Enhancing a BPSK receiver by employing a practical parallel network with Stochastic resonance

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Abstract: Stochastic resonance (SR) is a noise-enhancement phenomenon that enables the detection of sub-threshold signals by adding noise and using nonlinear systems. This paper explores the applicability of SR in a BPSK receiver with sub-threshold signals. Although received signals are amplified as a result of the nonlinear behavior of the receiver, they are somewhat distorted. This results in the lower performance of SR receivers in comparison with linear receivers. Employing a parallel network of SR systems is expected to solve this problem. The present theoretical analysis demonstrates that in a certain noise intensity range, the output of the network can fully describe an input sub-threshold signal, and hence, the performance close to that of the linear receivers can be obtained. The effectiveness of the SR receiver was also demonstrated through a numerical example of the bit error rate (BER). However, achieving good BER performance requires an infinite number of arrayed SR systems, which is not realistic in practical systems. A design framework for an SR network with a finite number of elements and an appropriate noise intensity that can realize BER performance close to that in linear systems is also provided.

Key Words: Stochastic resonance (SR), parallel network, bit error rate (BER)

1. Introduction
In the situation where a signal to be detected is lower than the detection limit of the system and/or device, noise plays an important role in signal detection. In the framework of Stochastic resonance (SR), because of the nonlinearity in the system, such a sub-threshold signal can be stochastically amplified by adding noise, allowing the shape of the signal to be discerned [1–4]. An example of SR is found in biology; experiments have demonstrated that animals use surrounding noise to sense signals buried in
noise [5, 6]. The mechanism of such amplification has been addressed theoretically and experimentally in past research [2, 7, 8]. The applicability of this interesting phenomenon to various fields, including system control [9], imaging [10, 11], nano-devices [12–14], localization [15], and fault diagnosis [16, 17], has also been discussed.

In wireless communication systems, transmitted signals are attenuated so that the amplitude of the received signal is often smaller than the detection limit. One example of a device in which this occurs is an analog-to-digital converter which is often installed at the front end of a receiver. This device has its own resolution or least significant bit (LSB) for the input signal. This parameter is referred to as the detection limit or threshold, and with a high resolution, such as 20 bits, the threshold is typically on the order of 1μV or higher. Thus, a weak attenuated signal may be below the limit, resulting in detection failure. To solve this problem, the effectiveness of receivers exploiting SR has been investigated [18–24]. With the noise intensity set to the optimal value, it has been revealed that the digital phase of the transmitted signal can be obtained by the receiver [20]. However, the performance of simple SR systems is not better than that of the linear receiver, because of their nonlinearity and the intentional use of environmental noise [24]. To ensure that SR systems are suitable for practical use, their design must be reconsidered.

Several methods of enhancing the effect of SR have been proposed [25–28]. One impressive method is the parallel network of bistable excitable units proposed by Collins et al. [25]: when the network consists of a large number of nonlinear devices, the shape of sub-threshold signals can be fully recovered at the network output. A notable point about this method is that this advantage can be obtained without tuning the noise intensity. The effectiveness of this interesting method in the detection of sub-threshold signals has been numerically evaluated [26, 27]. Such an advantage has been found in various fields, including biological systems [25, 29] and nano-devices [13, 14]. Employing a parallel network in the receiver may bring its performance closer to that of the linear receiver. However, using a sufficiently large number of parallel devices may pose a problem in practical applications.

The contribution of the present study is twofold. First, the potential of using a parallel network in the receiver for binary phase-shift keying (BPSK)-modulated signals was revealed. The present theoretical analysis using the linear approximation method shows that in a certain noise intensity range, the output of the network can fully describe the input sub-threshold signal. This means that the proposed method should achieve a performance close to that of a linear system. The effectiveness of the SR receiver was also numerically demonstrated in terms of the bit error rate (BER). High performance can be obtained when an infinite number of units are deployed in the network, which is not realistic in practical systems.

Toward resolving this drawback, the second contribution of this study was to develop a design framework for a practical network giving the noise intensity and the corresponding finite number of network elements. Such a system should achieve a performance level close to that of a linear system in practical situations. Notably, the optimal noise intensity, which was analytically derived in this work, realized the minimization of the number of the network elements. This point is interesting because even in a parallel network, minimizing the network size requires the tuning of the noise intensity. This contradicts the well-known advantage of without tuning the noise intensity [25].

The remainder of this paper is organized as follows. Section 2 presents the system model and the features of the parallel network of SR systems, hereafter referred to as the parallel SR network. The behavior of the system with an infinite number of the network elements is theoretically analyzed in Sec. 3. Interestingly, this analysis gives a condition for the noise intensity at which the proposed SR receiver is effective. Such effectiveness is also demonstrated through the numerical example in Sec. 4. For the application of the proposed method in practical systems, a design framework for the number of array elements is proposed in Sec. 5. Finally, Sec. 6 presents the conclusions of this study.

2. System model

The system model of the receiver with the SR system installed is shown in Fig. 1. The transmitted signal is assumed to be a BPSK signal: 

\[ s(t) = \sum_i g(t - iT_s) \cos(2\pi f_c t + \pi(d_i - 1)/2), \]

where \( d_i \in \ldots \)
\{+1, -1\} is the ith binary data, \(T_s\) is the symbol duration, \(f_c\) is the carrier frequency, and \(g(t)\) is a rectangular pulse such that \(g(t) = 1\) when \(0 \leq t \leq T_s\) and \(g(t) = 0\) otherwise. The channel noise \(n(t)\) was assumed to be additive white Gaussian with a noise power density of \(N_0\). The received signal is \(r(t) \equiv A s(t) + n(t)\), where \(A\) is the amplitude of the received signal. In this study, \(A\) is assumed to be below the receiver sensitivity. As the signal passes through the parallel SR network, it is stochastically amplified due to the SR effect. The estimated data \(\hat{d}_i\) are obtained by coherent detection with the network output \(y(t)\).

The parallel SR network, which is employed to detect sub-threshold signals, is known to be effective for improving the response of the SR system [25]. A system diagram of the parallel SR network is presented in Fig. 2(a). The network is composed of a parallel array of SR systems, and in each of these systems, the received signal \(r(t)\) with added internal noise \(\xi_i(t)\) is input into a nonlinear system. The noise \(\xi_i(t)\) was assumed to be independent and identically distributed (i.i.d.) Gaussian noise with a mean of zero and a variance of \(\sigma^2\). The detection performance can be enhanced by increasing the number \(K\) of array elements.

In this study, a three-level device was employed as the nonlinear system [21, 22, 24]. The input–output characteristics of this system are shown in Fig. 2(b). The device has symmetrical thresholds and three output levels; analytically, the output of the nonlinear system is expressed as

\[
h(w) = \begin{cases} +1.0 & (w > \eta) \\ -1.0 & (w < -\eta) \\ 0 & \text{otherwise} \end{cases}
\]  

(1)

where \(\eta\) is the threshold. The output of the parallel SR network is then given as

\[
y_K(t) = \frac{1}{K} \sum_{j=1}^{K} h(r(t) + \xi_j(t)) = \frac{1}{K} \sum_{j=1}^{K} h(r_j(t)), \quad r_j(t) \equiv r(t) + \xi_j(t).
\]  

(2)

On the basis of the internal noise characteristics, the term \(r_j(t)\) is an i.i.d. Gaussian variable with a mean of \(r(t)\) and a variance of \(\sigma^2\). Note that the network output is scaled by the number \(K\) of array elements.

The transmitted data are estimated from the network output. In a focused system, simple coherent detection is used. The estimated data \(\hat{d}_i(t)\) are obtained by making a hard decision for the output of the correlator \(z_i\):

\[
\hat{d}_i = \text{sgn}(z_i)
\]  

(3)

\[
z_i \equiv \frac{1}{T_s} \int_{(i-1)T_s}^{iT_s} y_K(t) \cos(2\pi f_c t) dt
\]  

(4)

\[
\text{sgn}(a) = \begin{cases} +1.0 & (a > 0) \\ -1.0 & (a < 0) \end{cases}
\]  

(5)

3. Theoretical analysis of the network behavior

The behavior of the receiver based on the parallel SR network is theoretically analyzed in this section. As described by Collins et al. [25], the receiver behavior strongly depends on the number \(K\) of elements.
in the array. In this analysis, an array with an infinite number of elements is considered, and the required noise variance for the successful estimation of the transmitted data is discussed.

With an infinite array, the network output is expressed by the expected value as

$$y_\infty(t) \equiv \lim_{K \to \infty} \frac{1}{K} \sum_{j=1}^{K} h(r_j(t)) = E[r_j(t)]$$

(6)

where $E[r_j(t)]$ denotes the expected value with respect to the random variable $r_j(t)$. With the probability density function of the random variable $r_j(t)$ defined as $p(r_j(t))$, substituting Eq. (1) into Eq. (6) yields the network output as

$$y_\infty(t) = \int_{-\infty}^{\infty} h(r_j)p(r_j(t))dr_j(t)$$

$$= \int_{-\eta}^{-\eta} -p(r_j(t))dr_j(t) + \int_{-\eta}^{\eta} 0 \cdot p(r_j(t))dr_j(t) + \int_{\eta}^{\infty} p(r_j(t))dr_j(t)$$

$$= \frac{1}{2} \left\{ \text{erf} \left( \frac{\eta + r(t)}{\sqrt{2}\sigma} \right) - \text{erf} \left( \frac{\eta - r(t)}{\sqrt{2}\sigma} \right) \right\}$$

(7)

where $\text{erf}(\cdot)$ denotes the error function.

From Eq. (7), the output of the network does not describe the input waveform $r(t)$ because the output is given by the nonlinear error function. Under certain conditions, however, a linear response is obtained so that the shape of the input signal can be determined by the output. This point can be analytically discussed based on the linear approximation method. The error function can be expressed by the series

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(2n+1)} = \frac{2}{\sqrt{\pi}} \left\{ x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} \cdots \right\}.$$  

(8)

Because of the higher-order terms, this function is typically nonlinear. That is, if $x \gg 1$, the nonlinear terms included in Eq. (8), namely from the second term on, are non-negligible. From Eq. (7), this is the case where the threshold or the received signal is sufficiently large relative to the standard variance of the intentional noise. However, when $x \ll 1$, the function is described only by the linear term:

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} x.$$  

(9)

Substituting Eq. (9) into Eq. (7) demonstrates that the network output linearly describes the input signal $r(t)$ as

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**Fig. 2.** Parallel SR network.

(a) System diagram.

(b) Input-output characteristics of the nonlinear system (three-level device).
The condition \( x \lesssim 1 \) corresponds to \( \eta + A \lesssim \sqrt{2}\sigma \). Therefore, under the condition given in Eq. (11), a network with a sufficiently large number of array elements outputs the original received signal, enabling the successful estimation of the transmitted data.

Figure 3 shows an example of the input and output waveforms of the parallel SR network with an infinite number of parallel elements. For the sake of simple illustration, a received signal with \( d_i = +1 \) and \( A = 1.0 \mu V \) and without channel noise \( n(t) \) was considered. The threshold of the device was set to \( \eta = 1.5 \mu V \). When the noise intensity \( \sigma \) meets the condition given in Eq. (11), the output faithfully describes the input waveform, as shown in Fig. 3(a). The condition may also be satisfied by changing the threshold \( \eta \). This is effective when the tunable noise sources \( \xi_i(t) \) are not available in the receiver. In contrast, when \( \sigma \) is much smaller than \( \eta + A \), the condition in Eq. (11) is not satisfied, the nonlinear terms in Eq. (8) become non-negligible, causing the nonlinearity thus affects the network response. As shown in Fig. 3(b), the output waveform in this case is distorted relative to the input waveform.

4. Numerical examples of the performance of the SR receiver

This section demonstrates the communication performance of the SR receiver with the parallel SR network through numerical examples. The performance measure considered here is the BER, \( p_{\text{err}} \equiv \Pr[\hat{d}_i \neq d_i] \), which is the probability of an erroneous estimation. The results of the numerical examples are shown in Fig. 4. The parameter settings used in the evaluation are given in Table I. The considered

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulation scheme</td>
<td>BPSK</td>
</tr>
<tr>
<td>Channel noise ( n(t) ), internal noise ( \xi_j )</td>
<td>AWGN</td>
</tr>
<tr>
<td>( E_b/N_0 )</td>
<td>6.0 dB</td>
</tr>
<tr>
<td>Noise bandwidth</td>
<td>5.0 MHz</td>
</tr>
<tr>
<td>Signal bandwidth</td>
<td>1.0 MHz</td>
</tr>
<tr>
<td>Carrier frequency ( f_c )</td>
<td>1.0 MHz</td>
</tr>
<tr>
<td>Received signal amplitude ( A )</td>
<td>1.0 ( \mu V )</td>
</tr>
<tr>
<td>Device threshold ( \eta )</td>
<td>1.5 ( \mu V )</td>
</tr>
<tr>
<td>Number of trials in the simulation</td>
<td>( 10^5 )</td>
</tr>
</tbody>
</table>
Fig. 4. BER performance of SR receiver composed of a parallel SR network. The number of array elements is $K \in \{1, 10, \infty\}$. The BER curve for the theoretical BPSK was obtained by plotting Eq. (12).

As shown in Fig. 4, the BER performance improved with increasing number $K$ of array elements. This effect comes from the ability of the summing network to recover a weak input signal at the output [25]. When $\sigma > 1.0 \mu V$, the BER at $K = \infty$ approaches that for the theoretical BPSK BER (dotted line in Fig. 4). As theoretically discussed in Sec. 3, in this case, the response of the network is linear. This means that the SR receiver can realize the theoretical limit (linear receiver) even if the input is a sub-threshold signal. In contrast, when $\sigma < 1.0 \mu V$, the nonlinear response degrades the BER performance even with an infinite number of array elements.

5. Discussion on the application to practical systems

The analytical and numerical discussions in Secs. 3 and 4 demonstrate that an SR receiver with an infinite number of array elements can achieve a BER close to the theoretical BPSK BER, especially with a sufficiently large noise intensity that satisfies Eq. (11). However, in practical systems, it is not possible to realize a network with an infinite number of array elements. In this section, to enable the application of the proposed SR receiver to practical systems, a design framework for the network is provided.

To obtain a BER close to the infinite array case, Fig. 4 shows that the required number of array elements depends on the noise intensity $\sigma$. For example, when $\sigma \ll 1.0 \mu V$, a small number of array elements should achieve a BER similar to that in the case of an infinite array because there is little difference between the curves for $K = 10$ and $K = \infty$. When $\sigma \gg 1.0 \mu V$, the large difference between these two curves indicates that a large number of array elements is necessary to achieve good BER performance. Because the noise intensity $\sigma$ is a tunable parameter in the receiver, the intensity at which the SR receiver realizes a BER similar to the theoretical BER in the infinite array case with the smallest possible number of array elements was derived.
For a theoretical discussion, an error term $\varepsilon$ was introduced to describe the difference between the network outputs between with infinite and finite numbers of array elements. Chebyshev’s inequality, which is well known in the theorem of weak law of large numbers, is introduced for the present analysis. The resulting expression,

$$\Pr\left[|y_K(t) - y_\infty(t)| \geq \varepsilon\right] \leq \frac{\sigma^2}{K\varepsilon^2}, \quad (13)$$

indicates that for a given $K$, as the noise intensity decreases, the chance that the error $\varepsilon$ is observed also decreases. This means that at a small noise intensity, this error is rarely observed. In combination with the linear response condition, this fact indicates that to minimize the number of array elements, the noise intensity should be as small as possible while still meeting the condition given by Eq. (11), i.e.,

$$\sigma \approx \frac{\eta + A}{\sqrt{2}}. \quad (14)$$

The required number of array elements was then obtained through numerical simulation, and the results are shown in Fig. 5. In this figure, the curves are plotted as a function of the degradation factor $\alpha > 1$, which represents the degree to which the resulting BER $p_{\text{err}}$ was degraded relative to $p_{\text{err}}^{\text{BPSK}}$. To clearly understand the behavior when $K < 10$, the inset in Fig. 5 focuses on this region. Note that the scale of the vertical axis in the main figure is linear, whereas that in the inset is logarithmic.

The preferred region where the BER is close to the theoretical BPSK BER with a small number of array elements is in the bottom left corner of the figure. As demonstrated by this plot, $p_{\text{err}}$ depends on the number $K$ of array elements. As the degradation factor increases, the required number of array elements is reduced. It is also shown in Fig. 5 that the required number of array elements strongly depends on the noise intensity. As analytically predicted by Eq. (14), the optimal intensity is approximately $\sigma = 1\mu V$; in this case, low degradation is achieved with the smallest number of array elements.
elements. As discussed in Sec. 3, in the region where $\sigma \gg 1\mu V$, the nonlinear response of the network degrades the BER. In contrast, in the region where $\sigma \ll 1\mu V$, the linear response yields the theoretical BSPK BER, but only when the number of array elements is large. Equation (13) indicates that as the noise intensity increases, smaller error $\varepsilon$ (or equivalently, a smaller degradation factor) is obtained in the case of large $K$. Again, in practical systems, the noise intensity should be determined according to Eq. (14), and the number of array should then be set to achieve the required BER.

6. Conclusions
In the present study, the performance of an SR receiver comprising a parallel network for communication with sub-threshold signals was assessed. The present theoretical analysis revealed that in a certain noise intensity range, the output of the network can fully describe an input sub-threshold signal. The effectiveness of the proposed SR receiver was also demonstrated through a numerical example of the BER dependence on the number of array elements and the noise intensity. The proposed method can realize BER performance close to that in linear systems. However, such performance requires an infinite number of array elements, which is not realistic in practical systems. A design framework for a practical network with a finite number of elements and an appropriate noise intensity was theoretically presented. The results demonstrated the outstanding performance of the proposed SR receiver with a parallel network even in practical systems.

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