^+ \pi^- + 2 \left\{ \left( K^- K^+ \right) \left( \pi^+ \pi^+ \right) \left( \pi^- \pi^- \right) \right\} + \left( K^0 \bar{K}^0 \right) \left( \pi^+ \pi^- \right) \left( \pi^- \pi^+ \right) \right\} = 8940.0 \text{ MeV/c}^2,
\]

where, \( M_{3\eta_0(\text{theor.)}} = 3 \times \left( 2980.3 \pm 1.2 \right) = 8940.9 \pm 3.6 \text{ MeV/c}^2 \) (37)

Here each central meson is \( 2\eta_0 \) or \( 3\eta_0 \). Therefore, we understand that antiparticle of SM Higgs boson is also SM Higgs boson itself. Developed inner shell (inner -‘basic structure’) is shown in Fig.5. And in Fig.6 we describe a conceptual model for spin and parity of SM Higgs boson from above discussion.

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**Fig.5  Developed inner shell of tr-O**

**Fig.6  Conceptual model for spin and parity of SM Higgs boson**
4. CONCLUDING REMARKS

So far, we have derived a formula and shown the mass value of top quark and of bound top quark-pair, then explained relevant structure of SM Higgs particle that it is to be a composite scalar meson whose mass composes a recursive truncated-octahedron structure of all spin 0, pseudo-scalar meson (parity: -1)'s masses in two phases. The parity of SM Higgs boson (+1) will be come from its recursive (double) structure. The result is to be expected to examine under the forthcoming experiments. And, next problem would be to explain Higgs Mechanism itself from the structure (geometry) above, which is now proceeded to study.

5. APPENDICES

-A: Infinitesimal Hyperboloid in Multi-dimensional Space

We have seen that top quark to weak boson and to other quark decay was described on an infinitesimal hyperboloid of one sheet in multi dimensional space with \((\sqrt{w',w''}, \sqrt{bb}, \sqrt{ss}, \sqrt{dd}, \sqrt{zz'}, \sqrt{cc}, \sqrt{uu'})\) coordinates as explained in Sec.2.2.2. Its behavior is shown in Fig.A1, only for \((\sqrt{w',w''}, \sqrt{zz'}, \sqrt{bb'})\)-coordinates from technical reason of difficulty to describe. Similar figure of infinitesimal hyperboloid may be shown also for \(\sqrt{ss}, \sqrt{dd}, \sqrt{cc}\) and \(\sqrt{uu'}\)-coordinate respectively both instead of \(\sqrt{bb'}\)-coordinate above. On the plane of \((\sqrt{w',w''}, \sqrt{zz'}, \sqrt{cc})\)-coordinates there is a micro elliptic mass curve which has been shown and is equivalent one in ref. 1, around origin (0).

![Fig.A1 Behavior of SM Higgs and top quark in an infinitesimal hyperboloid of one sheet](image)

-B: Mass Triangles for Top Quark

Since we have seen that top quark and virtual-W(bsd)'s, -Z(cu)' particles should have both equal \(\gamma\), to maintain Lorentz invariance as eq.(14), we shall hereafter discuss with their rest masses, dropping out \(\gamma\)'s from their relativistic masses in top quark triangles below, etc. with remembering eq.(13).

From eq.(15) we can describe mass triangles for virtual W(bsd)' and Z(cu)'

![Fig.B1 Mass Triangles for W(bsd)' and Z(cu)'](image)
particles as shown in Figs.B1(a) and (b) respectively.

Here as numbers of both W boson and \((b + s + d)\)-quarks should be equal by referring top quark decay to weak boson and other quarks, we write \(N'\) as total numbers of particles;

\[
\frac{n_{W'} + n_{(b+s+d)}}{N'} = \frac{1}{2} + \frac{1}{2} = 1.
\]

Also for Z boson and \((c + u)\)-quarks,

\[
\frac{n_{Z} + n_{(c+u)}}{N^*} = \frac{1}{2} + \frac{1}{2} = 1.
\]

From Figs.B1(a) and (b), it will be understood that each mass quantity of virtual W(bsd) and Z(cu), to which \(\cos \theta\) is multiplied respectively, and each mass quantity of other sides to which \(\sin \theta\) is multiplied respectively, contribute to hypotenuses (resultant masses). Therefore we can generally write these processes,

\[
M_{W(bsd)} \cos \theta + n_{(b+s+d)} M_{(b+s+d)} \sin \theta = m_{W}, \quad M_{Z(cu)} \cos \theta + n_{(c+u)} M_{(c+u)} \sin \theta = m_{Z},
\]

by which Figs.B2(a) and (b) are described. Then each mass triangles: Figs.B1(a) and (b) should be equivalent to that of Figs.B2(a) and (b) respectively since these right-angled triangles should have an equal angle such that \(\theta_{ai} = \theta_{a2}, \theta_{bi} = \theta_{b2}\) to make same rates of contribution to corresponding hypotenuses (resultant masses) which should have an equal value (side) respectively. Thus the remainder sides should also be equal respectively. So with considering also eqs.(B1) and (B2), we obtain,

\[
\eta_{W} = \sqrt{2k_{bad}} = 1/2, \quad \eta_{(b+s+d)} = \sqrt{2k_{bad}} K_{bad} = 1/2 \quad \therefore k_{bad} = 1/(\sqrt{2}), \quad K_{bad} = 1.
\]

\[
\eta_{Z} = k_{cu} = 1/2, \quad \eta_{(c+u)} = \sqrt{2k_{cu}} K_{cu} = 1/2 \quad \therefore k_{cu} = 1/\sqrt{2}.
\]

-C: A short discussion for the physical interpretation of eqs.(27),(28),(29)

As shown in eqs.(27),(28),(29), the SM Higgs boson mass is to be a little smaller than virtual bound top quark-pair mass \(M_{(tt)} = M_{t}/\sqrt{2}\), by the amount for emitting one photon from \((tt)\). On the other hand, the dynamical strong coupling theory of top quark condensation\(^1\) predicts that Higgs boson mass is twice of top quark mass, namely, \(M_{H} = M_{t}\). We consider that the former (eqs.(27),(28),(29)) describes a 'solidification' phenomenon around at crystallization point \(T_{cryst}\) from Quark-Gluon Plasma (QGP) state into a more stable state via a bound top quark-pair \((tt)\) while the latter would describe one into a molecular-like state of two top quarks at condensing point \(T_{cond}\), where \(T_{cryst} < T_{cond}\). Details regarding the crystal from \((tt)\) above will be discussed in next paper.

REFERENCES


u} = 2M_{q}M_{t}/\sqrt{M_{q}^{2} + M_{t}^{2}} = 120.611 \text{GeV}^{2}$ is shown.


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