Paper

An automated design procedure for class-E amplifier

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Abstract: Class-E amplifier is one of the switching amplifiers. It is difficult to determine the circuit parameters satisfying the switching conditions. Although the Newton method was proposed to determine the passive elements, it requires a good initial solution that is obtained by using an analytical expression of the class-E amplifier. Even if the circuit configuration is slightly changed, the analytical expression must be re-derived. In this paper, the passive element value determination of class-E amplifier is defined as an optimization problem. This problem is solved by some methods which do not rely on the initial solution obtained by using the analytical expression. Furthermore, the objective function value for the optimization is efficiently calculated via a behavioral model of the class-E amplifier, which eases the optimum circuit design computationally and economically. Thus, an automated design procedure for the class-E amplifier will be presented in this paper.

Key Words: simplex method, Newton–Raphson method, Resonant Power Converters

1. Introduction

The class-E switching-mode circuits are used as valuable components in many applications, e.g., radio transmitters, switching mode-dc power supplies, devices of medical applications, and so on [1]. Because of the class-E switching, both zero voltage switching (ZVS) and zero derivative switching (ZDS), the class-E switching circuits can achieve high power-conversion efficiency at high frequencies. However, the class-E amplifier design is complicated because the two switching conditions must be satisfied simultaneously on the steady-state waveform.

The analytical expressions are very useful for the class-E amplifier design. Many analytical descriptions for the class-E amplifier design have been presented [2–8]. Early designs assumed an ideal switch, infinite output network Q (i.e., sinusoidal output current), and an RF choke in the dc supply

lead (i.e., constant current drive) [2, 3]. Later work allowed finite output Q [4, 5], drain-current fall time [6], and nonlinear parasitic capacitance on the active device [7, 8]. Although these analytical expressions are very useful, we cannot guarantee that the class-E amplifier satisfies the class-E ZVS and ZDS conditions rigorously.

In [1, 9], an optimization procedure for the class-E amplifier design is proposed using Newton method. In this procedure, the design problem of the class-E amplifier is defined as solving a set of nonlinear equations that constrains the passive element values including in the circuit and the class-E ZVS and ZDS conditions. The Newton method, more precisely, the Newton–Raphson method is applied to find the solution. Newton–Raphson method provides an accurate solution, but it requires a good initial solution to converge. Thus, the designers have to calculate the initial solution using the above analytical expressions of the class-E amplifier. Even if the design is slightly changed, the analytical expression must be reproduced; thus, this procedure using the Newton–Raphson method is not fully automated. Alternatively, a black-box optimization procedure is presented in [11], in which the ZVS and ZDS conditions are calculated by a commercial circuit simulator. Then, particle swarm optimization (PSO), which is a metaheuristic algorithm [10], is applied to find the solution. To evaluate the class-E ZVS and ZDS conditions, the steady-state response must be calculated. Unfortunately, the class-E amplifier is a high output Q circuit, which makes the transition time very long. Thus, the steady-state analysis of the class-E amplifier is very costly, even if a simulator which directly calculates the steady-state response is used. As a result, the optimization procedure using PSO requires approximately half a day in order to determine only two parameters. Even worse, the class-E amplifier design is not complete by determining a set of design parameters once. To know information such as power-conversion efficiency, distortion, and so on, many design candidates must be provided using the optimization. Although the class-E amplifier is a small-scale circuit, the design thus is computationally very expensive. Therefore, if one intends to execute it reasonably, many computational resources are required, which is economically expensive.

In this paper, an improved optimization procedure for the class-E amplifier design is presented. This is a fully automated design procedure that does not depend on a good initial guess for the optimization. Thus, the analytical expressions of the class-E amplifier are not required. To reduce the computational cost for the optimization, a behavioral model of the class-E amplifier is provided. Then, an efficient steady-state analysis can be done, and the ZVS and ZDS conditions of the class-E amplifier are efficiently evaluated. Next, the passive element values determination of the class-E amplifier is defined as an optimization problem through preliminary optimization tests. Here, we show that it is difficult to design the class-E amplifier by using ZVS and ZDS conditions only, i.e., these conditions precisely correspond to the class-E frequency multipliers. An objective function, which constrains the frequency of output waveform, is introduced in order to prevent to converge into the solutions which correspond to the class-E frequency multipliers. After applying a grid search to the problem with the objective function that constrains the frequency, the simplex method [20] and the Newton–Raphson one are applied to find the global minimum point that satisfies the class-E ZVS and ZDS conditions. The grid point whose objective function value is the smallest within all grid points is used as an initial solution for these methods. We also show that the simplex method with an initial solution, which is near the origin in the parameter space, finds the global minimum point more efficiently than the methods with the grid search. In [13], Independent-minded PSO (IPSO), which is an enhanced version of PSO [12], is used for finding the design parameters of the class-E amplifier. As IPSO is a metaheuristic algorithm, we may expect that IPSO is not necessarily faster than the proposed methods, but it provides a better solution. Surprisingly, however, it will be shown in the numerical examples that the proposed procedure is more reliable than the IPSO-based design one.

To provide many candidates of the class-E amplifier, we must calculate the design parameters repeatedly changing the design specification gradually. Then, to find a series of the solutions efficiently, a curve tracing algorithm using Newton–Raphson method is proposed. The proposed optimization procedure neither requires many computational resources nor relies on the analytical expressions that designers with expert knowledge of the class-E amplifier can only derive. Although the mathematical and computational expertise is necessary to develop the design automation tool, once developed,

designers can use this tool to design the class-E amplifier. This procedure can be also extended to other resonant power amplifier designs [16].

This paper is organized as follows. In Section 2, an efficient steady-state analysis of the class-E amplifier and the problem definition for the passive element values determination are presented. In Section 3, a curve tracing algorithm is presented in order to provide many sets of design parameters efficiently, in which the simplex method and the Newton–Raphson one with/without grid search are provided, which are a component of the curve tracing algorithm. In Section 4, some examples, which illustrate the effectiveness of the optimization algorithms, and design examples are presented. Finally, conclusions are drawn in Section 6.

2. Optimization of class-E amplifier

2.1 Circuit topology and principle operation

Figure 1(a) shows a circuit topology of the class-E amplifier. The class-E amplifier consists of dc-supply voltage source V_D , dc-feed inductor L_C , n -channel MOSFET S as a switching device, shunt capacitor C_S , and a series resonant circuit L_0 - C_0 - R . Figure 2 shows example steady-state waveforms of the class-E amplifier. The MOSFET turns on and off by the driving signal D_r . When the switch is in the on-state, the voltage across the MOSFET is zero and the current flows through the MOSFET. While the switch is in the off-state, the difference of the currents through the dc-feed inductor and resonant network flows through the shunt capacitor, which generates a pulse-shape voltage waveform as shown in Fig. 2. In the nominal operation of the class-E amplifier, the switch turns on when both the switch voltage and its derivative are zero simultaneously on the steady-state. These switching conditions are referred to as the class-E ZVS and ZDS conditions, which are respectively expressed as

$$v_s(\theta_{\text{on}}) = 0, \quad (1)$$

Fig. 1. A class E amplifier, (a) circuit model with MOSFET and (b) circuit model with an ideal switch.

Fig. 2. Typical steady-state waveforms of class-E amplifier.

$$\left. \frac{dv_s}{d\theta} \right|_{\theta=\theta_{on}} = 0, \quad (2)$$

where $\theta = \omega t = 2\pi ft$ is the angular time and f is the operating (switching) frequency. Additionally, θ_{on} is the angular time at the switch turn-on instant. By satisfying the class-E ZVS and ZDS conditions, switching losses are reduced and high power-conversion efficiency at high frequencies can be achieved. The passive elements should be determined uniquely to satisfy the class-E ZVS and ZDS conditions, which is not an easy task because the two conditions need to be filled simultaneously on the steady-state.

2.2 Circuit equations and waveform expressions

The resonant filter of the class-E amplifier usually has a high output Q value. Therefore, it takes a long transition time for the class-E amplifier to reach the steady-state. As the class-E ZVS and/or ZDS conditions are defined on the steady-state operation, it is not easy to find the passive element values achieving the class-E ZVS and ZDS conditions, using the transient analysis. With circuit simulators such as SPICE, we can carry out transient analysis, in which sophisticated MOSFET models are available. Then, the optimization to find the passive element values using SPICE is a black-box optimization, but it suffers from its heavy computational cost. Even though circuit simulators, which have a tool that steady-state waveforms can be obtained efficiently, are used, the computation is still costly for the optimization [11]. Therefore, the MOSFET in the circuit of Fig. 1(a) is modeled as an ideal switch as shown in Fig. 1(b). By using the ideal switching model, the circuit equations are expressed by piece-wise linear differential equations. Therefore, the solutions can be derived explicitly and we use them for obtaining the steady-state waveforms.

In this paper, the design parameters of the class-E amplifier are given as follows [9]:

- 1) $\omega = 2\pi f$: the operating (switching) angular frequency,
- 2) $\omega_0 = 2\pi f_0 = 1/\sqrt{L_0 C_0}$: the resonant angular frequency,
- 3) $Q = \omega L_0 / R$: the loaded quality factor,
- 4) $A = f_0 / f = \omega_0 / \omega$: the ratio of the resonant frequency to the operating frequency,
- 5) $B = C_0 / C_S$: the ratio of the capacitance of the resonant circuit capacitor to that of the shunt capacitor,
- 6) $H = L_0 / L_C$: the ratio of the inductance of the resonant circuit inductor to that of the dc-feed inductor, and
- 7) D : the switch-off duty ratio of the switch, in which $0 < D < 1$.

Additionally, we assume that

- a) the switching device works as an ideal switch; zero on-resistance, infinite off-resistance, and zero switching time,
- b) shunt capacitance C_S includes MOSFET parasitic capacitances,
- c) all the passive elements including parasitic capacitors of MOSFET are linear and have no parasitic resistance, and
- d) the switch turns on and off at $\theta = 2\pi$ and $\theta = 2\pi D$, respectively.

By applying these parameters and putting assumptions, the circuit equations can be formulated by

$$\left\{ \begin{array}{l} \frac{d}{d\theta} \frac{Ri_C}{V_D} = \frac{H}{Q} \\ \frac{v_S}{V_D} = 0 \\ \frac{d}{d\theta} \frac{Ri_o}{V_D} = -\frac{1}{Q} \left(\frac{Ri_o}{V_D} + \frac{v}{V_D} \right) \\ \frac{d}{d\theta} \frac{v}{V_D} = A^2 Q \frac{Ri_o}{V_D} \end{array} \right. \quad \text{for } 0 \leq \theta \leq 2\pi D, \quad (3)$$

$$\left\{ \begin{array}{l} \frac{d}{d\theta} \frac{Ri_C}{V_D} = \frac{H}{Q} \left(1 - \frac{v_S}{V_D} \right) \\ \frac{d}{d\theta} \frac{v_S}{V_D} = A^2 B Q \left(\frac{Ri_C}{V_D} - \frac{Ri_o}{V_D} \right) \\ \frac{d}{d\theta} \frac{Ri_o}{V_D} = \frac{1}{Q} \left(\frac{v_S}{V_D} - \frac{Ri_o}{V_D} - \frac{v}{V_D} \right) \\ \frac{d}{d\theta} \frac{v}{V_D} = A^2 Q \frac{Ri_o}{V_D} \end{array} \right. \quad \text{for } 2\pi D < \theta \leq 2\pi. \quad (4)$$

In the above equations, (Q, H, D) is given as a design specification and $\mathbf{X} = [A, B]^T$ is set as the design parameters. Note that the circuit parameters, V_D , R , and f , are given a priori. If $\mathbf{X}$ is determined, all the passive elements are determined; thus, the problem to find $\mathbf{X}$ is referred to as the passive element values determination of the class-E amplifier.

The system with (3) and (4) is a linear-time-invariant system. Thus, the solutions of (3) and (4) are expressed via the eigenvalue decomposition of the matrices

$$\boldsymbol{\alpha}_1 = \begin{bmatrix} -\frac{1}{Q} & -\frac{1}{Q} \\ A^2 Q & 0 \end{bmatrix}$$

for (3)¹ and

$$\boldsymbol{\alpha}_2 = \begin{bmatrix} 0 & -\frac{H}{Q} & 0 & 0 \\ A^2 B Q & 0 & -A^2 B Q & 0 \\ 0 & \frac{1}{Q} & -\frac{1}{Q} & -\frac{1}{Q} \\ 0 & 0 & A^2 Q & 0 \end{bmatrix}$$

for (4), respectively. By using the eigenvalues and eigenvectors, the solutions $\mathbf{x}(\theta) = [x_1(\theta), x_2(\theta), x_3(\theta), x_4(\theta)]^T = [Ri_C/V_D, v_S/V_D, Ri_o/V_D, v/V_D]^T$ of (3) and (4) are obtained as

$$\begin{aligned} \mathbf{x}_{\text{on}}(\theta) &= \gamma_1(\theta) \mathbf{x}(0_-) + \phi_1(\theta), \\ \mathbf{x}_{\text{off}}(\theta) &= \gamma_2(\theta) \mathbf{x}(2\pi D) + \phi_2(\theta), \end{aligned} \quad (5)$$

where

$$\begin{aligned} \gamma_1(\theta) &= \begin{bmatrix} 1 & 0 & \mathbf{0} \\ 0 & 0 & \\ \mathbf{0} & \tilde{\gamma}_1(\theta) \end{bmatrix}, \\ \phi_1(\theta) &= \begin{bmatrix} \frac{H}{Q} \theta & 0 & 0 & 0 \end{bmatrix}^T, \\ \tilde{\gamma}_1(\theta) &= \mathbf{S}_1 \text{diag}\{e^{\lambda_{1,1}\theta}, e^{\lambda_{2,1}\theta}\} \mathbf{S}_1^{-1}, \\ \gamma_2(\theta) &= \mathbf{S}_2 \text{diag}\{e^{\lambda_{1,2}\theta}, \dots, e^{\lambda_{4,2}\theta}\} \mathbf{S}_2^{-1}, \\ \phi_2(\theta) &= \mathbf{S}_2 \text{diag}\left\{ \frac{e^{\lambda_{1,2}\theta} - 1}{\lambda_{1,2}}, \dots, \frac{e^{\lambda_{4,2}\theta} - 1}{\lambda_{4,2}} \right\} \mathbf{S}_2^{-1} \begin{bmatrix} \frac{H}{Q} & 0 & 0 & 0 \end{bmatrix}^T. \end{aligned}$$

¹The solutions of the first and second equations of (3) are $(H/Q)\theta$ and 0, respectively.

Here, $\lambda_{1,1}, \lambda_{2,1}$ and $\lambda_{1,2}, \dots, \lambda_{4,2}$ are respectively eigenvalues of α_1 and α_2 , and S_1 and S_2 are matrices whose columns are the corresponding eigenvectors. Note that the continuity at $\theta = 0$ is not guaranteed, because (3) is a descriptor system rather than a set of ordinary differential equations². Thus, the solution $\mathbf{x}_{\text{on}}(\theta)$ depends on $\mathbf{x}(0_-)$ [15].

As (4) is a set of ordinary differential equations, the solution's continuity holds at $\theta = 2\pi D$. Using (5), the condition is expressed as $\mathbf{x}(2\pi D) = \mathbf{x}_{\text{on}}(2\pi D) = \gamma_1(2\pi D)\mathbf{x}(0_-) + \phi_1(2\pi D)$. On the other hand, the steady-state condition is provided as $\mathbf{x}(0_-) = \mathbf{x}_{\text{off}}(2\pi) = \gamma_2(2\pi)\mathbf{x}(2\pi D) + \phi_2(2\pi)$. Combining the two conditions, we can obtain the initial values as

$$\mathbf{x}(0_-) = (\mathbf{I} - \gamma_2(2\pi)\gamma_1(2\pi D))^{-1} (\gamma_2(2\pi)\phi_1(2\pi D) + \phi_2(2\pi)), \quad (6)$$

where $\mathbf{I}$ is an identity matrix. The steady-state response of the class-E amplifier can be calculated by using (5) with the initial values of (6).

The class-E ZVS condition (1) is divided by V_D . Then, the ZVS condition is evaluated at $\theta_{\text{on}} = 0_-$ as

$$x_2(0_-) = 0. \quad (7)$$

The ZDS condition (1) is multiplied by $C_S R/V_D$. Then, the relationship, $C_S R/V_D dv_S/dt = R/V_D (i_C(\theta) - i_o(\theta))$, holds in the off-state. Thus, the class-E ZDS condition is evaluated as

$$x_1(0_-) - x_3(0_-) = 0. \quad (8)$$

The behavioral modeling in this subsection is based on the state space modeling [14]. We extend this method to the case where the idealized circuit in the on-state is expressed by a descriptor system. When the idealized circuit has a more complicated structure, more generalized modeling can be used [15]. Therefore, the proposed optimization method can be applied to other types of resonant power amplifier designs [16].

2.3 Problem definition

In this subsection, we define the problem of the passive element values determination for the class-E amplifier. Before providing the definition, preliminary tests were performed. To achieve the ZVS and/or ZDS conditions of (7) and (8), the tested objective functions were defined as

$$f_1(A, B) = (x_2(0_-))^2, \quad (9)$$

$$f_2(A, B) = (x_1(0_-) - x_3(0_-))^2, \quad (10)$$

$$f_3(A, B) = (x_2(0_-))^2 + (x_1(0_-) - x_3(0_-))^2, \quad (11)$$

where $f_1(A, B)$, $f_2(A, B)$, and $f_3(A, B)$ were used to find the global minimum points that satisfy the ZVS, ZDS, and both ZVS and ZDS conditions, respectively. In the dynamic equations of (3) and (4), a design specification (Q, H, D) is fixed and the design parameters (A, B) are changed for the optimization. Then, the initial values $\mathbf{x}(0_-)$ of (6), which provide a steady-state response, depend on (A, B) . This implies that (A, B) changes the objective function values of $f_1(A, B)$, $f_2(A, B)$, and $f_3(A, B)$, implicitly. If a minimization algorithm is applied to each problem associated with $f_1(A, B)$, $f_2(A, B)$, and $f_3(A, B)$, respectively, the passive element values of the class-E amplifiers that satisfy the ZVS, ZDS, and both ZVS and ZDS conditions, respectively, can be found.

Preliminary optimization tests were run, in which the class-E amplifier expressed by (3) and (4) was analyzed, set to $Q = 30$, $H = 0.001$, and $D = 0.5$. As an algorithm for the minimization, PSO was applied to find the global minimum points, restricted as $A, B \in [0.1, 2.0]$. In Appendix, PSO is explained in a part of IPSO, which is an extended version of PSO, where the relationship between a particle's position of PSO and the design parameters of the class-E amplifier is expressed as $\mathbf{X} \equiv (x_1, x_2) \equiv (A, B)$. For PSO, the parameters were set as Table I in which the values of M , w ,

²The second line of (3) is not a differential equation. Then, (3) is called a descriptor system, the solution's continuity of which is not guaranteed, differently from ordinary differential equation cases.

Table I. Parameters used in the methods in this paper.

methods	content	symbols	value
PSO/IPSO	population size	M	24
	inertia weight	w	0.729
	acceleration coefficients	c_1, c_2	1.494
	maximum number of iterations	N_0	5,000
	termination tolerance on function value of (11)	Θ_1	10^{-10}
PSO/IPSO	lower limit of searching interval	$x_{\min}$	
SMPLX-G1/SMPLX-G2/NEW-G2	upper limit of searching interval	$x_{\max}$	
IPSO	cooperative coefficient	K	0.1
SMPLX-G1/SMPLX-G2/NEW-G2	division number of searching interval	N_1	
SMPLX-G1/SMPLX-G2/SMPLX	maximum number of iterations	N_0	5,000
	maximum number of function evaluations	N_2	10,000
	termination tolerance on function value of (11)	Θ_1	10^{-10}
	termination tolerance on solution variation	Θ_2	10^{-10}
SMPLX	a small positive number in (15)	ϵ_1	
NEW-G2/Curve Tracing	a small positive number in (20)	ϵ_2	
	termination tolerance on function value of (11)	Θ_1	10^{-10}
	maximum number of linear search iterations	N_3	10
	maximum number of Newton–Raphson iterations	N_4

Fig. 3. Class-E ZVS and/or ZDS conditions found by PSO.

c_1 , and c_2 are typical as standard PSO, and N_0 was set to 100,000 in order to find the global minimum points surely. The particles' initial positions were randomly assigned in the region. Figure 3(a) shows the global minimum points found by PSO, in which 2000 trials were run using $f_1(A, B)$ and $f_2(A, B)$, respectively, and 100 trials were run using $f_3(A, B)$. When an objective function value is less than 10^{-10} , which is Θ_1 of Table I, PSO was terminated. We found 1991 and 1965 global minimum points that satisfy the ZVS and ZDS conditions, respectively. From Fig. 3(a), the ZVS or ZDS condition appears to be satisfied not only at isolated points but on curves in the parameter space. For 32 trials, PSO found the same point that satisfies both the ZVS and ZDS conditions; thus, this point is considered to be an intersection point of two curves that respectively satisfy the ZVS and ZDS conditions.

Fig. 4. Steady-state waveforms obtained by analyzing the circuits that satisfy the ZVS condition, (a) switch voltages and (b) the enlarged responses around $\theta = 2\pi$ of (a).

Fig. 5. Steady-state waveforms obtained by analyzing the circuits that satisfy the ZDS condition, (a) switch voltages and (b) the enlarged responses around $\theta = 2\pi$ of (a).

Extending the searching area as $A, B \in [0.01, 30.0]$, PSO was again applied, in which 2000 trials were run with each (9) and (10) and 100 trials were with (11). Figure 3(b) shows the global minimum points found by PSO, in which the ZVS and/or ZDS conditions are fulfilled at many points. Selecting the two points $zvs(1)$ and $zvs(2)$, we calculated the corresponding steady-state responses of the class-E amplifier. Figures 4(a) and 4(b) show the steady-state responses of the switch voltage $\frac{v_s}{V_D}$. As shown in Fig. 4(b), the responses are zero at $\theta = 2\pi$; thus, we can confirm that the ZVS condition of (7) is satisfied on the steady-state responses. Figures 5(a) and 5(b) show the steady-state responses of the switch voltage $\frac{v_s}{V_D}$ which satisfy the ZDS condition. The time derivatives of the responses are zero at $\theta = 2\pi$; thus, the condition (8) is satisfied on the steady-state responses. Selecting the two points $zvs\&zds(1)$ and $zvs\&zds(2)$ of Fig. 3(b) at which both the ZVS and ZDS conditions are satisfied, the steady-state responses of the switch voltage were calculated. As shown in Fig. 6(b), the responses and their time derivatives are both zero at $\theta = 2\pi$; thus, both the ZVS and ZDS conditions are satisfied on the steady-state responses. However, the fundamental frequency is not only 1 [Hz] but a positive integer [Hz]; therefore, these circuits are not class-E amplifiers and are referred to as class-E frequency multipliers. From this fact, the optimization problem with the objective function $f_3(A, B)$ has many global minimum points if the parameter space is not limited. Therefore, we need another criterion to obtain the class-E amplifier.

To prevent from being trapped to the frequency multiplier solutions, consider the following objective function:

$$f_4(A, B) = \frac{1}{|V_1|} \sum_{k=0, k \neq 1}^8 |V_k|, \quad (12)$$

where V_k is the k -th frequency component of the output voltage waveform v_o . The frequency com-

Fig. 6. Steady-state responses obtained by analyzing the circuits which satisfy both the ZVS and ZDS conditions, (a) steady-state waveforms of switch voltages, (b) the enlarged responses around $\theta = 2\pi$ of (a).

ponents are calculated by fast Fourier transform, where 16 sampled points in the time-domain are taken into account. If the DC component is eliminated, (12) is called total harmonic distortion. Note that the fundamental frequency of the output waveform is approximately 1 [Hz] when the value of $f_4(A, B)$ is smaller than 1.0.

From the discussion so far, we need to find a point that minimizes $f_1(A, B)$, $f_2(A, B)$ and $f_4(A, B)$ to determine the passive element values of the class-E amplifier. Further, the condition $A, B > 0$ is required, because the frequencies f_0 and f and the capacitances C_0 and C_S are positive. A point which minimizes the three functions simultaneously does not exist. However, as the solution curves of $f_1(A, B)$ and $f_2(A, B)$ can exist, the points which minimize $f_1(A, B)$ and $f_2(A, B)$ simultaneously can be obtained by using $f_3(A, B)$ which is no-weighted sum of $f_1(A, B)$ and $f_2(A, B)$. As the output waveform contains any distortion, $f_4(A, B)$ does not become very small. Thus, the function $f_4(A, B)$ should not be added to $f_3(A, B)$.

Thus, the passive element values determination of the class-E amplifier is defined as a multi-objective optimization problem:

$$\begin{aligned} &\text{minimize} && f_3(A, B), f_4(A, B), \\ &\text{subject to} && A, B > 0. \end{aligned} \quad (13)$$

If there exists a tradeoff between $f_3(A, B)$ and $f_4(A, B)$, finding the Pareto solutions [19] would be a good strategy. However, even if $f_3(A, B)$ is small, i.e., a class-E multiplier is found, $f_4(A, B)$ is not necessarily large. This is because the class-E frequency multiplier may include a class-E amplifier. Therefore, as tradeoffs do not exist in this problem, we should not think to find the Pareto solutions. Additionally, the passive element values determination gives priority to the minimization with $f_3(A, B)$ in which the objective function value must be approximately zero. Therefore, after finding many global minimum points associated with $f_3(A, B)$, the point that has the smallest value of $f_4(A, B)$ would be selected. However, this is not efficient, because there are infinite global minimum points associated with $f_3(A, B)$.

Thus, we take the following strategy to determine the passive element values. First, the optimization associated with $f_4(A, B)$ is run. Next, the obtained minimum point is used as the initial solution to the optimization associated with $f_3(A, B)$, in which a computationally efficient method to find the global minimum point to a design specification is used. As we must calculate many global minimum points changing the design specification, all necessary points are calculated by applying a curve tracing algorithm.

3. Optimization methods

3.1 Overview of automated design procedure

The automated design procedure proposed in this paper is to find the design parameters $\mathbf{X} = [A, B]^T$ that satisfies the class-E ZVS and ZDS conditions of (7) and (8). Both the conditions are written by

Fig. 7. Flow chart of the automated design procedure.

$$\mathbf{g}(\mathbf{X}, Q, H, D) = \mathbf{0}, \quad (14)$$

where $\mathbf{g} \equiv [g_1, g_2]^T = [x_2(0_-), x_1(0_-) - x_3(0_-)]^T$ is associated with (6) and the solution is put as $\mathbf{X} = [A, B]^T$. Here, (Q, H, D) is a design specification stated in Sect. 2.2. The solution curve of $\mathbf{X} = [A, B]^T$ that satisfies (14) is referred to as the design curve that depends on (Q, H, D) . The design curve depending on the loaded quality factor Q only is very useful for the designers when (H, D) is fixed. Giving a specification (Q, H, D) and solving the optimization problem by a method, we can find a point $\mathbf{X}_0$ on the design curve. Then, changing the loaded quality factor gradually and tracing the design curve from a point $\mathbf{X}_0$, we can know the whole characteristics that satisfy both the class-E ZVS and ZDS conditions.

Figure 7 shows an overview of the automated design procedure. The point $\mathbf{X}_0$ is found by the simplex method with/without grid search or the Newton–Raphson method with grid search. The curve tracing can be done by using the Newton–Raphson method without grid search. In this section, we will explain each method step by step.

3.2 Grid searches

Generally, a small constrained optimization problem is solved by a grid search and a minimization algorithm [21]. Thus, applying a grid search to the objective function $f_4(A, B)$, the point whose objective function $f_4(A, B)$ is minimum over all the grid points is found. Using this point as an initial solution, a minimization algorithm is applied to find the global minimum point for the objective function $f_3(A, B)$.

The grid search is referred to as **Grid Search 1**, which is a simple two-dimensional search. The computational complexity is $O(N_1^2)$, in which N_1 is the number of divisions for each coordinate. Thus, when N_1 is large, the computational cost cannot be ignored. As the objective function $f_4(A, B)$ is a multimodal function, for example, $N_1 = 1000$ is used, which means that the grid search becomes

costly. Thus, the two coordinates are searched sequentially, which is referred to as **Grid Search 2**, and the computational complexity is $O(N_1)$.

3.3 Simplex method

As derivative is not analytically available for the objective function $f_3(A, B)$, we use the simplex method [20]. In this method, a simplex is composed of $m + 1$ nodes, in which m represents the number of design variables. Thus, the simplex is a triangle for $m = 2$. The procedure continues until the simplex projection onto the function surface rolls into a minimum point at which the $m + 1$ nodes are sufficiently close together. To realize convergence, three operations, reflection, expansion, and contraction are used, evaluating the objective function values. In our implementation, we use the MATLAB function `fminsearch()`, which is the simplex method [20]. Finding a grid point by using **Grid Search 1** or **Grid Search 2** and providing it as an initial solution, the simplex method is applied, which corresponds to **Simplex Method with Grid Search** of the flowchart of Fig. 7. Then, these algorithms are referred to as SMPLX-G1 and SMPLX-G2 which use **Grid Search 1** and **Grid Search 2**, respectively. The MATLAB function `fminsearch()` has two stopping criteria Θ_1 and Θ_2 that are termination tolerances on the function value of (11) and the solution variation, respectively. In our implementation, we take as $\Theta_1 = \Theta_2 = 10^{-10}$ as listed in Table I. The objective function value of (11) being less than Θ_1 implies that the simplex method could find the global minimum point. However, satisfying the solution variation within Θ_2 implies that the simplex method is trapped into a local minimum point without finding the global minimum point. Thus, the simplex method must be stopped, fulfilled with being less than Θ_2 .

Furthermore, we found the initial solution (A^*, B^*) , which is suitable for the simplex method with the objective function $f_3(A, B)$, as follows:

$$(A^*, B^*) = \epsilon_1(1, 1), \quad (15)$$

where ϵ_1 is a small positive number. This method corresponds to **Simplex Method without Grid Search** of the flowchart of Fig. 7 and is referred to as SMPLX. As this procedure does not require a grid search, it is very efficient. The parameter ϵ_1 must be empirically determined.

3.4 Newton–Raphson method

The motivation of this paper is on the difficulty to find an initial solution for the Newton–Raphson method. In the previous subsection, the point found by a grid search is used as an initial solution for the simplex method. Fortunately, the grid point can also be used as an initial solution for the Newton–Raphson method.

Equation (14) is compactly written by

$$\mathbf{g}(\mathbf{X}) = \mathbf{0}. \quad (16)$$

After applying a grid search, we solve (16) by the Newton–Raphson method, in which the point obtained from a grid search is used as an initial solution for the Newton–Raphson method as $\mathbf{X}_0 = [A^*, B^*]^T$. To ensure the convergence of the Newton–Raphson method, the linear search with the direction

$$\mathbf{d} = -\{\mathbf{g}'(\mathbf{X}_n)\}^{-1} \mathbf{g}(\mathbf{X}_n) \quad (17)$$

is used, where $\mathbf{g}'(\mathbf{X}_n)$ is the Jacobian matrix. We find the smallest integer $m \geq 0$ such that

$$\|\mathbf{g}(\mathbf{X}_n + 2^{-m} \mathbf{d})\| < \|\mathbf{g}(\mathbf{X}_n)\|. \quad (18)$$

Then, the solution is written by

$$\mathbf{X}_{n+1} = \mathbf{X}_n + 2^{-m} \mathbf{d}. \quad (19)$$

The linear search (18) is truncated until N_3 iterations. If m is not found, put $m = 0$.

The elements of the Jacobian matrix are obtained by giving a small perturbation ϵ_2 to the design parameters and calculating (6). For example, $\partial g_1 / \partial X_1$, which is the (1,1) element, is calculated by

$$\frac{\partial g_1}{\partial X_1} = \frac{g_1(A + \epsilon_2, B) - g_1(A, B)}{\epsilon_2}. \quad (20)$$

This method corresponds to **Newton–Raphson Method with Grid Search** of the flowchart of Fig. 7. When **Grid Search 2** is used, the Newton–Raphson method is referred to as NEW-G2. Note that the Newton–Raphson method itself is not necessarily useful for finding a set of design parameters, but is effectively incorporated with the curve tracing algorithm provided in the next subsection in order to calculate many sets of design parameters changing the design specification.

3.5 Design curve tracing

After applying the Simplex method or the Newton–Raphson method, the curve tracing is started with the initial solution $\mathbf{X} = [A, B]^T$, which is given by one of these methods, as shown in Fig. 7. Then, the solution $\mathbf{X} = [A, B]^T$ is used as the initial solution of the Newton–Raphson method (**Newton–Raphson Method without Grid Search** of Fig. 7) as $\mathbf{X}_0 = \mathbf{X}$. After the Newton–Raphson method converges, the solution is given as the initial solution for the Newton–Raphson method at the next Q value. Changing the loaded quality factor Q gradually, we can trace the design curve until Q_{end} . Then, all the necessary values of (A, B) s are obtained; thus, the passive element values of the class-E amplifier can be determined for every needed specification.

4. Results

All the results throughout this paper were performed on a computer with a 3.7-GHz Intel Xeon E5-1620 CPU and 32 GB of memory, using MATLAB R2017a.

4.1 Methods with/without grid searches

The grid searches of **Algorithms 1** and **2** for $[x_{\text{max}}, x_{\text{min}}] = [0.01, 100]$ were used with the simplex method, in which the specification was given as $Q = 30$, $H = 1$, and $D = 0.5$. Table II shows the found points obtained by the grid searches associated with the objective function (12). As the points obtained by **Grid Search 1** are the same as those by **Grid Search 2**, the final points found by SMPLX-G1 are the same as those by SMPLX-G2. Thus, the function values of (11) and (12) for SMPLX-G1 are identical to those for SMPLX-G2. The underline indicates that an appropriate minimum point as the class-E amplifier is not obtained. Note that the parameters Θ_2 , N_0 , and N_2 are set to terminate the simplex method and these parameters do not improve the performance of the simplex method. Only when $N_1 = 1000$, the simplex method provided a global minimum point suitable for the class-E amplifier. When $N_1 = 10$, the simplex method did not converge. For the other

Algorithm 1 Grid Search 1

1. Divide $[x_{\text{min}}, x_{\text{max}}]$ uniformly for each ordinate, in which N_1 is the number of divisions.
 2. Find the point whose objective function value of (12) is the smallest within all the grid points (A_i, B_j) , $i, j = 0, 1, \dots, N_1$ and put it as (A^*, B^*) .
-

Algorithm 2 Grid Search 2

1. Divide $[x_{\text{min}}, x_{\text{max}}]$ uniformly for each ordinate, in which N_1 is the number of divisions.
 2. Find the point whose objective function value of (12) is the smallest within the grid points (A_i, x_{min}) , $i = 0, 1, \dots, N_1$ and put it as (A^*, x_{min}) .
 3. Find the point whose objective function value of (12) is the smallest within the grid points (A^*, B_j) , $j = 0, 1, \dots, N_1$ and put it as (A^*, B^*) .
-

Table II. Function values of (11) and (12) obtained from the simplex method with grid searches and the computational times of these methods.

(a)

	Grid Search 1		SMPLX-G1		
N_1	minimum point	time [sec.]	(11)	(12)	time [sec.]
1000	(1.0099, 0.11000)	2436.64	2.8×10^{-11}	0.021	0.065
500	(1.0099, 0.01)	583.62	5.1×10^{-2}	<u>2.650</u>	0.071
100	(1.0099, 0.01)	22.76	5.1×10^{-2}	<u>2.650</u>	0.073
10	(0.01, 100.0)	0.27	1.0×10^{-1}	<u>0.259</u>	3.061

(b)

	Grid Search 2		SMPLX-G2		
N_1	minimum point	time [sec.]	(11)	(12)	time [sec.]
1000	(1.0099, 0.11000)	4.359	2.8×10^{-11}	0.021	0.074
500	(1.0099, 0.01)	2.175	5.1×10^{-2}	<u>2.650</u>	0.088
100	(1.0099, 0.01)	0.445	5.1×10^{-2}	<u>2.650</u>	0.079
10	(0.01, 100.0)	0.054	1.0×10^{-1}	<u>0.259</u>	3.114

Table III. Values of (12) obtained from SMPLX, SMPLX-G2, and NEW-G2, (a) $H = 1$, (b) $H = 0.001$, (c) $H = 10^{-10}$.

(a)

	SMPLX-G2 /NEW-G2	SMPLX	SMPLX-G2 /SMPLX	NEW-G2
Q	N_1	ϵ_1	(12)	
30	1000	0.01	0.021	0.021
20	1000	0.01	0.032	0.032
10	1000	0.01	0.061	<u>0.76</u>
5	1000	0.01	0.114	0.114
3	1000	0.05	0.176	0.176
2	1000	0.05	0.244	0.244

(b)

	SMPLX-G2 /NEW-G2	SMPLX	SMPLX-G2 /SMPLX	NEW-G2
Q	N_1	ϵ_1	(12)	
30	1000	0.01	0.022	<u>0.801</u>
20	1000	0.01	0.033	0.033
10	1000	0.01	0.066	0.066
5	1000	0.01	0.133	0.133
3	1000	0.01	0.225	0.225
2	500	0.05	0.349	0.349

(c)

	SMPLX-G2 /NEW-G2	SMPLX	SMPLX-G2 /SMPLX	NEW-G2
Q	N_1	ϵ_1	(12)	
30	1000	0.9	0.022	<u>2.758</u>
20	1000	0.1	0.033	0.033
10	1000	0.1	0.066	0.066
5	1000	0.1	0.133	0.133
3	1000	0.1	0.225	0.225
2	500	0.1	0.350	0.350

Fig. 8. Steady-state switch and output waveforms, v_s/V_D and Ri_o/V_D , respectively, (a) $H = 1$, (b) $H = 0.001$, (c) $H = 10^{-10}$.

cases, the simplex method converged within 200 function evaluations; thus, the computational times are very short except for $N_1 = 10$. When $N_1 = 100$ and $N_1 = 500$, the simplex method terminated with filling the condition Θ_2 , which implies that the simplex method was trapped to a local minimum. As **Grid Search 2** is much faster than **Grid Search 1**, SMPLX-G2 is preferable to SMPLX-G1.

Table III shows the values of (12) obtained by SMPLX, SMPLX-G2 and NEW-G2, in which ϵ_1 is set for SMPLX and N_1 is for SMPLX-G2 and NEW-G2. For SMPLX-G2 and NEW-G2, $[x_{\max}, x_{\min}] = [0.01, 100]$ were also assumed. From Table III, SMPLX provides the same values of (12) as SMPLX-G2. However, NEW-G2, which is the Newton–Raphson method provided in Sect. 3.4, did not converge for some cases where $N_4 = 2000$ was set, which implies that the simplex method is superior to the Newton–Raphson method.

For SMPLX-G2 and NEW-G2, we must set $x_{\min}$, $x_{\max}$, and N_1 , and the grid search is required. For SMPLX, we have only to set ϵ_1 and any grid search is not required. Thus, SMPLX is the most effective for solving this problem and we use SMPLX for the design examples in the following subsections. Figure 8 shows the normalized switch and output voltage waveforms by V_D , which are obtained by using the parameters provided by SMPLX. From all the switch voltage waveforms at $\theta = 2\pi$, the ZVS and ZDS conditions are satisfied for all the cases; thus, the passive elements of class-E amplifiers were adequately determined.

4.2 Design Example 1

We used the example provided in [18] to confirm the accuracy of the curve tracing algorithm in Sect. 4, where $H = 0$ is assumed. However, as we cannot assign infinite L_C to solve the circuit equations numerically, $H = 1.0 \times 10^{-10}$ was used.

We calculated some factors depending on the loaded quality factor, in which Q was divided in three regions: $[1.7879, 3]$, $[3, 20]$, and $[20, 5 \times 10^5]$. For $[1.7879, 3]$, the solution at $Q = 3$ was first calculated

Fig. 9. Numbers of iterations of Newton–Raphson method needed to find the solutions, (a) $Q \in [1.7879, 3)$, (b) $Q \in [3, 20)$, (c) $Q \in [20, 5 \times 10^5]$.

Table IV. Some factors depending on loaded quality factor Q .

Q	PR/V_D^2		$C_S \omega R$		$C_0 \omega R$	
	Reference [18]	Proposed	Reference [18]	Proposed	Reference [18]	Proposed
∞	0.57680	0.57680	0.18360	0.18360	0	0.00002
20	0.56402	0.56397	0.19111	0.19111	0.05313	0.05313
10	0.54974	0.54963	0.19790	0.19790	0.11375	0.11375
5	0.51659	0.51662	0.20907	0.20907	0.26924	0.26924
3	0.46453	0.46452	0.21834	0.21834	0.63467	0.63467
2.5	0.43550	0.43550	0.22036	0.22036	1.01219	1.01219
2	0.38888	0.38888	0.21994	0.21994	3.05212	3.05212
1.7879	0.35969	0.35980	0.21770	0.21770	∞	3,157.1

In the proposed method, $Q = 5 \times 10^4$ was used instead of ∞ to calculate each factor.

by the simplex method, and the characteristics curve was traced by the Newton–Raphson method in Sect. 3.5. For the simplex method (SMPLX), the initial solution was given as $(A, B) = (10^{-2}, 10^{-2})$. In the Newton–Raphson method, Q was decremented with 10^{-1} until 2, 10^{-2} until 1.8, 10^{-3} until 1.79, 10^{-4} until 1.7881, and 2.0×10^{-5} until 1.7879. As a perturbation for calculating the elements of Jacobian matrix in (20), $\epsilon_2 = 10^{-3}$ was used until 1.79, $\epsilon_2 = 10^{-5}$ until 1.7881, and $\epsilon_2 = 10^{-10}$ until 1.7879.

For $[3, 20]$, the solution at $Q = 3$ was first calculated by the simplex method. Incrementing Q with 0.5, we calculated the design curve. For $[20, 5 \times 10^5]$, the solution at $Q = 20$ was first calculated by the simplex method, and Q was incremented with 10^2 until 10^3 , 10^3 until 10^4 , and 5×10^3 until 5×10^4 . The perturbation $\epsilon_2 = 10^{-3}$ was taken for the Newton–Raphson method.

The Newton–Raphson method was terminated when $f_3(A, B) < \Theta_1 = 10^{-10}$ or $N_4 = 20000$. The other parameters are listed in Table I. Note that the parameters for the curve tracing algorithm are set to obtain the design parameters at strange Q 's. Figures 9(a) and 9(c) show the numbers of iterations needed to find the solution at each step, in which the intervals #1, #2, and #3 respectively imply that an uniform incremental of Q was used within the interval. When Q is small or large, a large number of iterations are required for the Newton–Raphson method to converge, because the design parameters (A, B) abruptly change when Q is small in Fig. 9(a) and the incremental Q is taken largely when Q is large in Fig. 9(c).

To compare with the results [18], the output (average) power P and voltage V_o are respectively defined as

$$P = \frac{V_o^2}{R}, \quad V_o = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} v_o^2(t) dt},$$

where the integral related to V_o was calculated by a trapezoidal rule.

Table IV shows values of PR/V_D^2 , $C_S \omega R$, and $C_0 \omega R$ on Q , where $R = 5[\Omega]$ and $V_D = 5[\text{V}]$ were

Fig. 10. Design characteristics of Example 2, (a) $H = 0.001$ and (b) $H = 1$.

assumed, and the last two values were calculated as

$$C_S \omega R = \frac{1}{A^2 B Q}, \quad C_0 \omega R = \frac{1}{A^2 Q}.$$

The results obtained from the proposed method are almost identical to [18].

4.3 Design Example 2

As the second design example, we consider the case of $H \neq 0$, whereas $H \approx 0$ is assumed in the previous design example. All the parameters (A, B) depending on Q were calculated by the curve tracing algorithm, in which the initial solution to trace the curve was calculated by the simplex method.

Figure 10 shows the design characteristics. To obtain them, the solution at $Q = 5$ was first calculated by the simplex method with the initial solution $(A, B) = (10^{-2}, 10^{-2})$ and the design curve was traced to both directions using the Newton–Raphson method. For $H = 0.001$, incrementing Q with 0.5 and oppositely decrementing with 0.5 until $Q = 2$ and 0.01 until $Q = 1.79$, we traced the curve. At $Q = 1.78$, the Newton–Raphson method did not converge. For $H = 1$, incrementing Q with 0.5 and oppositely decrementing with 0.5 until $Q = 2$ and 0.01 until $Q = 0.37$, we traced the curve. At $Q = 0.36$, the Newton–Raphson method did not converge. Therefore, we can calculate the design characteristics for the case of $H \neq 0$ except for strange Q values.

4.4 PSO and IPSO

Finally, PSO and IPSO with the objective function $f_3(A, B)$ were applied to find the passive element values of the class-E amplifier. With several $x_{\min}$ and $x_{\max}$, PSO and IPSO were evaluated over 100 trials, in which the design specification was given as $Q = 10$, $H = 1$, and $D = 0.5$. If the objective function value of (11) is less than Θ_1 within N_0 iterations and the global best position is $(p_{g1}, p_{g2}) = (0.9813986, 0.1675134)$, the circuit satisfies the ZVS and ZDS conditions and the class-E amplifier is obtained. On the other hand, when the global best position is not $(0.9813986, 0.1675134)$, the circuit corresponds to a class-E frequency multiplier.

Table V shows the performances of PSO and IPSO, in which “amplifier” indicates the percentage that the class-E amplifier is obtained and “multipliers” indicates the percentage that class-E frequency multipliers are obtained. PSO found the appropriate global minimum point as the class-E amplifier in only one trial and often converged into the global minimum points that correspond to the class-E frequency multipliers. IPSO could not find any global minimum point. From this fact, we can say that the passive element values of the class-E amplifier are not easily determined with the objective function (11) only. From the experiment results, we conclude that the proposed optimization procedure is more reliable than a metaheuristic method such as PSO.

Table V. Rates [%] that PSO and IPSO found the minimum points that correspond to the class-E amplifier or frequency multipliers.

$x_{\min}$	$x_{\max}$	PSO		IPSO	
		amplifier [%]	multipliers [%]	amplifier [%]	multipliers [%]
0.01	100	0	0	0	0
0.01	100	0	11	0	0
0.1	10	0	22	0	0
0.1	1	1	0	0	0

5. Conclusions

We have presented an automated design procedure for class-E amplifier. First, an efficient steady-state analysis of the class-E amplifier is presented, which provides a behavioral model that is used to evaluate the class-E ZVS and ZDS conditions efficiently. Without relying on the analytical expressions of the class-E amplifier, the passive element values determination is defined as an optimization problem. Using the simplex method, a minimization algorithm is proposed, which is more reliable than the IPSO-based method [13]. To calculate the design parameters changing the design specification, the curve tracing algorithm is provided using the Newton-Raphson method. Consequently, the proposed optimization procedure neither requires many computational resources nor relies on the analytical expressions that the designers with expert knowledge of the class-E amplifier can only derive.

We can apply the proposed method to other resonant power converter and amplifier designs [16]. Moreover, the proposed method can be easily linked to a circuit simulator [17]; then, the nonlinear effects of active devices are taken into account within a circuit simulation level.

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Appendix

In this section, we explain IPSO [13] which is applied to solve the optimization problem with the objective function $f_3(A, B)$. IPSO is an extension of PSO which thus is a special case of IPSO.

In PSO, there are multiple potential solutions, which are called “particles”. At each step, each particle flies toward its own past best position (*pbest*) as well as the best position among all the particles (*gbest*). Thus, all the particles of (standard) PSO are always fully-connected and affect one another. IPSO particles possess independence; thus, IPSO includes isolated particles that are not connected to any particle, and the connections among particles are stochastically determined at every step. Each particle has two kinds of information, position and velocity. The position and velocity of particle i are represented by $\mathbf{X}_i = (x_{i1}, x_{i2})$ and $\mathbf{Y}_i = (y_{i1}, y_{i2})$, respectively. Figure A-1 shows a pseudo code, in which $d = 1, 2$ indicates a coordinate of particle’s position. Steps of IPSO are summarized as follows:

(Step1) (Initialization) Let the iteration step n be 0. Initialize the particle position $\mathbf{X}_i$ ($x_{id} \in [x_{\min}, x_{\max}]$), which is randomly selected in the parameter space, its velocity as $\mathbf{Y}_i = \mathbf{0}$, and $\mathbf{P}_i = (p_{i1}, p_{i2}) = \mathbf{X}_i$.

(Step2) Decide a connection relationship for every coordinate. In the d -th coordinate, each particle i is connected to the others with a probability K ; $i \in S_{Cd}$, where S_{Cd} is a set of particles connected to the swarm in the d -th coordinate. In other words, the particle i is isolated from the swarm with a probability $1 - K$, and the particle i does not interact with the others. $K \in [0.01, 0.1]$, which is a constant cooperative coefficient, is effective for most problems [12].

(Step3) Evaluate the objective function $f_3(\mathbf{X}_i)$ for each particle i . Update the personal best position (*pbest*) as $\mathbf{P}_i = \mathbf{X}_i$, if $f_3(\mathbf{X}_i) < f_3(\mathbf{P}_i)$.

(Step4) Let $\mathbf{P}_l$ represent the best position *lbest* with the local best *pbest* among the connected particles in $S_C = \cup_d S_{Cd}$. The standard PSO always uses *gbest* for updating the particle information. However, IPSO uses *lbest* instead of *gbest*, because the connection relationships change at every step.

```

Objective function  $f_3(\mathbf{X}_i)$ ,  $\mathbf{X}_i = (x_{i1}, x_{i2})^T$ 
Initialize particle's position  $\mathbf{X}_i$  and velocity  $\mathbf{Y}_i$ , ( $i = 1, 2$ )
Initialize  $pbest$   $\mathbf{P}_i$  with a copy of  $\mathbf{X}_i$ , ( $i = 1, 2$ )
Iteration count  $n \leftarrow 0$ 
while ( $(n < N_0 \text{ iterations})$  AND  $(f_3(\mathbf{P}_g) > \Theta_1)$ ) do
  for  $i = 1$  to  $M$ , all  $M$  particles do
    Evaluate the objective function at  $\mathbf{X}_i(n+1)$ 
    for  $d = 1$  to  $2$ , all the coordinates do
      Connect/isolate the  $d$ -th component of the particle  $i$  with/from the swarm
    end for  $d$ 
    Update  $pbest$   $\mathbf{P}_i$ 
  end for  $i$ 
  Update  $lbest$   $\mathbf{P}_l$ 
  Update  $gbest$   $\mathbf{P}_g$ 
  for  $i = 1$  to  $M$ , all  $M$  particles do
    for  $d = 1$  to  $2$ , all the coordinates do
      Calculate  $y_{id}(n+1)$  and  $x_{id}(n+1)$  using (22) and (23)
    end for  $d$ 
  end for  $i$ 
   $n \leftarrow n + 1$ 
end while

```

Fig. A-1. Pseudo code of IPSO.

Update $lbest$ as $\mathbf{P}_l = (p_{l1}, p_{l2})$ according to

$$l = \arg \min_i f_3(\mathbf{P}_i), \quad i \in S_C. \quad (\text{A-1})$$

Note that the particles isolated from the swarm do not affect the new $lbest$, even if $gbest$ is the minimum $pbest$ among all the particles including the isolated particles.

(Step5) Update $\mathbf{Y}_i$ and $\mathbf{X}_i$ according to

$$y_{id}(n+1) = \begin{cases} wy_{id}(n) + c_1 r_1 (p_{id} - x_{id}(n)) \\ \quad + c_2 r_2 (p_{ld} - x_{id}(n)), & \in S_{Cd} \\ wy_{id}(n) + c_1 r_1 (p_{id} - x_{id}(n)), & \text{otherwise} \end{cases} \quad (\text{A-2})$$

$$x_{id}(n+1) = x_{id}(n) + y_{id}(n+1), \quad (\text{A-3})$$

where w is the inertia weight, c_1 and c_2 are positive acceleration coefficients, $c_1 = c_2$ in general, and r_1 and r_2 are uniform random numbers $\in [0.01, 0.1]$, which are represented as $U(0, 1)$. In (A-2), we stochastically determine whether each particle is affected by $lbest$ or not, by using the cooperativeness K . When $K = 0$, all the particles move depending only on $pbest$, and when $K = 1$, the algorithm is entirely the same with (standard) PSO.

(Step6) Let $n = n + 1$, and go back to **(Step2)** if any of stopping conditions ($n == N_0$ OR $f_3(\mathbf{P}_g) < \Theta_1$) are satisfied, where $\mathbf{P}_g = (p_{g1}, p_{g2})$ is $gbest$.

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