Data Assimilation of an Earthquake Generation Cycle Model on a 2-D Fault Using Interseismic Data

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To develop a quantitative earthquake forecasting model, we experimentally estimated frictional parameters at a model plate interface during an earthquake generation cycle using realistic synthetic observational data. We used a 2-D fault model based on a rate- and state-dependent friction law to generate displacement data at observation points located above the fault plane during the latter half of an interseismic period, when fault slip is stable and slip rate is nearly constant. We estimated the likelihood of friction law parameters using sequential importance sampling (a particle filter). We accurately determined $A - B$, which represents the rate dependence of steady-state frictional stress, for an aseismic slip area around a circular seismic slip area using typical noise levels and observation point spacings. However, the characteristic slip distance $d_c$ we estimated for the aseismic slip area contained large estimation errors. We found that the likelihood distribution depended on the noise level of the data rather than on the interval between observation points. Moreover, we found two ranges of $A - B$, one with high likelihood and the other with low likelihood, even when the noise level was too high or the data period was too short for detailed estimation. The likelihood data provided little information for determination of $d_c$, indicating that aseismic sliding during the interseismic period is much more sensitive to $A - B$ than to $d_c$. These results indicate that interseismic displacement data are useful for estimating friction parameters, especially $A - B$ values.

1. INTRODUCTION

To forecast earthquakes using observational data (e.g., crustal deformation data) in quantitative modeling of the earthquake generation cycle, the model parameters that determine the frictional properties on a fault have been estimated using data assimilation methods. These studies estimated frictional parameters using data on aftershocks, which are aseismic slips that occur around earthquake source areas following large earthquakes. Because aftershocks are more stable than coseismic slips, their use simplifies these analyses.

Values of the frictional parameters used in an earthquake generation cycle model can also be estimated using data mainly from unsteady slips (e.g., earthquakes and aftershocks). Most observations, however, are made during interseismic periods, when no earthquake events occur, because the coseismic period lasts only a few minutes at most and the duration of aftershock can be from...
a few months to a few years. The recurrence time of large earthquakes ranges from decades to several hundred years. Therefore, it is necessary to understand the sensitivity of frictional parameters to interseismic observational data, so that modeling parameters can be estimated during the interseismic period, well before the next earthquake. We therefore examined whether frictional parameter values can be estimated from interseismic data alone.

For data assimilation, we used sequential importance sampling (SIS)\(^6\) (a particle filter) and synthetic interseismic crustal deformation data to calculate the likelihood distribution of the friction law parameters in the parameter space. Among the several assimilation methods available, this is the most suitable method for nonlinear models without system noise, such as the model we describe in sections 2 and 4. For the original particle filter described by Evensen (2003)\(^7\), resampling without system noise results in degeneracy of the ensemble members. Although for the Ensemble Kalman Filter\(^8\) this degeneracy does not occur, the filtering procedure with application of Kalman gain may cause inconsistency in the model equations. The adjoint method\(^9\) is useful for quick searches of optimum parameter values; however, it is only a local optimization procedure and the entire likelihood distribution cannot be obtained.

In the following sections, we introduce our simulation model and discuss the process of data assimilation. Then, we consider the implications of our results for the use of presently available observations of frictional parameters to predict earthquakes.

2. CALCULATION OF SLIP HISTORY ON A FAULT IN AN ELASTIC MEDIUM

Fig. 1. Model used in this study. (a) Two-dimensional planar fault on the xy-plane in an infinite, uniform elastic medium (modified from Kato, 2003\(^7\)). The fault is loaded by a constant relative plate velocity \(V_{pl}\) in the x-direction. (b, c) Spatial distributions of the true values of (b) \(d_c\) and (c) \(A - B\) on the fault. Large dots in (c) are observation points at which the cumulative slip history \(u_i\) is shown in Fig. 2.

Fig. 1a illustrates a planar square fault in an infinite, uniform elastic medium, following the approach used by Kato (2003)\(^7\), used here to model an earthquake fault at the boundary between a continental plate and an oceanic plate subducting beneath it. On the fault plane, stress increases depending on the slip deficit with respect to the relative plate velocity, which models the strain at the plate boundary caused by subduction of the oceanic plate. When the shear stress reaches the static frictional strength of the plate boundary, slip displacement releases the slip deficit.

The elastic medium is shear-loaded such that two elastic blocks are displaced in the x-direction at a constant rate \(V_{pl}\) across the fault (Fig. 1a). The square fault is divided into 65 536 (256 \(\times\) 256) square cells, each 100 m \(\times\) 100 m. Only slip \(u\) in the x-direction and shear stress \(\tau_{xx}\) are considered. Other slip
and stress components are ignored because they are small; for simplicity, \( \tau_{xy} \) is denoted by \( \tau \) hereafter.

The shear stress \( \tau_i \) at the center of cell \( i \) is related to slip as follows:

\[
\tau_i = \sum_j k_j (V_{pl,i} - u_j) - \frac{G}{2c} V_i,
\]

where \( i \) is the cell number, \( u_j \) is the amount of slip, \( i \) is time, \( k_j \) is the change in shear stress on cell \( i \) per unit of slip on cell \( j \), \( V_j \) is slip velocity, \( V_{pl} \) is relative plate velocity (10 cm/yr; \( \sim 3.18 \times 10^9 \) m/s), \( G \) is rigidity \( (3 \times 10^{10} \) Pa), and \( c \) is shear-wave velocity \( (3 \times 10^7 \) m/s). The second term on the right side of eq. (1) represents seismic radiation damping. In this model, the periodic boundary condition is assumed and a 2-D fast Fourier transfer technique is applied for efficient computation of stress due to fault slip.

The slip is assumed to obey the composite rate- and state-dependent friction law (9) (eqs. 2 and 3 below) because it has been shown to successfully reproduce various slip patterns at a plate boundary:

\[
\tau_i = \tau_0 + A_i \ln(V_i/V_0) + B_i \ln(V_0 \theta_i/d_c),
\]

\[
d\theta_i/dt = \exp(-V_i/V_c) - (V_i/d_c) \ln(V_i/d_c),
\]

where \( \theta_i \) is the state variable and \( A_i, B_i, d_c, \) and \( V_c \) are constants that characterize frictional properties. \( V_0 \) is an arbitrarily chosen reference velocity, and \( \tau_0 \) is a reference stress corresponding to the steady-state frictional stress at \( V = V_0 \). The values of \( V_0 \) and \( \tau_0 \) do not affect the simulated slip behavior in our model. Following Kato and Tullis (2001) (12), we used \( V_c \) of \( 10^{-8} \) m/s in all our simulations, based on experimental data for granite surfaces (13).

The frictional parameters, \( A_i, B_i, \) and \( d_c \) determine sliding behavior. When \( A - B > 0 \), the frictional characteristic shows velocity strengthening, which leads to stable sliding at a constant slip velocity unless the stress is perturbed. When \( A - B < 0 \), the frictional characteristic shows velocity weakening, which leads to stick-slip motion. \( d_c \) is the characteristic slip distance associated with the evolution of the slip surface state. The smaller \( d_c \) is, the more unstable the slip tends to be.

We used the initial condition that each cell slides stably with \( V_i = 0.9 V_{pl} \). Numerical simulations were done by solving eqs. (1) to (3), where eqs. (1) and (2) were time derivatives, and the temporal change in the cumulative slip distance \( u_i \) on the cell was

\[
du_i/dt = V_i.
\]

Simulations were calculated by the fifth-order Runge-Kutta method with an adaptive time step (14). Thus, we determined the temporal changes of \( \tau_i, \theta_i, V_i, \) and \( u_i \).

Figs. 1b and 1c show the spatial distributions of \( d_c \) and \( A_i - B_i \), assumed here to be true values. We assumed that \( A - B \) and \( d_c \) within and outside the circular patch were constant, a uniform value of \( A = 1.0 \) MPa over the fault, and that \( A - B \) and \( d_c \) within the patch were true values. Thus, we assumed that \( B \) and \( d_c \) outside the patch \( (B_o, d_{co}) \) were unknown, and investigated how accurately they could be estimated using synthetic observational data.

### 3. FORWARD SIMULATION

Before carrying out data assimilation, we performed the forward simulation described above for each frictional parameter set \( (B_o, d_{co}) \) in the search ranges of the data assimilation. Subsequently, we
calculated synthetic data using the simulation result for one of these parameter sets, and used all of the simulation results repeatedly in the data assimilation for each of three cases (described below). The parameter search ranges and true values that we used are listed in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Search range</th>
<th>True value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_o$</td>
<td>$0.0 &lt; B_o &lt; 1.0$</td>
<td>0.5</td>
</tr>
<tr>
<td>$d_{co}$ (m)</td>
<td>$0.001 &lt; d_{co} &lt; 1$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Fig. 2. Cumulative slip history $u(t)$ at observation points 1 to 6 shown in Fig. 1c, derived using $(B_o, \log d_{co}) = (0.5, -1)$.

Fig. 2 shows the histories of cumulative slips $u_t$ simulated using true values at points 1 to 6 (Fig. 1c), which were at intervals of 2 km on the fault plane. Afterslip outside the patch was a response to stress perturbation caused within the patch by the earthquake. The recurrence time was about 11 years for this case.

Synthetic data at each observation point were calculated from this simulation result as follows. For observation points 1' to 6', located 4 km above the fault plane (Fig. 3a), each observation point was displaced owing to the slip deficit ($u_{di} = u_t - V_{sl})$ on the fault plane. Thus, the displacement in the x-direction $u_{di}$ was calculated using $u_{di}$ at all cells, following Maruyama (1964)\(^{15}\), for the geometric setting of Fig. 1a. The interval between the observation points $\Delta x$ (2 km) and the 4 km distance of the observation points from the fault plane were set proportional to the radius of the patch in this model (6 km) to reflect the typical spacing of GPS observation points and the typical length and depth of large earthquake faults in Japan. We prepared synthetic observational data $y_i$ for the latter half of the interseismic period by adding noise to $u_{di}$. We set the standard deviation ($\sigma$) of the data to 2 cm, which is the noise level in most GPS daily observational data recorded in Japan\(^{16}\).

$(\Delta x, \sigma) = (2 \text{ km}, 2 \text{ cm})$ was set as the reference case (Case I); $(\Delta x, \sigma) = (2 \text{ km}, 4 \text{ cm})$ as Case II, and $(\Delta x, \sigma) = (4 \text{ km}, 2 \text{ cm})$ as Case III. The small gray dots in Fig. 3b are the locations of the observation points for Case III, for which the number of observation points was 36, one-fourth the number used for Case I (144 points; Fig. 3a). Fig. 3c shows example records of $u_{di}$ at observation points 1' to 6' (4 km above points 1 to 6), and Fig. 3d shows $y_i$ with two levels of added noise, superimposed on $u_{di}$ at
observation points 1' to 6' during the latter half of the interseismic period. We used the data of Cases I–III for the data assimilation, described below, and compared the results.

Fig. 3. Fault model showing locations of observation points with an interval Δx of (a) 2 km and (b) 4 km. (c) Histories of displacements $u_{di}$ at observation points 1' to 6', which are 4 km above points 1 to 6. (d) Synthetic observation data for displacement histories $y_{k}(t)$ at points 1' to 6' with displacements $u_{di}$ during the last half of the interseismic period. Lines show the displacements $u_{di}$ (as in Fig. 3c). Large symbols indicate $y_{k}(t)$ with noise of $\sigma = 2$ cm added to $u_{di}$, and small symbols indicate $y_{k}(t)$ with noise of $\sigma = 4$ cm added to $u_{di}$.

4. DATA ASSIMILATION PROCESS

We calculated the likelihood distribution of $(B_p, d_{eq})$ by comparing observational data $y$ with the simulation results $u_{d}$ for the parameter sets shown in Table 1. We made the following assumptions:

1. The observational equation was defined as $Y_k = H X_k^{(j)} + \sigma_k$ at time $t = k dt$ ($k = 1, \ldots, M$), where $Y_k = (y_1, \ldots, y_M)^T$ is the observational data vector for data number $k$, $dt$ is the sampling rate, $H$ is the observation matrix, $\sigma_k$ is the observational noise vector, $X_k^{(j)} = (x_1^{(j)}, \ldots, x_M^{(j)})^T$ is the state
matrix, $X_i^{(j)}$ is the state vector for cell $i$ that contains all variables including $u_a^{(j)}$ calculated using the frictional parameter set $(B^{(j)}, d_c^{(j)})$ ($j = 1, \ldots, N$), and $j$ is the particle number. The observational noise vector $\varepsilon_k$ has a Gaussian distribution with zero mean and is independent at each time $t$; thus, the covariance matrix of $\varepsilon_k$ is diagonal. Here, $H$ is a linear map from $u_a^{(j)}$ to $y_i$ for all $i$ that provides the correspondence relation between them. The original sampling rates of the observational data $Y$ and the simulation results $HX^{(j)}$ varied. Thus, $Y_k$ and $HX^{(j)}$ were calculated from $Y$ and $HX^{(j)}$, respectively, in order to use the same sampling rates to calculate the likelihoods of the frictional parameters.

2. System noise was set to zero in the SIS procedure; in other words, the simulation model was time independent, and all variables were undisturbed in the time domain. Because we assumed that data were available for the entire observation period, we did not need to estimate the frictional parameter set progressively in the SIS procedure. Thus, in the steps given below, we modified the process about data number $k$. In addition, the simulated history of displacement $u_a^{(j)}$ can be calculated for a period longer than the data period, so we can choose any period as $X^{(j)}_k$ in the simulated history. Moreover, the period of the simulated earthquake cycle depends on the frictional parameter set $(B^{(j)}, d_c^{(j)})$.

The data assimilation procedure thus consisted of the following steps.

(1) Generate frictional parameter sets $(B^{(j)}, d_c^{(j)})$ ($j = 1, \ldots, N$).

(2) Calculate $X^{(j)}_l$ for $j = 1, \ldots, N$ at time $t' = ltdt$ ($l = 1, \ldots, L$).

(3) Calculate the likelihood distribution in the frictional parameter space.

(a) Calculate the likelihood $w^{(j)}$ for $j = 1, \ldots, N$.

(i) Choose the period comparing $Y_k$ and set as $X^{(j)}_k$. In other words, substitute $X^{(j)}_{k+1}$ ($l = m, \ldots, m + M - 1$) for $X^{(j)}_{k+1}$ ($l = k, \ldots, M$).

(ii) Calculate the likelihood $w^{(j)}_m$ for $m = 1, \ldots, L - M + 1$ with eq. (5):

$$w^{(j)}_m = \frac{1}{\sqrt{2\pi\sigma M}} \exp\left[ -\frac{1}{2\sigma^2} \sum_{k=1}^{M} \left\| Y_k - HX^{(j)}_k \right\|^2 \right]. \quad (5)$$

(iii) Choose the maximum value of $w^{(j)}_m$ as the likelihood $w^{(j)}$ for $j = 1, \ldots, N$.

(b) Sum $w^{(j)}$; that is, calculate $W = \sum_{j=1}^{N} w^{(j)}$. 
(c) Calculate the normalized likelihood \( \tilde{w}^{(j)} = w^{(j)} / W \) such that it sums to 1 and \( 0 \leq \tilde{w}^{(j)} \leq 1 \).

5. RESULTS AND DISCUSSION

Fig. 4 shows the distribution maps of the normalized likelihood \( \tilde{w}^{(j)} \) and the true value in the frictional parameter space for Cases I–III. For Case I, \( \tilde{w}^{(j)} \) is largest at points near the true value (Fig. 4a). Thus, we can accurately determine frictional parameters for typical noise levels and typical spacing of observation points. For Cases II and III, the observational conditions were worse than those of Case I. For Case III (Fig. 4c), the distribution of \( \tilde{w}^{(j)} \) is similar to that of Case I, although the frictional parameter values where \( \tilde{w}^{(j)} \) is largest are somewhat different from the true values. On the other hand, for Case II (Fig. 4b), there are no localized large values of \( \tilde{w}^{(j)} \). These results indicate that a high noise level hides the detailed information needed to estimate the optimum frictional parameters. Thus, if the observation points are located uniformly relative to the true parameter distribution, the likelihood distribution depends on the noise level of the data rather than on the interval between the observation points. In contrast, for all three cases \( \tilde{w}^{(j)} \) is low (white areas) mainly where \( B_o \) is large. Thus, the range of possible frictional parameters can be restricted even when the noise level of the interseismic data used is high.

Fig. 4. Distribution maps of the normalized likelihood \( \tilde{w}^{(j)} \) in the frictional parameter space \((B_o, \log d_e)\) outside the circular patch for (a) Case I (interval between the observational points \( \Delta x \), standard deviation of the data \( \sigma \) = (2 km, 2 cm), (b) Case II (2 km, 4 cm), and (c) Case III (4 km, 2 cm). \( \tilde{w}^{(j)} \) was calculated from the simulated displacement \( u_{ds} \) and synthetic data for the latter half of the interseismic period. The true value \((B_o, \log d_e) = (0.5, -1.0)\) is indicated by a broken black line; \((B_o, \log d_e) = (0.5, -0.75)\) (see Fig. 6a) is indicated by a gray line; and \((B_o, \log d_e) = (-0.625, -3.0)\) (see Fig. 6b) is indicated by a solid black line.

Figs. 5.1 and 5.2 show the distributions of the normalized likelihoods of \( B_o \) and \( d_e \), respectively, for
the three cases. For $B_o$, $\tilde{w}^{(j)}$ is high around the true value of 0.5 for Cases I and III, and the $\tilde{w}^{(j)}$ distributions resemble a Gaussian distribution (Fig. 5.1). The weighted average of $\tilde{w}^{(j)}$ is close to the true value in both cases. In contrast, for Case II, the highest $\tilde{w}^{(j)}$ is only about one-third of the highest values of the other cases, although it is located close to the true value. However, for Case II, the weighted average is very different from the true value.

On the other hand, for $d_{co}$ there are no localized large values of $\tilde{w}^{(j)}$ for any values of the frictional parameters for all cases, indicating that these data contain little information with which to estimate $d_{co}$ (Fig. 5.2). The weighted averages are near the middle of the search range, and they depend on the range. Thus, they cannot be estimated from the data.

![Fig. 5. Distributions of normalized likelihoods $\tilde{w}^{(j)}$ in the frictional parameter space outside the circular patch (shown in Fig. 4) for $B_o$ (Fig. 5.1) and $d_{co}$ (Fig. 5.2). (a) Case I ($\Delta x$, $\sigma$) = (2 km, 2 cm), (b) Case II (2 km, 4 cm), and (c) Case III (4 km, 2 cm).]

We explain the dependency of $\tilde{w}^{(j)}$ on the frictional parameters as follows. Fig. 6 shows the simulated displacements at observation points 1' to 6' for low $\tilde{w}^{(j)}$ using parameter values near the true values (Fig. 6a) and those for high $\tilde{w}^{(j)}$ using parameter values far from the true values (Fig. 6b), along with simulated displacements $u_{dh}$ in the latter half of the interseismic period. Comparison of the simulated displacement with that using the true value in Fig. 6a (but not in Fig. 6b) shows that the slopes of the two lines for each observation point differ. The difference in $\tilde{w}^{(j)}$ is caused by the differences in the slopes of the displacement between the simulation result and the data with noise. The
slope of the displacement at an observation point depends on the stick-state, that is, the difference between the slip velocity $V$ on the fault and the relative plate velocity $V_p$. The stick-state depends on the frictional parameters on the fault (especially $A - B$). Therefore we can estimate the frictional parameters from the slopes of the displacements at the observation points. The resolution of $d_c$ is lower than that of $B_c$ because aseismic slip during an interseismic period is much more sensitive to $A - B$ than to $\alpha$.

Fig. 7 shows the distribution maps of the normalized likelihood $\tilde{w}^{(j)}$ in the frictional parameter space determined from the data from the last quarter of the interseismic period for Cases I–III. In all cases, no localized space with high $\tilde{w}^{(j)}$ is apparent, which shows that detailed estimation of frictional parameters is difficult with data from a short interval within the interseismic period. However, two areas, one with high and one with low likelihood, appear in the parameter space, indicating that the data can be used to reduce the possible parameter range.

![Figure 6](image-url) Fig. 6. Simulated displacements at observation points 1'–6' (broken lines) for (a) low $\tilde{w}^{(j)}$ using parameters near the true values ($B_c$, $\log d_c$) = (0.5, -0.75) (indicated by the gray outline in Fig. 4a) and (b) high $\tilde{w}^{(j)}$ using parameter values far from the true values ($B_c$, $\log d_c$) = (0.625, -3.0) (indicated by the solid black outline in Fig. 4a), with superimposed simulated displacements $u_{di}$ for the latter half of the interseismic period (solid black lines corresponding to the lines shown in Fig. 3d).

![Figure 7](image-url) Fig. 7. Distribution maps of the normalized likelihood $\tilde{w}^{(j)}$ in the frictional parameter space ($B_c$, $\log d_c$) outside the circular patch for (a) Case I ($\Delta x$, $\sigma$) = (2 km, 2 cm), (b) Case II (2
km, 4 cm), and (c) Case III (4 km, 2 cm). $\tilde{w}^{(j)}$ was calculated from simulated displacement $u_d$ and synthetic data from the last quarter of the interseismic period.

Finally, we consider the limitations of our model and assimilation method and prospects for future research. We assumed two areas with uniform parameter values, although the real parameter distribution is more complex. Future research will need to consider the spatial variation of frictional parameters. This spatial variation will influence the slip–time function on the fault, and this will be reflected in the displacement history at observation points. Therefore, we will be able to estimate the spatial variation of frictional parameters if we account for the spatial variation of displacement history at the observation points. Using data from the interseismic period, as in this study, we expect that the spatial parameter variations can be accounted for, but the localization of the observation points may make the estimation difficult. Moreover, many earthquake cycle simulations for many frictional parameter distributions will be necessary, and this requires much processing time. Thus, SIS should be used for rough estimations of the parameter distribution, and another less time consuming assimilation method (e.g., the adjoint method) should be used for detailed estimation.

6. SUMMARY

If our assumed model is adequate, afterslip frictional parameters can be estimated from GPS data presently recorded in Japan by using data from the latter half of the interseismic period, during which slip behavior is almost steady state. The likelihood distribution depends on the noise level of the data rather than on the spacing of observation points. Two ranges of $A - B$, one with high and one with low likelihood, were obtained using data from just one-fourth of the interseismic period or data with a high noise level. However, it was more difficult to constrain $d_c$ because aseismic sliding during the interseismic period is much more sensitive to $A - B$ than to $d_c$.

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