Planar Geometrical Analysis for Design of the Shortest Incision to Open the Dura Mater

—Technical Note—

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Abstract

Quadrilateral dural window is opened with a conventional incision design, such as a pair of diagonal lines or a rectangular shape, but the total cutting length is not the shortest possible. Shorter incision length will have a lower risk of dysraphia associated with cerebrospinal fluid (CSF) liquorhea or related CSF infection. We propose a new and effective dural incision design with the shortest cutting length for quadrilateral dural openings. We investigated the design of the dural incision using a simple planar geometrical figure. We discovered the shortest network design to connect the four vertices of the quadrilateral. The shortest network design was formed of five line segments with two three-pronged interconnections (TPIs) with the same angle of 2π/3 between any two lines (2π/3-TPI). In practice, first we must draw a quadrilateral W horizontally then add two equilateral triangles outside W. Using a 2π/3-bent wire, the 2π/3-TPIs are traced on the path connecting the outward vertices of the equilateral triangles. Using this method, we can reduce the incision length by 10% from conventional designs using a pair of diagonal lines.

Key words: brain surgery, dural incision, dural opening, shortest incision design

Introduction

Adopting the shortest dural cutting line possible during brain surgery will avoid unnecessary suturing of dura mater, and lead to lower risk of dysraphia associated with cerebrospinal fluid (CSF) liquorhea or related CSF infection. Standard dural incision involves opening of a quadrilateral dural window by cutting the dura mater first in a semicircular shape and then in a pair of auxiliary lines, a rectangular line or a pair of diagonal lines. Such dural incision techniques are taught by mentors or textbooks and regarded as the standard.5,6 However, the total cutting length of these standard designs vary and do not adopt the shortest length possible. The dural incision design with the shortest length has not yet been discussed, although some technical notes to prevent CSF leakage at dural closure have been published.1–4 To open a quadrilateral dural window, the four vertex points must be connected with a cutting line. Discovering how to connect the four points with the shortest possible line segments is called “Steiner’s tree problem.” The solution is to create a network with two three-pronged interconnections (TPIs). However, this approach to the problem has not been evaluated in operative procedures.

The present study discusses the most effective dural incision design with the shortest cutting length for opening of a quadrilateral dura and demonstrates a concise intraoperative method based on a simple planar geometrical figure.

Method

The most effective opening design for quadrilateral dural window surgery was studied by connecting the four vertices of the quadrilateral with line segments. The objective was to determine the dural in-
Fig. 1 Correlations between Patterns I, II, IV, and III. Moving P₁ onto A, P₁ and P₂ onto A and D, respectively, or merging P₁ and P₂ together results in Pattern II, Pattern I, or Pattern IV. (a) The quadrilateral W is set horizontally and defined as A, B, C, and D in a counterclockwise direction. (b) Necessary condition for the solution of Problem 1: The incision must consist of five line segments forming Pattern III (heavy solid lines), AP₁,BP₁, CP₂, DP₂, and P₁P₂, where both P₁ and P₂ are points of the area limited by W. Open circle indicates three-pronged interconnection.

Fig. 2 Design of the shortest network. If we set two equilateral triangles, ΔAEQB and ΔDFQC, at both sides of W and construct the equilateral triangles ΔAQ₁P₁ and ΔDQ₂P₂, the path from E to F (the polyline of EQ₁ + dQ₁P₁ + dP₁P₂ + dP₂Q₂ + dQ₂F) is equal to L (a, b). The shortest length of L is the straight line connecting E and F (c). Thus, the network design in (d) is the shortest (Property 1). Note that the dura mater is cut with the design (e). Panel (f) shows the polygon AEQP₁ in (c). ∠P₁BA = ∠Q₁EA = ∠P₁EA. The four vertices of the quadrilateral AEQP₁ are located on a circle. If ∠EP₁B = ∠EAB = π/3 and ∠AP₁Q₁ = π/3, then ∠AP₁B and the exterior ∠AP₁Q₁ must be 2π/3 (Property 2).

The shortest line segment connecting two points is a straight line. Thus, the solution to Problem 1 must consist of five line segments, AP₁, BP₁, CP₂, DP₂, and P₁P₂, with a set of P₁ and P₂ within the quadrilateral W (Fig. 1b).

The design of the shortest network is as follows. First, the equilateral triangles (Δ) ΔAEQB and ΔDFQC are constructed outside W followed by the equilateral triangles ΔAQ₁P₁ and ΔDQ₂P₂ (Fig. 2a). Then, ΔAQ₁E and ΔDQ₂F are rotated by π/3 into ΔAP₁B and ΔDP₂C, respectively. Therefore, BP₁ = EQ₁ and CP₂ = FQ₂. Furthermore, AP₁ = Q₁P₁ and DP₂ = Q₂P₂. Then, the total network distance, \( L = dAP₁ + dBP₁ + dCP₂ + dDP₂ + dP₁P₂ \), is the distance of the path from E to F (the polyline of EQ₁ + Q₁P₁ + P₁P₂ + P₂Q₂ + Q₂F) (Fig. 2b). The path from E to F is shortest when it is straight (Fig. 2c). Hence, L is shortest when P₁ and P₂ are on the straight line EF (Property 1) (Fig. 2d). The W-shaped dural window is opened with the design in Fig. 2e. Under this condition, ∠P₁BA = ∠Q₁EA = ∠P₁EA, and the four ver-
Fig. 3 Analysis of other cases. Case 1 has one three-pronged interconnection with the same angle of $2\pi/3$ between any two lines (2$\pi$/3-TPI) (P2) and one bent point (P1) of EF (a). Case 2 has no 2$\pi$/3-TPI and two bent points (P1 and P2) of EF (b). Case 3a has no 2$\pi$/3-TPI and two bent points (P1 and P2) of EF (c). Case 3b has one 2$\pi$/3-TPI (P2) and one bent point (P1) of EF (d).

Fig. 4 Practical application of the method. Left: Select a quadrilateral W (trapezoidal broken line in this case) and draw an equilateral triangle with one side sharing the lateral line segment of W using 3-0 silk thread with a marker at its end. Right: Determine the 2$\pi$/3-TPIs on the line segment by applying the 2$\pi$/3 bent wire on both ends of the lateral line segment. This simple manipulation is enough to discover the 2$\pi$/3-TPIs (refer to Fig. 3).

**Discussion**

We compared the total lengths of the dural incision in this method with several conventional designs employed by many neurosurgeons. We assumed that the size of the dural window was a 10 × 15 cm rectangle (Fig. 5A). Our method requires 32.32 cm of dural incision (Fig. 5E), whereas a conventional method using a pair of diagonal lines requires 36.06 cm (Fig. 5C). Our method requires an overall incision, which is 3.74 cm shorter or is almost 10% shorter. Other common methods using a rectangular or semicircular incision together with additional lines required longer incisions (35 cm and 33.55 cm, respectively) than our method (Fig. 5B, D).

Our method still contains a problem to be solved. At the beginning of our procedure to determine the shortest network design, we attach two equilateral triangles to the lateral line segments of the quadrilateral W (Case A). However, a pair of equilateral triangles may be formed in other cases. The triangle pair can be placed on the top and bottom line segments of W (Case B). We could not obtain a proof of design requires that P1 or P2 be located on a vertex of W. Thus, the solutions do not always have a pair of 2$\pi$/3-TPIs on line EF, as shown in Fig. 3.

To apply this design procedure in surgical practice, first we must draw a hexagon AEBCFD by adding two equilateral triangles outside W and drawing line segment EF. Next, judge the condition of W by the positional relationship of line segment EF to W. If 2$\pi$/3-TPIs are present, a wire template with an angle of 2$\pi$/3 on the lateral line segment of W will identify the locations of the 2$\pi$/3-TPIs on the path from E to F (Fig. 4).
the conditions for shorter networks. However, we can estimate intuitively that Case A is more favorable than Case B from the following argument.

Let $L_1$ be the distance between the outward vertices of the adopted equilateral triangles for Case A, and let $L_2$ be the distance for Case B. Assuming in a rectangle, $ABCD (AB = CD = m, AD = BC = am; 1 < a)$, $L_1$ and $L_2$ are $(\sqrt{3} + a)m, (1 + \sqrt{3}a)m$, respectively. Therefore, $\lim_{a \to 1} L_1/L_2 = 1, \lim_{a \to \infty} L_1/L_2 = 1/\sqrt{3}$. Hence, $1/\sqrt{3} < L_1/L_2 < 1, L_2$ is greater than $L_1$.

This new method has some problems in practice. Closure of the dural window requires closure of the tripartite dural incision, which may be more troublesome compared to a rectangular incision. Proximity to the dural venous sinuses and location of underlying pathology may restrict use of our incision design. Nonetheless, the approximately 10% shorter incision length will certainly reduce the risk of CSF leakage.

This study examined the design for dural incision by considering a planar geometrical figure and determined the design of the dural incision with the shortest cutting length for quadrilateral openings. By employing several planar geometrical analyses, we found the shortest network design to connect four vertex points with a set of five line segments with two $2\pi/3$-TPIs. If the anatomical or pathological conditions of the dural window do not cause restrictions, the present procedure should lower the risk of dysraphia associated with CSF liquorhea or related CSF infection.

**Conflicts of Interest Disclosure**

The authors have no personal financial or institutional interest in any of the drugs, materials, or devices in the article. All authors who are members of The Japan Neurosurgical Society (JNS) have registered online Self-reported COI Disclosure Statement Forms through the website for JNS members.

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