Analysis of Thrust Performance for Paddling Locomotion

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Optimal ways of locomotion in paddling are analyzed theoretically and verified by observing locomotion of webbed or broad feet of turtles swimming in a water channel. In the optimal locomotion, a paddle is placed perpendicular to a moving direction of turtles. And a direction of paddling motion may vary according to an angle of attack of the paddle whereas lift-to-drag ratio is the highest at maximum efficiency and fluid dynamic coefficient \( C_D(=\sqrt{C_L^2 + C_D^2}) \) is maximum at maximum thrust.

1. Introduction

Paddling is used by many swimming creatures as a primary propulsive device such as webbed feet of water fowls, bristled legs of aquatic insects and webbed or broad feet of aquatic mammals, some species of Amphibia and Reptilia. It is a propelling locomotion by reciprocal motion of any corresponding driving tools or devices. Unlike snake as a continuous movement, the reciprocal motion of paddling consists of a power stroke and recovery stroke. It is generally defined that a power stroke generates forward thrust by fully spread paddles, while a recovery stroke minimizes unwanted drag during returning back to an initial position for the next routine.

Aigeldinger et al. 1) studied a rapid escape behavior of utilizing paddles by mallard (Anas platyrhynchos) duckling. Velocity, angles, and frequency of paddling strokes were measured experimentally. They concluded paddling motions of their webbed feet were generate both thrust and lift force. Fish et al. 2) investigated metabolic rate of platypus (Ornithorhynchus anatinus Shaw) while paddling or swimming in water submerged. These studies, however, were done physiologically rather than bio-mechanically. Among other studies, sweeping, a similar reciprocal motion to paddling, was analyzed by Azuma, one of the authors et al. 3) They investigated sweeping motion of "Ro," an oriental scull, hydro-dynamically. Their study showed that the operation of "Ro" by an expert was conducted with optimal efficiency. Taking their study into considerations, it is assumed that paddling locomotion by animals should be optimal at almost any occasions, that is, efficiency is primal in a normal action and thrust force is in urgent.

In this study, two kinds of turtles are investigated concerning towing drag tests and free swimming tests to analyze their ways of paddling. One is a reeve's turtle (Chinemys reevesii) as a representative of pond turtle and a slow swimmer. The other is a soft-shelled turtle (Pelodiscus sinensis), which is a fast swimmer.
Nomenclature

\( AR \) : Aspect Ratio
\( C_{DP} \) : Parasite drag coefficient of the body
\( C_D \) : Drag coefficient of the paddle
\( C_L \) : Lift coefficient of the paddle
\( C_P \) : Power coefficient of the paddle
\( C_R \) : Coefficient of resultant force
\( C_T \) : Thrust coefficient of the paddle
\( D \) : Drag force of the paddle
\( D_B \) : Parasite drag of the body
\( L \) : Lift force of the paddle
\( P \) : Power of the paddle
\( R \) : Resultant force of \( L \) and \( D \)
\( S \) : Area of the paddle
\( S_B \) : Cross sectional area of the body
\( T \) : Thrust force of the paddle
\( U \) : Driving velocity of the paddle
\( V \) : Advance velocity of the body
\( \alpha \) : Angle of attack to the paddle
\( \beta \) : Slip angle
\( \gamma \) : \( \gamma = \tan^{-1}(L/D) \)
\( \delta \) : Driving angle of the paddle
\( \rho \) : Density of water
\( \theta \) : Tilt angle of the paddle

\( \cos(\theta + \alpha) \)

\( \alpha \) : Angle of Attack
\( \beta \) : Slip Angle
\( \gamma \) : Gradient Angle of \( L/D \)
\( \delta \) : Driving Angle
\( \theta \) : Tilt Angle of Paddle

II Steady Paddle Translation

As shown in Fig. 1, an inclined paddle by tilt angle \( \theta \) is moved diagonally with driving velocity \( U \) and driving angle \( \delta \) while the body moves with advancing velocity \( V \). Thrust force \( T \), power \( P \) and efficiency \( \eta \) can be defined by using them as follows:

\[
T = \frac{1}{2} \rho W^2 S C_R \cos(\gamma + \alpha - \theta) = \frac{1}{2} \rho W^2 S C_{\gamma} \tag{1}
\]

\[
P = \frac{1}{2} \rho W^2 U S C_R \cos(\gamma - \beta) = \frac{1}{2} \rho W^2 S C_{\gamma} \tag{2}
\]

\[
\eta = TV / P = \cos(\gamma + \alpha - \theta) / ((U/V) \cos(\gamma - \beta)) \tag{3}
\]

where

\[
L = \frac{1}{2} \rho W^2 S C_L \quad D = \frac{1}{2} \rho W^2 S C_D \tag{4a, b}
\]

\[
C_R = \sqrt{C_L^2 + C_D^2} \tag{4c}
\]

\[
\gamma = \tan^{-1}(L/D) \tag{5}
\]

In the above variables, the following geometrical relations are established:

\[
W / V = (U/V) \cos \beta - \sqrt{1 - (U/V)^2 \sin^2 \beta} \tag{6}
\]

\[
\tan(\theta - \alpha) = (U/V) \sin \beta / ((U/V) \cos \beta - (W/V)) \tag{7}
\]

\[
\delta = \theta - (\alpha + \beta) \tag{8}
\]

Eqs. (6) and (7) are reduced to the following equation:

\[
\cos(\theta + \alpha) \cos(\gamma + \alpha - \theta) = \frac{1}{2} \rho W^2 S C_{\gamma} \tag{9}
\]
\[ \beta = \sin^{-1}\left(\sin(\theta - \alpha)/(U/V)\right) \] (9)

In a trimmed state, thrust force \( T \) must be equal to parasite drag of the body, \( D_p \)

\[ T = \sqrt{2} \rho W^2 S C_T = D_p = \sqrt{2} \rho V^2 S_p C_{DP} \] (10)

where \( S_p \) is a drag area of the hull or driven body. Let us introduce an exemplified paddle assumed to be a flat and rectangular plate whose aspect ratio \( AR \) is 0.68. The hydrodynamic characteristics of the plate is shown in Fig. 2.

General characteristics in Figs. 3(a)(b)(c)(d) are calculated performance of general paddling creatures. Figure 3(a) represents thrust efficiency \( \eta \) for a specified value of drag ratio \( S_p C_{DP}/SC_T = 5.0 \) as a function of tilt angle \( \theta \) and angle of attack \( \alpha \) in a trimmed state. Maximum efficiency \( \eta_{max} \) is obtained at \( \theta = 90^\circ \) and \( \alpha = 12^\circ \), where lift-to-drag ratio is maximum in the characteristic curve shown in Fig. 2.

In order to show the state of paddling locomotion easily, angle of attack \( \alpha \) is converted to driving angle \( \delta \) by Eqs. (8) and (9), and thrust efficiency \( \eta \) is redrawn in Fig. 3(b) based on tilt angle \( \theta \) and driving angle \( \delta \). Both of the shapes of Figs. 3(a) and (b) are convex to the upper part and each has a peak. Similarly, Figs. 3(c) and (d) represent thrust coefficient \( C_T \) by the parameters \( \theta - \alpha \) and \( \theta - \delta \) respectively. Maximum thrust can be obtained at \( \theta = 90^\circ \) and \( \alpha = 42^\circ \), where resultant force of lift and drag is maximum in the specific curve of Fig. 2. Figures 3(e) and (f) demonstrate power coefficient \( C_p \) and driving velocity ratio \( U/V \) in the same situation where two parameters are \( \theta \) and \( \delta \). Power coefficient \( C_p \) has a high peak and driving velocity ratio \( U/V \) has a bottom peak. The latter means that paddle driving velocity \( U \) must be 2.3 times (the value of bottom peak) faster than body velocity \( V \) in order for the body to move forward.

![Figure 2. Hydrodynamic force acting on low-aspect ratio plates](image)

![Figure 3. General characteristics of paddling for an exemplified paddle at \( S_p C_{DP}/SC_T = 5.0 \)](image)
Maximum values of characteristics $\eta$, $C_T$ and $C_p$ in Figs. 3(a)~(f) are determined as a function of drag ratio $S_pC_{DP}/SC_D$, as shown in Fig. 4(a)~(d).

The results are:

(i) Maximum efficiency $\eta_{\text{max}}$ can be given when $\theta=90^\circ$ and $\alpha=12^\circ$ at any drag ratio $S_pC_{DP}/SC_D$, where lift-to-drag ratio is maximum, $\gamma_{\text{max}}=[\tan^{-1}(L/D)]_{\text{max}}$.

(ii) Maximum efficiency $\eta_{\text{max}}$ decreases from $\eta_{\text{max}}=1$ at a point of $S_pC_{DP}/SC_D=W/V=0$ and $U/V=1$, in other words, the size of the paddle becomes smaller, when the drag-area ratio increases.

(iii) Maximum thrust can be obtained at $\theta=90^\circ$ and $\alpha=42^\circ$ where coefficient $C_T$ is maximum at any drag ratio $S_pC_{DP}/SC_D$. The point is that where resultant force of lift and drag on the paddle is maximum.

(iv) Each value of maximum thrust is always smaller than that of maximum efficiency.

Figure 3. General characteristics of paddling for an exemplified paddle at $S_pC_{DP}/SC_D=5.0$
Figure 4. Optimal paddling

Figure 5. Turtles used in observation
III. Materials and Methods

Experimental objects were a reeve’s and a soft-shelled turtle shown in Fig. 5(a), (b) respectively. Reeve’s turtle has a hard oval carapace having three keels in the back with broad feet and soft-shelled turtle has a soft, relatively flat and circular shape carapace with webbed feet. Both were 18 cm in length.

A water channel has a measurement section made of plexy glass. The size of the water channel is 300 mm x 300 mm x 1200 mm. The maximum velocity of water flow in this channel is 0.6 m/s. Observation can be performed from both bottom and left side against the running stream. Water temperature in experimental tests is adjusted at 30°C, which is desirable for active motion of these reptiles.

Free Swimming Test

There are two swimming conditions applied to the turtles in the water channel. One is that each turtle was left in a stream of the circulating water channel in order to investigate the turtle’s maximum speed in a natural steady swimming condition. For this test, the water speed is controlled accordingly for the position of the swimming turtles being stayed at a part of the test section. The other is that let each turtle swim freely in a still water to examine its normal velocity. Each swimming form was recorded by VCR from the both bottom and left sides simultaneously by using four mirrors as shown in Fig. 6.

Towing Test

The purpose of this test was to measure hydrodynamic drag of each turtle. A balance system for measuring the drag was made with a spring scale balance as shown in Fig. 7. Trim angles of testing objects were adjusted to be horizontal by using thin fishing lines. Each turtle was fixed onto a flat plate to eliminate any movement of the turtle such as a movement of its body and feet to obtain a steady state drag. The velocity of the water flow was set at 0.35 m/s, the maximum swimming speed of the soft-shelled turtle resulted from the free-
swimming test. A drag of the flat plate itself was measured as well as a drag of the live turtle with the plate. Then, the excessive drag of the flat plate without turtle was subtracted from the respective datum. In order to verify this method, a dead body of a different kind of turtle was used. As a result, a datum from the drag of the live turtle without excessive drag of the plate has no difference from the one from the drag of the dead turtle without excessive drag of the plate. Parasite drag coefficient of the body $C_{DP}$ was obtained by normalizing a value of the speed of running water in the channel and a value of the cross sectional area of each turtle.

Test Results

Table 1 shows results of the free and the towed swimming tests. The maximum swimming speed of the soft-shelled turtle is 0.35 m/s and that of the reeve's turtle was 0.2 m/s by the free-swimming test in a circulating water flow. By the towing test, parasite drag coefficient of the soft-shelled turtle is obtained as $C_{DP}=0.52$ and that of the reeve's turtle is $C_{DP}=0.77$ respectively. Drag coefficient $C_D$ of the paddle of each turtle in this analysis was assumed to be the same $C_{DP}$ of its body. To define it, drag coefficients of half sphere and round oval examined by Hoerner were referred.

According to a biological research, turtles control their buoyancy by changing volume of air in their lung in order to sink or float. In the observa-

<table>
<thead>
<tr>
<th></th>
<th>Max Speed</th>
<th>Parasite Drag</th>
<th>Paddle Drag</th>
<th>Forward Tilted Angle</th>
<th>Cross Sectional Area of Body</th>
<th>Paddle Area</th>
<th>$S_D C_{DP}/S_C_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unit</td>
<td>$m/s$</td>
<td>$C_{DP}$</td>
<td>$C_D$ (assumed)</td>
<td>$\text{cm}^2$</td>
<td>$\text{cm}^2$</td>
<td></td>
</tr>
<tr>
<td>Reeve's turtle</td>
<td>0.20</td>
<td>0.77</td>
<td>0.77</td>
<td>-25°</td>
<td>70.5</td>
<td>3.8</td>
<td>7.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.52</td>
<td>0°~12°</td>
<td>57.9</td>
<td>10.1</td>
<td>13.5</td>
</tr>
</tbody>
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Table 1. Aspects of swimming turtles
tion of free-swimming test, the swimming position in depth of the reeve’s turtle sometimes stayed almost 10cm deep under the surface without any paddle locomotion. The soft-shelled turtle stayed at the bottom of the water channel most of the time without moving. Therefore, locomotion of paddles is utilized not for buoyancy but for generating thrust force when they swim in level.

The swimming form or locomotion of paddling, tested in a still water, was recorded by VCR and analyzed by a PC system. Figures 8(a) and (b) show trajectories of the paddling locomotion of the reeve’s turtle and the soft-shelled turtle respectively. In the bottom view of Fig. 8(a) and (b), the dots are differentiated in size in order to show time elapse of the paddle movement. The dots become larger as the paddle moves. In Figs. 8, both power and recovery strokes were illustrated. The only recovery stoke of the side view in Fig. 8(a) is not because of unsuccessful recording.

The diagram of paddle locomotion in the side view of Fig. 8(b) indicates that soft-shelled turtle swims more efficiently compared with the swim of reeve’s turtle. Concerning soft-shelled turtle, both locomotion planes of fore and hind legs are on the same level on the side of the carapace. Both legs transmit thrust force to its body efficiently.

On the other hand, locomotion planes of fore and hind legs of reeve’s turtle are not on the same level. The only locomotion plane of the hind feet passes over near the gravity center of the body and the hind feet fully generate thrust force.

As shown in Fig. 9, it is observed that the body of the reeve’s turtle was forward-tilted with a negative pitch when it is swimming parallel to the water surface at maximum speed. The pitch angle for reeve’s turtle is $-25^\circ$. For the soft-shelled turtle, it is from $0^\circ$ to $-12^\circ$ that varied according to its weight. Configuration of the carapace causes upward lift while swimming. The forward-tilted posture seems to
balance the upward lift against downward lift generated by the posture to maintain their swimming depth. Fore legs of the reeve’s turtle may be utilized for balancing its body by paddling lower than its belly carapace.

The side-view of Fig. 8(a) shows that tilt angle $\theta$ of the hind paddle of reeve’s turtle indicates almost 90° to the advancing direction of the body in power stroke to generate thrust force. The tilt angles $\theta$ of paddles of both fore and hind feet of soft-shelled turtle are 90° to the advancing direction of the body in power stroke as well shown in the side view of Fig. 8(b).

On the other hand in the recovery stroke, the paddles of soft-shelled turtle were inclined backwards to minimize unwanted drag.

As mentioned in section II, the optimal tilt angle $\theta$ should be kept 90°. Therefore, the analysis agrees to the actual locomotion of paddling.

Velocity transition of the fore and hind feet and that of the center of the body are shown in Figs. 10(a) and (b). In the theoretical analysis, each driving velocity ratio $U/V$ and the driving angle $\delta$ at an optimal state are related to the angle of attack $\alpha$. In observation, driving angle $\delta$ of the paddle movement was so three-dimensional that a values of the driving angle $\delta$ could not be taken this time. Therefore, the only driving velocity ratio $U/V$ was determined in order to confirm our analysis. The maximum velocity $U$ of fore and hind feet, and the averaged velocity $V$ of the body of each turtle in Fig. 10(a) and (b) are taken to calculate for driving velocity ratio $U/V$ as observed data. Then plotted over the optimal $U/V - S_{f}C_{D}/S_{C}C_{D}$ curves in Fig 4(c), now shown as Fig 11 in order to confirm the state of paddling locomotion. The plotted point A for hind feet of reeve’s turtle is close to the thrust maximum curve: $T_{\text{Max}}$. The swimming condition of reeve’s turtle here is observed as under quick motion. As shown in Fig. 10(a), the average velocity of the body here is 0.17m/s which is close to its maximum speed 0.20 m/s resulted, in the free-swimming test shown in Table 1.

For that reason, the locomotion mode might be considered under the maximum thrust condition. Similarly, plotted points of B and C of soft-shelled turtle are placed closely to the efficiency maximum curve: $\eta_{\text{Max}}$. The soft-shelled turtle swam gently in observation. In Fig. 10(b), the average velocity of the body is 0.18 m/s, which is almost a half of its maximum speed, 0.35 m/s, in Table 1.

Therefore, the observed locomotion of the soft-shelled turtle is satisfactory in the efficiency maximum mode. The point of the fore feet of reeve’s turtle, not plotted here, is far from any of the optimal curves because the fore feet of the turtle is not utilized only for a thrust force as stated earlier. This study also states that this analysis agrees with the observation as well.

IV. Conclusions

The paddling locomotion was analyzed theoretically and the data taken from the actual swimming observation of turtles were compared with the analytical solutions. The following analyzed results agree to the observed facts.
1. The paddle plane is kept perpendicular to a direction of motion of the body.
2. A driving velocity ratio $U/V$ is different at the maximum efficiency and at the maximum thrust.

And the following is predicted by the analysis:

3. For the maximum efficiency, the angle of attack $\alpha$ against the paddle plane should be the angle $\alpha$ where the lift-to-drag ratio is maximum, and for the maximum thrust, it is the one where the fluid dynamic coefficient $C_R (= \sqrt{C_L^2 + C_D^2})$ is maximum.

References