Effect of a Single Particle on Water-Tube Interaction SubJECTED TO Axial Impact Loading

Kollika NGUON, Kazuaki INABA, Tatsuro HORI and Kikuo KISHIMOTO

Department of Mechanical Engineering and Sciences, Tokyo Institute of Technology, Tokyo, Japan

This study numerically and experimentally investigated the effect of a single solid particle on the interaction between water and an elastic tube due to a water hammer. The investigation was conducted by impacting a steel cylinder projectile (at a speed of 2.4 m/s) onto the surface of a water-filled polycarbonate tube to initiate the water hammer propagating through both the tube and water. We examined the effect of a single cylindrical particle on the impact response for different particle radii and materials (steel, aluminum, and polycarbonate). Computational and experimental strains observed in front of the particle were in good agreement; the radial tube displacement was higher when the particle was made of stiffer material. In the experiment, the tube response in the middle of the particle was strongly affected by the size of the particle rather than the particle’s material. To evaluate the maximum impact response of hoop strains around the particle, a new model was proposed that treats the particle as a rigid body, and it successfully captured the experimental tendencies.

1. INTRODUCTION

During transportation processes, solid particles in the form of slurry—including coal, limestone, iron concentrate, and hazardous materials—can be released into the environment by fluid–structure interaction (FSI) accidents [1]. Eventually, impact-loaded water may interact with the pipe wall to cause displacement and escalating structural damage because of the instant variation in pressure; this is called a “water hammer” [2, 3]. In engineering practice, a liquid-filled pipe experiences such an event when it is subjected to an abnormal flow resulting from, for example, the sudden closure of check valves. A water hammer can cause pipelines to break if the pressure is high enough.

FSI analyses of cases where the contained particles are tiny have been successively reported for four decades [1, 4–6]. In the majority of these works, propagation of transients in axisymmetric elastic and viscoelastic fluid-filled pipes were analyzed using various one-dimensional theories; techniques for determining the relevant mechanical parameters of the system components were developed for slender, straight, prismatic, and thin-walled circular cross-section pipes [7]. Skalak’s extended theory of water hammer describes axisymmetric wave propagation in straight liquid-filled pipes. Tijsseling reviewed two mathematic models: a complex one and simple one [8]. The simple model is popular with researchers investigating fluid–structure interaction in liquid-filled pipes [9] as it permits coupling the propagation of pressure waves in the liquid and axial stress waves in the pipe wall without considering the reflections from the pipe ends.

However, in some industrial applications, the transportation of large particles in the form of slurry is becoming increasingly necessary [1]. Examples include the pumping and transportation of mining materials through pipelines. Various forms of radioactive contamination may cause hydrogen explosions, which would result in damages to the system during the transportation process [3]. Physically, the mixture in these cases is non-homogeneous because of the large size of the particles. There has been
speculation over how the tube responds to pressurized fluid and how the coupling characteristics can be estimated if the same volume fraction of particles is replaced by a large single solid. To the best of our understanding, this condition remains a challenging issue in engineering practice since no clear explanation of this case has yet been documented. Therefore, the present work examined the effect of a large solid particle on the tube response and water pressure; it is an extension of the FSI homogeneous mixture discussion published in our previous paper [7].

2. EQUATIONS FOR WATER HAMMER IN HETEROGENEOUS FLOW COUPLED WITH IMPEDANCE MISMATCHING THEORY

A water hammer, which is also referred to as a pressure surge or fluid transient phenomenon, is a spectacular form of unsteady flow generated by abrupt changes to steady flow conditions in liquid-filled pipe systems. In the majority of cases, these changes are due to the rapid closing or opening of valves or to the stopping or starting of pumps [9]. The first work on water hammers has been attributed to Ménabréa [10]. Although several other publications appeared in the second half of the nineteenth century, the present water hammer theory is based on classical investigations by Joukowsky [11]. In his theoretical treatment, Joukowsky derived the formula that bears his name:

$$\Delta p = \rho_f C \Delta u$$  \hspace{1cm} (1)

where $\Delta p$ denotes the change in pressure; $\Delta u$, the change in velocity; $\rho_f$, the mass density of the fluid; and $C$, the speed of sound in the fluid. The sound in an unconfined fluid is described by eq. (2):

$$C_f = (K_f/\rho_f)^{1/2}$$  \hspace{1cm} (2)

where $K_f$ is the bulk modulus of the fluid, and $\rho_f$ is the density of fluid. If the tube is stiff enough, the unconfined sound speed is used to evaluate the pressure jump across the water hammer.

FSI coupling in a water hammer with an elastic tube depends on the geometry and properties of the fluid and solid. In a tube with circular cross section, the Young’s modulus of the pipe material $E_p$ and diameter $D$ with wall thickness $e$ are also used to estimate the wave speed; this is usually referred to as the Korteweg speed $C_K$ [12]:

$$C_K = C_f \left[1 + (DK_f/eE_p)^{1/2}\right].$$  \hspace{1cm} (3)

The water hammer wave speed in eq. (3) is used to obtain the Joukowsky pressure, which is the pressure jump across the water hammer. Therefore, evaluation of the wave speed is the key to estimating the impact response of the tube wall in a water hammer. Han et al. [1] proposed the similar eq. (4) for the wave propagation speed in a heterogeneous mixture of water and particles:

$$C_H = \sqrt{\frac{\left(c_p, 1-c_p\right) K_f}{\rho_s, \rho_f}} \sqrt{1 - c_p \rho_s/\rho_f} \sqrt{\frac{dE_f}{dE_p}}$$  \hspace{1cm} (4)

where $c_p$ is the volume fraction of the particle; $\rho_s$, the particle density; and $E_s$, the Young’s modulus of the particles.

The relations of the incident stress wave $\sigma_I$, reflected stress wave, $\sigma_R$, and transmitted stress wave $\sigma_T$ between two media A and B in solid bars are described by the impedance mismatching theory [13]:
\[
\sigma_T = \frac{2\rho_B C_B}{\rho_B C_B + \rho_A C_A} \quad (5)
\]

\[
\sigma_R = \frac{\rho_B C_B - \rho_A C_A}{\rho_B C_B + \rho_A C_A} \quad (6)
\]

where \( \rho \) and \( C \) with subscripts of medium A or B are the density and longitudinal wave speed, respectively. By considering two regions (in front of the particle, or medium A, and around the particle, or medium B), we can extend the above stress relations into water hammer pressures and strains by

\[
\frac{\Delta P_T}{\Delta P_R} = \frac{2\rho_B C_B}{\rho_B C_B + \rho_A C_A} = \frac{\varepsilon_B}{\varepsilon_1} \quad (7)
\]

\[
\frac{\Delta P_R}{\Delta P_T} = \frac{\rho_B C_B - \rho_A C_A}{\rho_B C_B + \rho_A C_A} = \frac{\varepsilon_R}{\varepsilon_1} \quad (8)
\]

where \( \Delta P_T \) is the pressure jump in the water region (medium A) due to the incident water hammer wave; \( \Delta P_R \), the pressure jump due to the reflected wave in the water region (medium A); and \( \Delta P_T \), the pressure jump due to the transmitted wave in the region where a single particle or particles exist (medium B). Strains \( \varepsilon_1 \), \( \varepsilon_S \), and \( \varepsilon_B \) are hoop strains caused by the incident wave, reflected wave, and transmitted wave, respectively. Density \( \rho_A \) and wave speed \( C_A \) in medium A can be evaluated by water density and Korteweg speed \( C_K \). The pressure inside the tube can be translated into the hoop strain by Tijsseling's thick-wall theory for water hammer [14]; this conversion relation was experimentally confirmed in water hammer experiments [15]. We evaluated the density \( \rho_B \) and wave speed \( C_B \) in medium B by averaging the densities of water and a particle and Han's wave speed \( C_H \), and we compared the experimental and numerical results.

3. LABORATORY APPARATUS

The laboratory apparatus was designed for the water hammer test, as shown in Fig. 1. We tested a 1-m-long polycarbonate tube (a transparent tube material was chosen for visualizing the solid particle's position and movement) with an inner diameter of 52 mm and wall thickness of 4 mm. The tube was filled with water and sealed at both ends. A polycarbonate buffer plugging the top end of the tube was smoothly stroked by a steel projectile at a speed of 2.4 m/s; the projectile was accelerated by gravity (in a transparent guide tube) from a drop height of 300 mm. The buffer was virtually free from moving in the axial direction due to lubricated O-rings. As an advantage, the impulse energy from the projectile was directly transmitted through the buffer to the fluid. The displacement of the buffer caused compression of the contained water and generated water hammer wave propagation throughout the systems. The pressure was recorded by a piezoelectric transducer mounted in the center of the aluminum plug at the lower end of the tube. The radial tube displacement data were collected by the mean of strain gages mounted in the circumferential direction at various locations along the tube. As shown in Fig. 1, each strain gage was applied on the outer wall at 100-mm intervals except around the particle location, where the intervals were smaller. In contrast to the very fine mixture case, the solid large particle cannot bear a desirable position in a water-filled pipe due to its higher density with respect to the water density. Consequently, in each case, a particle (steel, Al and PC, all of different sizes; Table 1) was suspended from the buffer by an elastic rubber string. A high-speed camera was used to capture the motion of the solid particle.
Table 1 Dimensions and mechanical properties of specimens.

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Material</th>
<th>Diameter (mm)</th>
<th>Length (mm)</th>
<th>Young’s modulus (GPa)</th>
<th>Bulk modulus (GPa)</th>
<th>Density (kg/m³)</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube</td>
<td>PC</td>
<td>outer 60, inner 52</td>
<td>1000</td>
<td>2.45</td>
<td>4</td>
<td>1220</td>
<td>0.37</td>
</tr>
<tr>
<td>Particles</td>
<td>Steel</td>
<td>50</td>
<td>60</td>
<td>210</td>
<td>175</td>
<td>7850</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>Al</td>
<td>35</td>
<td>60</td>
<td>70</td>
<td>68</td>
<td>2700</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>PC</td>
<td>25</td>
<td>60</td>
<td>2.4</td>
<td>4</td>
<td>1200</td>
<td>0.37</td>
</tr>
</tbody>
</table>

4. COMPUTATIONAL MODELS

An explicit finite element code LS-DYNA was used to model a shorter tube (246 mm) with the same diameters and materials as the above experimental setups. Similarly, 36-mm-long solid-cylindrical particles made of steel, Al, and PC were suspended in the tube to resemble the experimental setup, although there was no actual string in the model (Fig. 2). For simplicity, we neglected gravity and replaced the buffer by the piston. We set the distance between the piston and the particle long enough to observe the incident and reflected waves. The elastic string in experiments was omitted in simulations. The tube was fixed at the bottom by a rigid wall, and the top end had a rigid piston at a distance of 216 mm. The fluid inside the tube was represented by two water columns and a cylindrical layer called water middle with the same inner cross-section as the particle diameter; water 1 was the solid column between the piston and particle, and water 2, which had the same length as water 1, was the lowest water column. The separation of these water columns was to ease analysis in the observation regions; the constraints between parts were carefully set. hoop strains of the tube that developed through elastic deformation
during the loading process were obtained from the observed displacements of nodes along the wall. Each
reported strain value was obtained by averaging eight strains circumferentially at the same axial location.
The pressure data in the water column elements were measured just beneath the observed nodes and not
averaged [7]. Three-dimensional first-order hexahedral solid elements were applied to all active parts
(see the number of elements in Table 2).

![Schematic diagram showing the model with a large particle (unit: mm): regions 1 and 2 correspond to medium A, and region 3 corresponds to medium B.]

Fig. 2 Schematic of the model with a large particle (unit: mm): regions 1 and 2 correspond to medium A, and region 3 corresponds to medium B.

Table 2 Size of elements on the outer layer of active parts (in mm)

<table>
<thead>
<tr>
<th>Element properties (mm)</th>
<th>Length (in z-axis)</th>
<th>Width</th>
<th>Thickness</th>
<th>Total elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube</td>
<td>3</td>
<td>2.356</td>
<td>1.33</td>
<td>19,680</td>
</tr>
<tr>
<td>Piston</td>
<td>3</td>
<td>2.042</td>
<td>2</td>
<td>2232</td>
</tr>
<tr>
<td>Particle</td>
<td>3</td>
<td>1.963</td>
<td>1.33</td>
<td>8604</td>
</tr>
<tr>
<td>Water 1</td>
<td>3</td>
<td>2.042</td>
<td>2</td>
<td>22,320</td>
</tr>
<tr>
<td>Water 2</td>
<td>4.5</td>
<td>2.042</td>
<td>2</td>
<td>11,160</td>
</tr>
<tr>
<td>Water middle</td>
<td>3</td>
<td>2.042</td>
<td>2</td>
<td>7680</td>
</tr>
</tbody>
</table>

4.1. Boundary conditions

In this computation, the stationary fluid was modeled so that all relations between the fluid and
structure were considered when the kinematic energy from the piston was excited. The impact process
was started at the initial speed of 2.4 m/s. Fixed nodes were set for the tube bottom to prevent
displacement of the tube off the rigid plate due to the impulse load generated by the acoustic wave.
Poisson coupling, which is known to be the most important liquid–pipe interaction problem, was
considered; friction was neglected between piston–tube, piston–fluid, fluid–particle, tube–particle, and
fluid–tube active parts.

4.2. Governing equations

In most cases, coupling between the fluid and structure was considered at the deformable fluid–
structure interface; thus, the fluid flow was characterized in a domain with moving boundaries. This
phenomenon can be described by the well-known arbitrary Lagrangian Eulerian (ALE) technique [7, 16]. Transformations between different formulations were based on pull-back and push-forward
operations in terms of the material deformation gradient, mesh deformation gradient, and their Jacobians. The fluid behavior was characterized by the conservations of mass, momentum, and energy. Since it was treated as an inviscid fluid, water was modeled as a null material. The structural behavior was governed by the conservation of momentum.

5. RESULTS

5.1. Experimental results

The water hammer, which is also called the primary wave, was generated initially by the projectile hitting the buffer within the tube. This wave characterized the coupling between the tube and fluid. In fact, expansion of this thick tube due to this coupling phenomenon could not be visually observed. Fortunately, the transparency of the tube (PC) permitted visualization of the moving solid particle by a high-speed camera. In this test, radial tube displacement data were recorded by means of strain gages mounted along the pipe wall (see Fig. 1). The observed results indicated that the water hammer wave propagated downward and reflected off the bottom closed end of the specimen tube. Since we focused on the frontal wave dynamics, we neglected the reflected waves and analyzed only the first propagation of the water hammer as the incident wave.

The magnitude of the incident wave changed depending on the drop height of the projectile. Because the magnitudes of the incident wave differed slightly in each test, we compared the peak hoop strains at the middle of the particle $\varepsilon_8$ as normalized by the incident hoop strains $\varepsilon_1$, as shown in Fig. 3. Each point was plotted by averaging nine tests conducted under the same test conditions, and the error bars indicate the standard deviation. In Fig. 3a, the hoop strain around the PC particle indicated the maximum peak strains in four cases: only water (without particle), PC with the polycarbonate particle, Al with the aluminum particle, and with the steel particle. These results indicate that there was not much difference in tube response (normalized strains were 1.39–1.5 for a 50-mm-diameter particle) for different particle materials. Similar results were obtained for the case of a smaller particle ($d = 35$ mm), as shown in Fig. 3b. Han’s theory [1] and the impedance mismatching theory do not explain our findings. In the majority of cases, the impact responses of the mixture of water and particles were estimated by assuming the homogeneous mixture and using the averaged bulk modulus and density of the water and particles. The mixture of water and steel was thus expected to indicate the maximum strains; however, our experimental observations revealed different behavior from that expected by classical theory. To develop a new theoretical model, we conducted simplified numerical simulations to compare them with experimental results, as discussed in the following section.

Fig. 3 Peak hoop strains in the middle of the particle normalized by the incident peak strains.
The strains in front of the particle were estimated to become large due to the interactions between the incident wave and the reflected wave off the particle surface. To check the reflection effect, we observed the maximum strains in front of the particle $e_s$ rather than the strains in the middle of the particle $e_m$. Figure 4 plots the peak strains in front of the particle $e_s$ normalized by the incident peak strains $e_i$. According to the impedance mismatching theory, the reflected wave off a stiffer material has higher amplitude. Therefore, the hoop strain with the steel particle should be higher than that with the aluminum. Fig. 4 shows that the particle size influenced the tube strains in front of the particle: strains produced with the 50-mm particle (Fig. 4a) were higher than those measured with the smaller particle (Fig. 4b). The obtained results were consistently acceptable with the theoretical estimations proposed by Han et al. [1]. These phenomena were also examined by a numerical approach.

![Graphs showing normalized hoop strains](image)

**Fig. 4 Peak hoop strains in front of the particle normalized by the incident peak strains.**

### 5.2. Numerical results

Similar to the experiment, the tube response was measured and observed in front and in the middle of the particle. Deformation in the region behind the particle was not fully investigated since the tube response was much smaller than those observed in front and in the middle of the particle. Axial strains were caused by the Poisson effect; therefore, the most important response was the hoop strains in the water hammer with a homogeneous elastic tube. Intuitively, a larger solid particle should obstruct the pressure wave and cause a bigger expansion of the tube in the region located in front of the particle. Therefore, the effects of the size and material of the solid particle on the variation in hoop strains and water pressures were examined.

With a very large particle ($d = 50$ mm), normalized strains measured in front of the particle exhibited the maximum strain with respect to the particle material (Fig. 5). Elastic deformation of the tube due to the water hammer wave was triggered by wave reflection off the particle; this produced a larger strain than that due to the incident wave. The steel particle produced the maximum normalized strain of 1.53, followed by Al. The corresponding water pressure (recorded beneath the observed nodes) became dominant with the metal particle; the steel case indicated a peak pressure of 2.17 MPa, while Al and PC particles produced pressures of 1.84 and 1.46 MPa, respectively as shown in Fig. 5b. The pressure generally depended on the initial impact velocity, and its variation in front of the particle depended on the particle stiffness.
In the same manner, the particle effect on the tube behavior can be anticipated for smaller particle sizes. As shown in Fig. 6a, the maximum peak strain was found with stiffer particles. The slight strain variation from one case to another for smaller particles was further examined numerically. Tube strains varied from 1.13 to 1.31 with increasing stiffness of 35-mm-diameter particles. Such a variation decreased with a smaller particle (25 mm) in Fig. 6b. This trend remained satisfactorily true for the slight decrease in pressure for the last two particle size cases.

The current findings imply that the particle material does not greatly influence the tube response observed in front of the particle; the particle diameter accounts for the main effect. A bigger particle provides a large cross-section, which produces a stronger reflection to obstruct the water hammer wave; thus, the maximum strains and pressure occurred with the largest particles.

In the middle of the particle, the very large particle permitted a very low pressure wave. The 50 mm diameter particle indicated very small strains. This weak water–tube coupling may be associated with the large reflection of the particle to the primary wave. This predicted response became clearer when the strains and pressures grew larger with a decrease in the particle size. The steel, Al, and PC particles all exhibited similar tube response magnitudes that were very close to that caused by the incident wave (Fig. 7). The peak hoop strains were higher with steel than with Al and PC.
6. DISCUSSION

The tube behavior observed by the mean of the variation in hoop strains (in front of the particle) showed agreeing trends for the experiment and computation results (Fig. 4). This trend agreed well with the coupling of Han’s theory and impedance mismatching theory, proving that waves off stiffer material were more significant. The numerical strains were higher than the experimental ones but very close to those estimated with the rigid core model where the particle was assumed to be the rigid body.

The newly proposed rigid core model considers the stiffening effect on the water that leads to large coupling between the tube and water. Since the size of the particle has a stronger effect on the tube response around the particle than the material, the speed of sound in a fluid was assumed to change with a reduction in the fluid area due to the presence of the rigid particle, which does not move in the tube. This assumption led to the following derivation of the wave speed:

\[
C_{Rigid} = \sqrt{\frac{\kappa_f/\rho_f}{1+\left(\frac{1}{1-\epsilon_p}\right)\left(\frac{\rho_f}{\rho_p}\right)}}
\]

where \(\epsilon_p\) is the volume fraction of the particle in medium B.

The above wave speed in medium B was used with the rigid core model to evaluate the hoop strain around the particle. This model predicted strain ratios of \(d = 50\ mm\) and \(d = 35\ mm\) of 1.59 and 1.18, respectively. The particle material had little influence on the elastic deformation of the tube in the middle of the particle; however, the tube response in this region depended on the particle size, as shown in Fig. 3. The hoop strains in front of the particle predicted by this theory differed from the experimental and numerical results (see Fig. 4). Since the model assumed that the particle does not move, we needed to evaluate strains in front and in the middle of the particle with two different theories. In our experiments and numerical simulations, we confirmed that heavier particles were slowly accelerated by passage of the water hammer wave due to inertia. In future work, it will be necessary to consider this particle motion to predict strains with a universal theory.

7. CONCLUSION

Strongly coupled fluid-structure interaction caused by axial impact loading of water and a polycarbonate tube was investigated experimentally and numerically in the presence of a single large
particle; different materials for the particle were evaluated. Tube strains recorded in front of the particle increased with stiffer particles of the same size. The two methods of analysis agreed in their results. The tube response in the middle of the particle was not very dependent on the material of particles with a small size. Since the tube strains and fluid pressure decreased with the particle size, we proposed a new rigid core model to predict the water hammer wave and impact tube response around the particle. We confirmed that the new model successfully estimated the hoop strain around the particle.

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REFERENCES