Load-Carrying Capacity of Laterally Confined RC Column Considering Buckling of Primary Rebar


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The fundamental equations estimating the compressive load-carrying capacity of RC columns with tie and/or spiral reinforcements are used all over the world, based upon the ultimate limit state design, but the common equations include both the elastic term and the plastic one; so, there is no unification concept of the ultimate limit state. In recent years, the high-strength type reinforcement (SBPD type) has been used frequently in the RC columns and beam in Japan. Now, the common equations can not apply to the case of the high-strength primary reinforcement of the RC column. The previous reports[1,2] have already dealt with the concrete's sharing capacity, the applicable range of the common equations and the generalized practical equation for the upper-bound load-carrying capacity considering the buckling effect of the primary rebars. Especially, this paper describes that the improvement of load-carrying capacity by virtue of the lateral confinement of tie bars depend on the buckling strength of primary rebars in assuming that those elemental buckling length equals the double pitch spacing.

1. INTRODUCTION

The fundamental equations estimating the compressive load-carrying capacity of RC columns are currently based on the ultimate limit state design method. These estimated capacities give the upper limits and indicate the standard of judgment on the ultimate design so as not to exceed these values in any case. Such a design method is able to contribute to the integrity of the human life and property by virtue of the durability for the larger loads as the falling rock and the great earthquake, if the maximum load-carrying capacity is obtained in spite of occurrence of wider cracks than allowable crack width and larger displacement and/or deformation. In general, the design load can be determined by the load factor design which estimates the accidentally large load due to multiplying the common load by a load factor. The ultimate limit state of the section failure is examined by comparing the design load with the design load-carrying capacity. In this sense, it is very important to estimate the design load-carrying capacity strictly. On the other hand, in keeping step with development of the quality of materials of RC member, the estimation equation for the compressive load-carrying capacity must be looked at again, in order to prevent a serious trouble caused by its misestimation. The previous papers[1,2] dealt with a suggestion of the generalized upper-bound equation and its theoretical background, considering the buckling of the primary rebars.

This paper deals with the estimation method of load-carrying capacity of the practical RC column to be dominant on the axial compression and confined laterally by the tie bars.

2. FUNDAMENTAL UPPER-BOUND EQUATION OF DESIGN LOAD-CARRYING CAPACITY

2.1. Common Equation

The upper limit for the design axial compressive load-carrying capacity $N_{lad}^*$ is calculated by Eq.(1) where tie reinforcement is assumed, and by Eq.(1) for spiral reinforcement or by Eq.(2) when it is less[3,4].

\[
N_{lad}^* = \left( 0.85 f'_{cd} A_c + f_{yd} A_{as} \right) / \gamma_b \quad (1)
\]

\[
N_{lad}^* = \left( 0.85 f'_{cd} A_c + f_{yd} A_{as} + 2.5 f_{yd} A_{ste} \right) / \gamma_b \quad (2)
\]

where, $f'_{cd}$ is the design compressive strength of concrete, $f_{yd}$ is the design compressive yield strength of axial
reinforcement, $A_r$ is the area of concrete section, $A_c$ is the area of core concrete, $A_{nc}$ is the total amount of axial reinforcement, $f_{yd}$ is the design tensile yield strength of spiral reinforcement, $A_{w}$ is the idealized cross-sectional area of spiral reinforcement ($= \pi d_w A_w / s$), $d_w$ is the diameter of core concrete, $s$ is the pitch of spiral reinforcement, $\gamma_s$ is the member factor (1.3), and 0.85 is the factor considering the strength reduction due to permanent loads, the strength difference between the test specimen and the structural concrete and so on.

Here,

\[
\begin{align*}
\sigma_{cd} &= 0.65\sigma_0 \quad \text{[when } \sigma_0 \leq 500\text{N/mm}^2]\] \\
&= 0.57\sigma_0 \quad \text{[when } \sigma_0 \geq 600\text{N/mm}^2]\end{align*}
\]

That is to say, the design compressive strength of concrete uses 57% to 65% of the characteristic compressive strength $\sigma_0$; so, this stress level corresponds to the proportional limit of the stress-strain relation of concrete\(^\text{1}\). Such a procedure is not consistent with the original meaning because of the reason of the complex type consisting of the allowable stress design or the serviceable limit state design and the ultimate limit state design.

2.2. Upper-Bound of Load-Carrying Capacity Considering Buckling

In case of the compression test of the RC column model, it is an experienced fact that the effect of primary rebars does not appear remarkably. This reason may depend on the performance that the primary rebars do not show the simple compressive strength perfectly but those result in the elastic failure because of those buckling. Figure 1 showing a damaged highway bridge pier when the Great Hanshin Earthquake Disaster in Japan, 1995, may mean a phenomenal fact that the earthquake load was not only too large but also the load-carrying capacity was too little beyond estimation. Figure 2 expresses the simplified buckling model of the reinforcement cage post the injury of the cover concrete of the real damaged bridge pier as illustrated in Fig. 1.

The load-carrying capacity considering the buckling of primary rebars depends on the buckling load given by the function of the slenderness ratio. The slenderness ratio $\lambda$ is denoted by Eq. (4)

\[
\lambda = \frac{l}{\phi / 4}
\]

where, $l$ and $\phi$ are the length and the diameter of the rebar, respectively. When both ends of rebar are pin-connections, the critical slenderness ratio $\lambda$ and the buckling stress $\sigma_b$ by Rankine’s equation\(^\text{6}\) are given by Eq. (5) and Eq. (6), respectively.

\[
\begin{align*}
\lambda &= \left( \frac{E}{f_{yd}} \right)^{1/2} \\
\sigma_b &= \frac{\pi^2 E}{\lambda^2 / 4}
\end{align*}
\]

![Fig. 1 An example of damaged pier rebars in RC column](image1)

![Fig. 2 Modeling of buckling of primary when great earthquake](image2)
where, $E_i$ is the modulus of elasticity, 200kN/mm², and $f_{cd}'$ is above mentioned. The upper-bound of load-carrying capacity $N'_{obs}$ considering the buckling effect, basically, can be expressed by Eq. (7)

$$N'_{obs} = A_s f_{ct} + A_k \sigma_u$$

where, $A_s$ is the core area of concrete.

In the practical design, the factor of strength: 0.85, and (the analytical factor of structure) × (the load factor) × (the factor of structure): 1.44, etc. should be considered.

3. EXPERIMENTAL VERIFICATION OF LOAD-CARRYING CAPACITY OF CONFINED RC COLUMN

3.1 Preparation of RC Column Model

D13 ($d=12.7$mm; SD type below 785N/mm² of the design yield strength, $f_{cd}'=333$ N/mm²) and D13 ($d=13.1$mm; SBPD type over 785N/mm² of the one, $f_{cd}'=1424$N/mm²) for the primary rebars, and U64(SBPD type) for the tie bar were used for preparation of the reinforcement cages. The specimen size of the column model and the core size were 150×150×530mm and 120×120mm, respectively. The nominal pitch spacings were five kinds of 25mm, 50mm, 75mm, 125mm, and 500mm. Figure 3 illustrates the examples of reinforcement cages in case of the SD type. The average compressive strength of the structural concrete with the maximum size of aggregate of 10mm was 39.4N/mm² at 28days underwater curing. The procedure placing concrete is first to fill it into the reinforcement cage, secondly to set down the filled cage into the mould for flexure, thirdly to pour the screened mortar into the part of covering and lastly enough to compact the whole to be in a body by the table type vibrator. The compression test was carried out by use of the 5000kN universal type testing machine.

3.2 Experimental Result

(1) General failure mode

Figures 4 and 5 show the failure modes in cases of the spacings 25mm, 50mm, 75mm, 125mm and 500mm for the SD type primary rebar and for the SBPD type one, respectively. In general, the crackings on the primary rebars and the spall-off of covering concrete are distinguished. The case of the spacing s=25mm in both the SD type and the SBPD type is the most ductile and the effective cross sectional area is never spalled off. The case of the spacing s=500mm is the most brittle and that the effective cross-sectional area happens to be deeply spalled off to the extent of about thirty percent as reported previously. The cases of the spacings s=50mm and 75mm are moderately ductile and have been already observed that the effective cross-section are spalled off only to some extent.

(2) Relation between load-carrying capacity and spacing of tie bars.

Figure 6 displays the relationship between the load-carrying capacity and the spacing of tie bars. In any case, the load-carrying capacity increases with decrease of the pitch spacing, furthermore, the use of the more practicably high strength primary rebar from the SD type of ordinary strength to the SBPD type of high-strength (cf. Figs. 10 and 11), is very advantageous to improve it. Such a general tendency is as similar as previously reported. Especially, a large attention must be paid to the fact that the load-carrying capacity gradually approaches an asymptote, that is, "the upper-bound of ultimate load-carrying capacity" in spite of the difference
Fig. 4  Failure modes of SD type RC column (s:mm)

Fig. 5  Failure modes of SBPD type RC column (s:mm)

Fig. 6  Relationship between load-carrying capacity and spacing of tie bars
in quality of primary rebars of RC columns, elucidated already by the previous report\(^1\). It deserves special mention that the high-strength effect of primary rebar is reflected under the lateral confinement by tie bars. Such an experimental fact indicates that the critical loads for the different materials are equal because the buckling load is independent of the strength of materials, if the size and the elastic modulus of columns are identical, when the pitch spacing widens beyond 300mm, the lateral confinement displays no longer its distinguished effect, and both curves for the SD type and the SBPD type approach the upper-bound load-carrying capacity, given by Eq (6) in place of Eq (1), as a fundamental design equation for the tied column, in spite of the difference of strengths of primary rebars.

4. COMpressive LOAD-CARRYING CAPACITY UNDER CONSIDERATION OF LATERAL CONFINEMENT

It can be understood that the lateral confinement begins to function effectively when the pitch spacing reduces below 300mm for the SBPD 1275 (\(G_\text{sd}=1424\text{N/mm}^2\)) and 100mm for the SD295 (\(G_\text{sd}=333\text{N/mm}^2\)) as shown in Fig 6, because the loss of core sectional area is restrained and the load-carrying capacity depends on the buckling stress of primary rebar (cf. Fig 11).

4.1 Effective Confinement Coefficient method

Mander et al.\(^6\) analyzed that the confined strength can be obtained by estimating the confining stress by virtue of the tie bars. The confining stress \(\Gamma_i\) is given by Eq (8)

\[
\Gamma_i = k_i \rho_i \tau_{ab}
\]

Where, \(\rho_i\) is the steel ratio of tie bar, expressed by \(2A_s/(b_2\cdot s)\), \(A_s\) is the area of tie bar, \(b_2\) is the side length of square cross-section of core concrete, \(s\) is the spacing between tie bars, \(\tau_{ab}\) is the yield strength of tie bar, and \(k_i\) is the effective confinement coefficient of comprehending the configuration of primary rebars and the spacing between ties, given by Eq (9)

\[
k_i = \frac{b_2}{\sum_{i=1}^{n} (w_i) \cdot s^2} \cdot \frac{1}{(1-s/2b_2)(1-\rho_{is})}
\]

where, \(w_i\) is the clear distance between primary rebars at any spacing, \(n\) is the number of primary rebars, \(s\) is the clear distance between tie bars, and \(\rho_{is}\) is the steel ratio of primary rebars to the concrete section.

Figure 7 exhibits the relationship between the confining stress ratio \(\alpha = \Gamma_i/\Gamma_c\) and the confined strength ratio \(\xi = \Gamma_c/\Gamma_{ac}\), in case of the isotropical confinement. Consequently, the strength of confined concrete can be estimated by the flow of calculation procedure as follows.

\[
k_i = \frac{\Gamma_i}{\Gamma_c} = \frac{\Gamma_c}{\Gamma_{ac}} = \frac{\xi}{\alpha}
\]

where, \(\Gamma_c\) is the structural concrete strength.

4.2 Buckling Method of Primary Rebar

It is understood that the "double spacings" between tie bars correspond to the buckling length of finite element primary rebar as shown in Fig. 8 concerning its buckling model, supposed from the phenomenal viewpoint. Thus, the load-carrying capacity of confined RC column not exceeding the pitch spacing, 300mm can be evaluated by Rankine's equation (14) by using the "double spacings" as the buckling length of primary rebar, that is, \(l = 2s\). The twelve times of the diameter of primary rebar, here, \(12\times13\text{mm} \approx 150\text{mm}\) is designated as the maximum pitch spacing by the design detail\(^1\). Figure 9 illustrates the relationship between the load-carrying capacity of confined column and the pitch spacing in cases of the SD type and the SBPD type. The "double pitch method" fairly well agrees with the experimental values as for both types as obviously from the concerned figure. On the other hand, the "Mander's method" is 1.4 to 1.6 times larger than the experimental values as for them. The above-

\[
\text{Ex: When } \alpha = 0.15 \text{ and } \Gamma_c = 30\text{N/mm}^2, \quad \xi = 1.81; \quad \text{so, } \Gamma_{ac} = 1.81 \times 30 = 54.3\text{N/mm}^2
\]

\[\text{Fig. 7 Relationship between confining stress ratio and confined strength}\]
mentioned tripartite load-carrying capacities, however, have a tendency to coincide with each other according to the expansion of pitch spacing and they ultimately result in conversing to the upper-bound load-carrying capacity as indicated in Fig. 6.

4.3 Load-Carrying Capacity Considering Structural Design Detail

The Standard Specification for Concrete in Japan ignores the confinement effect by virtue of tie bars in case of obtaining the load-carrying capacity of RC column in spite of reinforcing it practically, according to the design detail[3] as given by Eq. (10).

\[ s \leq 12 \phi \]  

where, \( s \) and \( \phi \) are above-mentioned.

Therefore, the slenderness ratio \( \lambda \) for the so-called “finite elemental buckling length” in case of \( l=2s=24 \phi \), can be expressed by Eq.(11) from Eq. (4)

\[ \lambda = \frac{2s}{(\phi/4)} = 96 \text{ (const.)} \]  

Thus, the “elemental buckling stress” can be obtained as follows from Eqs. (6) and (11), as displayed in Fig. 10 which composes the relationship between the buckling stress \( \sigma \), and the design yield strength of primary rebar \( f'_{yd} \) under the constructional condition of \( s=12 \phi \)

\[ \begin{align*}
\sigma & = f'_{yd} / (1+0.00467 f'_{yd}) \\

\end{align*} \]  


The buckling stress for the yield strengths 333N/mm² (SD type) and 1424N/mm² (SBPD type) are approximately 130N/mm² and 186N/mm², respectively; so, the difference between both of them is a fairly large value of 56N/mm². Therefore, it stands to reason that the strength of primary rebar is so high as to be advantageous to improve the load-carrying capacity of RC column.

The relative ratio \( \nu \) of the buckling stress \( \sigma \), for any yield strength to that \( \sigma_{yd} \) for the standard one is given by Eq.(13), and Fig.11 shows the relationship between the buckling stress and the design yield strength of primary rebar in case of \( s=12 \phi \) and \( l=2s \).

\[ \begin{align*}
\nu & = \sigma / \sigma_{yd} = \Phi f'_{yd} / (1+0.00467 f'_{yd}) \\
\Phi & = (1+0.00467 f'_{ydo}) f'_{ydo} \\

\end{align*} \]
Fig. 9  Relationship between load-carrying capacity of confined column and its pitch spacing
In Cases of SD type and SBPD type

Fig. 10  Relationship between buckling stress and yield strength of primary rebar when $s=12\sigma$ and $l=2s$
The extreme value \( v_m \) of Eq.(13) is as follows.

\[
v_m = \lim_{\varepsilon \to 0} v = 214.1 \Phi \tag{15}
\]

Now, assuming that \( f_{yd}^{\text{ub}} \) is 295N/mm² (SD295), \( v_m \) is 1.726, pointed together in Fig. 11. The kink obtained by the logarithmic representation of the curve in Fig. 11, that is, the singular point is \( f_{yd}^{\text{up}} = 490 \text{N/mm}^2 \) (SD490). As stated above, although the buckling stress is \( 295 \text{N/mm}^2 \times 1.73 \approx 510 \text{N/mm}^2 \) as the extreme value, imaginatively, it is in present practically \( 295 \text{N/mm}^2 \times 1.50 \approx 443 \text{N/mm}^2 \) < the yield strength \( f_{yd}^{\text{up}} = 1275 \text{N/mm}^2 \) (SBPD1275), and especially the effect of high-strength is remarkable under the condition \( f_{yd}^{\text{up}} \geq 490 \text{N/mm}^2 \).

6. CONCLUSIONS

(1) The present common equations for the upper-bound of compressive load-carrying capacity concerning both tied and spiral columns are contrary to the “ultimate limit state design concept”, giving the overestimate and then being undesirable for the column design.

(2) In general, the buckling behavior of primary rebar must be taken in the common equation for the upper-bound of load-carrying capacity of tied RC column as given by Eqs. (6) and (7), agreeing well with the experiment.

(3) The effective confinement coefficient method by Mander et al. supplies the excessive load-carrying capacity for the confined RC column.

(4) The present buckling method of primary rebar taking in the double pitch spacings as the buckling length on the basis of the phenomenal fact estimates well the load-carrying capacity for the confined RC column.

(5) The strength of primary rebar is so high as to be advantageous to improve the load-carrying capacity as expressed by Eq. (12), and/or by Eq. (13) if by virtue of the buckling stress ratio.

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