Invited Paper

Smooth gait transition in hardware-efficient CPG model based on asynchronous coupling of cellular automaton phase oscillators

Kentaro Takeda^{1a)} and Hiroyuki Torikai^{1b)}

¹ *Graduate School of Science and Engineering, Hosei University
3-7-2 Kajino-cho, Koganei-city, Tokyo 184-8584, Japan*

^{a)} *takeda@mail.nsci.jp*

^{b)} *torikai@hosei.ac.jp*

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Abstract: In this paper, a central pattern generator (CPG) model based on asynchronous coupling of cellular automaton (CA) phase oscillators for a hexapod robot is presented. The presented CPG model is composed of the CA phase oscillators whose discrete state transitions are triggered by multiple asynchronous clocks. Then, evaluation functions to quantify synchronization states for target gait patterns in the presented CPG model are introduced. Analyzing the synchronizations using the evaluation functions, this paper clarifies that the presented CPG model is suitable to perform smooth gait transitions for the hexapod robot than a CPG model whose discrete state transitions are triggered by a single clock (i.e., synchronous coupling). The presented CPG model is implemented in a field programmable gate array (FPGA); experiments verify that the hexapod robot mounted with the FPGA, in which the presented CPG model is implemented, can perform smooth gait transitions.

Key Words: central pattern generator (CPG), smooth gait transition, asynchronous cellular automaton, hexapod robot, field programmable gate array (FPGA)

1. Introduction

Legged animals perform complex yet stable locomotion and even adapt their rhythmic patterns for gait appropriate to their walking speed and certain types of terrain. It has been thought that such rhythmic patterns are produced by central pattern generators (CPGs), biological neural circuits found in the spinal cords, without sensory feedback [1]. The CPGs are mostly modeled by networks composed of nonlinear oscillators (e.g., Van der Pol oscillators [2], Hopf oscillators [3], Kuramoto oscillators [4], and spiking neurons [5]), which employ their synchronization properties to produce the rhythmic patterns. These models have been successfully utilized in gait generations for various types of legged robots [6]. Further, in recent years, CPG models have been applied in the field of medical engineering, e.g.,

Fig. 1. (a) Cellular automaton (CA) phase oscillator. (b) Synchronous coupling of CA phase oscillators. (c) Asynchronous coupling of CA phase oscillators.

non-invasive and invasive prostheses [7, 10]. For such practical applications, the CPG models must utilize fewer hardware resources and consume less power. However, bio-inspired models, including the CPG models, tend to increase the hardware resources required for circuit implementation due to their nonlinearities. Considering the nonlinear circuit and system theory, the bio-inspired models are classified into the following four classes based on continuousness and discontinuousness of state variables and time.

- *Class CTCS.* This is a nonlinear differential equation model of a bio-inspired system with a *continuous time and continuous states* (CTCS). A class CTCS bio-inspired model can be generally implemented in an analog nonlinear circuit, e.g., [10–16].
- *Class DTCS.* This is a nonlinear difference equation model of a bio-inspired system with a *discrete time and continuous states* (DTCS). A class DTCS bio-inspired model can be generally implemented in a switched capacitor circuit, e.g., [17–19].
- *Class DTDS.* This is a numerical integration model (in finite binary number representation) of a bio-inspired system with a *discrete time and discrete states* (DTDS). A class DTDS bio-inspired model can be generally implemented in a digital processor or a sequential logic circuit, e.g., [2–5, 7–9].
- *Class CTDS.* This is an asynchronous cellular automaton (CA) model of a bio-inspired system with a *continuous (state transition) time and discrete states* (CTDS). A class CTDS bio-inspired model can be generally implemented in an asynchronous sequential logic circuit, e.g., [20–23].

Our group has been developing CPG models for hexapod robots belonging to the class CTDS system, which utilize fewer hardware resources for circuit implementation than models belonging to the class DTDS system [20–23]. Figure 1 shows conceptual diagrams of our CPG model studied in this paper. As shown in Fig. 1(a), our CPG model is composed of CA phase oscillators implemented in a sequential logic circuit, which have flip-flops storing discrete state variables and logic gates realizing discrete nonlinear coupling functions. In particular, the state space of our CPG model is roughly discretized, i.e., the required bit length for the state space is very small. Accordingly, the logic gates for the nonlinear coupling function can also be designed to be small. However, in this study, we found that, due to the rough discretization, transitions of discrete variables do not work properly if they are triggered by a single clock (synchronous coupling of the CA phase oscillators as shown in Fig. 1(b)). In order to overcome this difficulty, we employed the asynchronous nature of the clocks, that is, the state transitions of the discrete variables are triggered by multiple asynchronous clocks (asynchronous coupling of the CA phase oscillators as shown in Fig. 1(c)). We also clarified that the CPG model based on the asynchronous coupling of CA phase oscillators can perform smooth gait transition for the hexapod robot while the CPG model based on the synchronous coupling of CA phase oscillators

Fig. 2. (a) Hexapod robot mounted with a field programmable gate array (FPGA), in which the presented CPG model is implemented. (b) Schematic diagram of the CA phase oscillator. (c) Network topology of the CPG presented model.

cannot. It should be noted that a CPG model belonging to the class DTDS system means that the oscillators composing a network are synchronously coupled, i.e., their state transitions are triggered by a single clock. Preliminary results can be found in our conference proceedings, which reported that our CPG model can produce some types of gait patterns and can be implemented in a field programmable gate array (FPGA) utilizing fewer hardware resources than a CPG model belonging to the class DTDS system [22, 23]. That is to say this paper studied the remaining important issues about the smooth transition between the gait patterns in our CPG model and comparisons of asynchronously coupled and synchronously coupled CA phase oscillators.

The significance of this paper is as follows.

Significance: In this study, we firstly demonstrate that roughly discretizing a state space of a CPG model in order to reduce hardware resources causes gait transitions failures. Then, this paper presents a CPG model based on asynchronous coupling of oscillators and demonstrates that the model can realize smooth transition between different gait patterns. This advantage of the presented model suggests that this study contributes to design future applications such as the non-invasive and invasive prosthetic devices [7, 10], and the bio-inspired robots [6], which can be implemented as a small-scale circuit and has low power consumption.

2. CPG model based on asynchronous coupling of CA phase oscillators

This section presents a central pattern generator (CPG) model based on asynchronous coupling of cellular automaton (CA) phase oscillators for a hexapod robot shown in Fig. 2(a). The presented CPG model consists of the CA phase oscillators, where a schematic diagram of each CA phase oscillator is shown in Fig. 2(b). As shown in this figure, each CA phase oscillator has the following discrete phase variable and discrete auxiliary variable.

Discrete phase variable:

$$\Phi_i \in \mathbb{Z}_N^+ \equiv \{0, \dots, N-1\}, \quad (1)$$

Discrete auxiliary variable:

$$P_i \in \mathbb{Z}_M^+ \equiv \{0, \dots, M-1\}, \quad (2)$$

where N and M are positive integers. Further, $i \in \{1, \dots, n\}$ represents an index for the oscillators with periodic boundary conditions (e.g., $\Phi_{n+1} = \Phi_1$ and $P_{n+1} = P_1$), where n is a positive integer representing the number of oscillators. State transitions of the discrete variables Φ_i and P_i are triggered by the following internal clock.

Internal clock:

$$Clk_i(t) \equiv \sum_{j=0}^{\infty} \delta(t - jT_i), \quad Clk_i \in \{0, 1\}, \quad (3)$$

where $t \in \mathbb{R}$ represents a continuous time, $T_i \in (0, \infty)$ represents a period of the i -th internal clock Clk_i , and δ represents the following unit impulse.

$$\delta(x) \equiv \begin{cases} 1 & \text{if } x = 0, \\ 0 & \text{if } x \neq 0, \end{cases} \quad \delta : \mathbb{R} \rightarrow \{0, 1\}. \quad (4)$$

Then, as shown in Fig. 3, the internal clock Clk_i triggers the following state transitions of the discrete phase variable Φ_i .

$$\text{If } Clk_i(t) = 1 \text{ and } P_i(t) \geq |H(\Delta\Phi_i^+)|, \text{ then } \Phi_i(t_+) := \begin{cases} \Phi_i(t) + 1 & \text{if } H(\Delta\Phi_i^+) \geq 0 \text{ and } \Phi_i(t) < N - 1, \\ 0 & \text{if } H(\Delta\Phi_i^+) \geq 0 \text{ and } \Phi_i(t) = N - 1, \\ \Phi_i(t) - 1 & \text{if } H(\Delta\Phi_i^+) < 0 \text{ and } \Phi_i(t) > 0, \\ N - 1 & \text{if } H(\Delta\Phi_i^+) < 0 \text{ and } \Phi_i(t) = 0, \end{cases} \quad (5)$$

where the symbol “ t_+ ” denotes “ $\lim_{\varepsilon \rightarrow +0} t + \varepsilon$ ” and the symbol “ $:=$ ” denotes an “instantaneous state transition” throughout the paper. Further, the discrete function H , which works as a coupling function as shown in Fig. 2(b), is defined as follows.

Discrete coupling function:

$$H(\Delta\Phi^+) \equiv \begin{cases} M - 1 & \text{if } h(\Delta\Phi^+) = 0 \text{ or } \lfloor h(\Delta\Phi^+)^{-1} \rfloor > M - 1, \\ -(M - 1) & \text{if } \lfloor h(\Delta\Phi^+)^{-1} \rfloor < -(M - 1), \\ \lfloor h(\Delta\Phi^+)^{-1} \rfloor & \text{otherwise,} \end{cases} \quad (6)$$

$$H : \mathbf{Z}_N^\pm \equiv \{0, \dots, \pm 2(N - 1)\} \rightarrow \mathbf{Z}_M^\pm \equiv \{0, \dots, \pm(M - 1)\},$$

where $\lfloor \cdot \rfloor$ denotes the following floor function.

$$\lfloor x \rfloor \equiv \max\{l \in \mathbb{Z} \mid l \leq x\}, \quad x \in \mathbb{R}. \quad (7)$$

The function h is defined as

$$h(\Delta\Phi^+) \equiv \Gamma N \sin(2\pi\Delta\Phi^+/N), \quad h : \mathbf{Z}_N^\pm \rightarrow \mathbb{R}, \quad (8)$$

where $\Gamma \in \mathbf{R}$ represents a coupling constant. $\Delta\Phi_i^+$ denotes

$$\Delta\Phi_i^+ \equiv \Phi_{i+1}(t) - \Phi_i(t) + \lfloor N\phi/2\pi \rfloor \bmod N, \quad (9)$$

where $\phi \in [0, 2\pi)$ is a parameter representing the phase difference of the discrete phase variables Φ_i and Φ_{i+1} . Hence, the CA phase oscillators are unidirectionally coupled in a loop arrangement, as shown in Fig. 2(c). It should be noted that if N and M are not so large, the discrete coupling function H can be implemented in look-up-tables (LUTs), which consume less hardware resources than arithmetic logic units. Further, as shown in Fig. 3, the internal clock Clk_i triggers the following state transition of the discrete auxiliary variable P_i .

$$\text{If } Clk_i(t) = 1, \text{ then } P_i(t_+) := \begin{cases} P_i(t) + 1 & \text{if } P_i(t) < |H(\Delta\Phi_i^+)|, \\ 0 & \text{if } P_i(t) \geq |H(\Delta\Phi_i^+)|. \end{cases} \quad (10)$$

Figures 4(a) and (b) show the examples of time waveforms of the CA phase oscillators. Further, Figs. 4(c) and (d) show the Cartesian coordinate representations of the discrete phase variables Φ_i on the unit circles for $t = 0$ and $t = 3$, where the i -th oscillator is plotted at

$$x = \cos(2\pi\Phi_i/N), \quad y = \sin(2\pi\Phi_i/N), \quad (11)$$

Fig. 3. State transitions of discrete phase variables Φ_i and discrete auxiliary variables P_i triggered by internal clocks Clk_i defined by Eqs. (5) and (10).

Fig. 4. (a) Example of time waveforms of discrete phase variables Φ_i . (b) Example of time waveforms of discrete auxiliary variables P_i . (c) Cartesian coordinate representation on unit circle for $t = 0$. The CA phase oscillator are plotted based on Eq. (11). (d) Cartesian coordinate representation on unit circle for $t = 3$. The parameter values are fixed as follows: $N = 18$, $M = 32$, $\Gamma = 10^{-2}$, $\phi = 0$, and $T_i = 10^3$ for all i .

as the black circle with the index. These figures show that the coupled CA phase oscillators exhibit in-phase synchronization. Depending on the clock periods T_i , the following definition is introduced.

Definition. The CA phase oscillators are said to be

$$\begin{cases} \text{synchronously coupled} & \text{if } T_i/T_j = \text{rational number} \text{ for all } i \text{ and } j, \\ \text{asynchronously coupled} & \text{if } T_i/T_j = \text{irrational number} \text{ for certain } i \neq j. \end{cases} \quad (12)$$

In the case of Fig. 4, the CA phase oscillators are synchronously coupled. The difference in behaviors between the synchronously and the asynchronously coupled CA phase oscillators is discussed in Subsection 3.2.

3. Synchronization in our CPG model for hexapod robot

This section shows that the presented CPG model can imitate typical gait patterns for the hexapod robot. By analyses using the evaluation function to quantify synchronization states, an advantage of the asynchronously coupled CA phase oscillators is clarified. Let us begin by introducing target gait patterns in the following subsection.

3.1 Target gait patterns

Figures 5(a) and (b) show gait diagrams [24] of a six-legged insect, where the labels L1–L3 and R1–R3 correspond to those shown in Figs. 2(a) and (c). These gait patterns, called tripod gait (fast) and wave gait (slow), are used as the target gait patterns to be imitated in this study. In the diagrams, the blue rectangle shows the moment when the leg is off the ground and moving forward, and the orange rectangle shows the moment when the leg is touching and crawling the ground. A pair of the blue rectangle and the orange rectangle forms a period τ as indicated by the black arrow in Fig. 5(a). In order to realize synchronization patterns for the coupled CA phase oscillators from the above target gait patterns, let us consider the following time-varying phases as shown in Figs. 5(c) and (d),

$$\varphi_i(t) \equiv 2\pi t/\tau + \psi_i \pmod{2\pi}, \quad \varphi_i \in [0, 2\pi), \quad (13)$$

where $\psi_i \in [0, 2\pi)$ represents an initial phase as indicated by the black arrow in Fig. 5(c). Let us also consider the following map,

$$\sigma(\varphi) \equiv \begin{cases} \text{“blue rectangle”} & \text{for } \varphi < \theta, \\ \text{“orange rectangle”} & \text{for } \varphi \geq \theta, \end{cases} \quad (14)$$

$$\sigma : [0, 2\pi) \rightarrow \{\text{“blue rectangle”, “orange rectangle”}\},$$

where $\theta \in [0, 2\pi)$ represents a threshold to determine a boundary between the “blue rectangle” and the “orange rectangle” as indicated by the black arrow in Fig. 5(c). Applying the map σ to the six time-varying phases ($\varphi_1(t), \dots, \varphi_6(t)$), a gait diagram can be obtained. For example, the diagram of the tripod gait shown in Fig. 5(a) can be obtained by applying the maps σ to the phases φ_i for a parameter case

$$(\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \theta) = (0, \pi, 0, \pi, 0, \pi, 0, \pi). \quad (\text{tripod gait}) \quad (15)$$

Figure 5(c) shows the time waveforms of the phases φ_i under the above parameter case, where the shadow rectangle shows the region when $\varphi_i(t) < \theta$, which corresponds to the “blue rectangle” in the diagram in Fig. 5(a). Further, the diagram of the wave gait shown in Fig. 5(b) can be obtained by applying the maps $\sigma(\varphi_i)$ to the phases φ_i for a parameter case

$$(\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \theta) = (0, 5\pi/3, 4\pi/3, \pi, 2\pi/3, \pi/3, \pi/3). \quad (\text{wave gait}) \quad (16)$$

Figure 5(d) shows the time waveforms of the phases φ_i under the above parameter case. Hence, the synchronization pattern of the time-varying phases φ_i shown in Figs. 5(b) and (c) should be generated by the presented CPG model for imitating the tripod gait and the wave gait. In the presented CPG model, the phase differences of the oscillators (e.g., the black arrow indicated in Fig. 5(c)) coincide with the system parameter ϕ . Figure 5(e) shows the time waveforms of the tripod gait generated by the presented CPG model, where the values of the parameter ϕ is

$$\phi = \pi. \quad (\text{tripod gait}) \quad (17)$$

Figure 5(g) shows Cartesian coordinate representations of the discrete phase variables Φ_i corresponding to Fig. 5(e) in a steady state. Further, Fig. 5(f) shows the time waveforms of the wave gait generated by the presented CPG model, where the values of the parameter ϕ is

$$\phi = \pi/3. \quad (\text{wave gait}) \quad (18)$$

Figure 5(h) shows Cartesian coordinate representations of the discrete phase variables Φ_i corresponding to Fig. 5(f) in a steady state. The next subsection investigates differences between the synchronously and asynchronously coupled CA phase oscillators for the cases in Eqs. (17) and (18).

Fig. 5. Two typical gait diagrams [24] and their target synchronization patterns. (a) Gait diagram of tripod gait (fast). (b) Gait diagram of wave gait (slow). (c) Target synchronization pattern for tripod gait defined by Eq. (13). The parameter values are described in Eq. (15). (d) Target synchronization pattern for wave gait defined by Eq. (13). The parameter values are described in Eq. (16). (e) Time waveforms of coupled CA phase oscillators for tripod gait. The parameter values are described in Eq. (17). The others are fixed as follows: $N = 36$, $M = 64$, $\Gamma = 4.347 \times 10^{-3}$, and $T_i = 4.347 \times 10^{-4}$ for all i . (f) Time waveforms of coupled CA phase oscillators for wave gait. The parameter values are described in Eq. (18). The others are fixed as the same values chosen in (e). (g) Cartesian coordinate representation on unit circle for $t = 5$ of time waveforms in (e). (h) Cartesian coordinate representation on unit circle for $t = 5$ of time waveforms in (f).

3.2 Synchronization analysis of asynchronously and synchronously coupled CA phase oscillators

In order to quantify the synchronization patterns of the coupled CA phase oscillators, the following instantaneous evaluation function for the tripod gait is introduced.

$$r_{\text{tripod}}(t) \equiv \frac{1}{n} \left| \sum_{i=1}^{n/2} \left(e^{j2\pi \frac{\Phi_{2i}(t)}{N}} + e^{j\left(2\pi \frac{\Phi_{2i+1}(t)}{N} + \pi\right)} \right) \right|, \quad r_{\text{tripod}} \in [0, 1], \quad (19)$$

where $j = \sqrt{-1}$. The instantaneous evaluation function r_{tripod} closes to 1 means that the coupled CA phase oscillators achieve the target synchronization pattern for the tripod gait. It should be noted that r_{tripod} only evaluates phase relationship of the oscillators, which does not necessarily mean that they are oscillating properly even if $r_{\text{tripod}} \approx 1$. Accordingly, the following instantaneous average velocity of the coupled CA phase oscillators is introduced.

$$v_{\text{ave}}(t) \equiv \frac{1}{n} \sum_{i=1}^n \sum_{k=1}^{K_i(t)} \frac{\Phi_i(t_{i,k}) - \Phi_i(t_{i,k-1})}{N(t_{i,k} - t_{i,k-1})} I_{i,k}(t), \quad v_{\text{ave}} \in \mathbb{R}, \quad (20)$$

where $K_i(t) \equiv \max\{k \in \mathbb{Z} \mid t_{i,k} \leq t\}$, $\Phi_i(t_{i,k}) - \Phi_i(t_{i,k-1})$ is calculated on the circle, $t_{i,k}$ represents the time when the k -th state transition of the discrete phase variable Φ_i is occurred, and $I_{i,k}$ is the following indicator function.

$$I_{i,k}(t) \equiv \begin{cases} 1 & \text{if } t \in [t_{i,k}, t_{i,k+1}), \\ 0 & \text{if } t \notin [t_{i,k}, t_{i,k+1}), \end{cases} \quad I_{i,k} : \mathbb{R} \rightarrow \{0, 1\}. \quad (21)$$

Figure 6(a) shows the instantaneous evaluation functions r_{tripod} and the instantaneous average velocity v_{ave} for ten simulation trials of the synchronously coupled CA phase oscillators, where the initial values are randomly chosen. In these trials, the four types of behaviors are observed as follows.

- (i) *Oscillation with target synchronization pattern.* In Fig. 6(a), the values of the instantaneous evaluation functions r_{tripod} for the positive v_{ave} in the steady states are drawn by the blue lines. Among them, the blue lines approaching 1 in steady states mean that the coupled CA phase oscillators exhibit the synchronization pattern for the tripod gait properly. For example, Fig. 6(b) shows the time waveforms and Cartesian coordinate representations of the discrete phase variables Φ_i in the trial indicated by the corresponding arrow in Fig. 6(a); the features of them are consistent with those shown in Figs. 5(d)–(h).
- (ii) *Oscillation with non-target synchronization pattern.* In Fig. 6(a), the values of the instantaneous evaluation function r_{tripod} drawn by the blue line that does not approach 1 in the steady state mean that the coupled CA phase oscillators do not exhibit the synchronization pattern for the tripod gait. For example, Fig. 6(c) shows the time waveforms and Cartesian coordinate representations of the discrete phase variables Φ_i in the trial indicated by the corresponding arrow in Fig. 6(a). As shown in this figure on the right, the CA phase oscillators exhibit three-phase synchronization, which is not the target synchronization pattern for the tripod gait.
- (iii) *Non-oscillation.* In Fig. 6(a), the values of the instantaneous evaluation functions r_{tripod} for $v_{\text{ave}} \approx 0$ in the steady states are drawn by the orange lines. The orange lines approach 1 in the steady states; however, all the CA phase oscillators stop oscillating. In this case, the coupled CA phase oscillators do not achieve the synchronization pattern for the tripod gait. For example, Fig. 6(d) shows the time waveforms and the Cartesian coordinate representations of the discrete phase variables Φ_i in the trial indicated by the corresponding arrow in Fig. 6(a). As shown in this figure, the whole CA phase oscillators do not oscillate in the steady state (left side).
- (iv) *Reverse oscillation.* In Fig. 6(a), the values of the instantaneous evaluation function r_{tripod} for the negative v_{ave} in the steady states are drawn by the green line. The green line does not approach 1 in the steady state; besides, all the CA phase oscillators are reversely oscillating. In this case, the coupled CA phase oscillators do not achieve the synchronization pattern for the tripod gait. For example, Fig. 6(e) shows the time waveforms and Cartesian coordinate representations of the discrete phase variables Φ_i in the trial indicated by the corresponding arrow in Fig. 6(a). As shown on the left side in this figure, all the CA phase oscillators evolve

Fig. 6. (a) Instantaneous evaluation function r_{tripod} defined by Eq. (19) for ten simulation trials of synchronously coupled CA phase oscillators from randomly chosen initial values. The parameter values are fixed as follows: $N = 36$, $M = 64$, $\Gamma = 4.347 \times 10^{-3}$, and $T_i = 4.347 \times 10^{-4}$ for all i . The others are described in Eq. (17). (b) Oscillation with target synchronization. The time waveforms and Cartesian coordinate representations on the unit circle correspond to the blue line not approaching 1 in (a). (c) Oscillation with non-target synchronization. The time waveforms and Cartesian coordinate representations on the unit circle correspond to the blue line not approaching 1 in (a). (d) Non-oscillation. The time waveforms and Cartesian coordinate representations on the unit circle correspond to the orange line in (a). (e) Reverse oscillation. The time waveforms and Cartesian coordinate representations on the unit circle correspond to the green line in (a).

Fig. 7. Instantaneous evaluation function r_{tripod} defined by Eq. (19) for ten simulation trials of asynchronously coupled CA phase oscillators from the same initial values as those chosen in Fig. 6. The parameter values are fixed as follows: $N = 36$, $M = 64$, $\Gamma = 4.347 \times 10^{-3}$, $T_i = 4.347 \times 10^{-4}$ for $1 \leq i \leq 5$, and $T_6 = 1.872\pi \times 10^{-4}$. The others are described in Eq. (17).

with negative slopes in the steady state that means the CA phase oscillators are reversely oscillating.

Remark 1 (advantage of asynchronous coupling in gait): Figure 7 shows the instantaneous evaluation functions r_{tripod} for ten simulation trials of the asynchronously coupled CA phase oscillators, where the parameter values (except for the parameter T_5 for the period of the clock Clk_5) and the initial values are the same as those chosen in Fig. 6(a). For all the trials, the coupled CA phase oscillators exhibit the synchronization pattern for the tripod gait where all the behaviors correspond to (i).

The characteristic above that the synchronously coupled CA phase oscillators may fail to synchronize is also observed in gait transitions; it is not observed in the asynchronously coupled CA phase oscillators. The result of the comparison on the gait transitions is shown below. For the wave gait, the following another instantaneous evaluation function to quantify the synchronization patterns of the coupled CA phase oscillators is introduced.

$$r_{\text{wave}}(t) \equiv \frac{1}{n} \left| \sum_{i=1}^n e^{j \left(2\pi \frac{\Phi_i(t)}{N} + \frac{\pi}{3}(i-1) \right)} \right|, \quad r_{\text{wave}} \in [0, 1]. \quad (22)$$

As with the one for the tripod gait in Eq. (19), the instantaneous evaluation function r_{wave} closes to 1 means that the coupled CA phase oscillators achieve the target synchronization pattern for the wave gait. Figure 8(a) shows the time waveforms of the synchronously coupled CA phase oscillators, where the values of the parameter ϕ is time variant as follows.

$$\begin{aligned} \phi &= \pi/3 \quad \text{for } 0 \leq t < 1.25 \quad \text{and} \quad 2.5 \leq t < 3.75, & (\text{wave gait}) \\ \phi &= \pi \quad \text{for } 1.25 \leq t < 2.5 \quad \text{and} \quad 3.75 \leq t \leq 5. & (\text{tripod gait}) \end{aligned} \quad (23)$$

Figure 8(b) shows the instantaneous evaluation functions r_{tripod} (solid line) and r_{wave} (dashed line), and the instantaneous average velocity v_{ave} corresponding to Fig. 8(a). For $0 \leq t < 1.25$, the synchronously coupled CA phase oscillators fail to synchronize to the target synchronization pattern, where the behavior is corresponding to (iii). For $1.25 \leq t < 2.5$, the synchronously coupled CA phase oscillators achieve the target synchronization pattern, where the behavior is corresponding to (i). For $2.5 \leq t < 3.75$, the synchronously coupled CA phase oscillators fail to synchronize to the target synchronization pattern, where the behavior is corresponding to (iv). For $3.75 \leq t \leq 5$, the synchronously coupled CA phase oscillators achieve the target synchronization pattern, where the behavior is corresponding to (i). On the other hand, Figs. 8(c) and (d) show the time waveforms, the instantaneous evaluation functions r_{tripod} and r_{wave} , and the instantaneous average velocity v_{ave} of the asynchronously coupled CA phase oscillators, where the parameter values (except for the parameter

Fig. 8. Gait transitions between wave gait and tripod gait. (a) Time waveforms of the synchronously coupled CA phase oscillators. The values of the parameter ϕ is time variant as described in Eq. (23). The other parameter values are the same as those chosen in Fig. 6. (b) Instantaneous evaluation functions r_{tripod} (solid line) and r_{wave} (dashed line) defined by Eq. (38), and instantaneous average velocity v_{ave} defined by Eq. (20) of the synchronously coupled CA phase oscillators. (c) Time waveforms of the asynchronously coupled CA phase oscillators. The values of the parameter ϕ_i is switched as described in Eq. (23). The other parameter values are the same as those chosen in Fig. 7. (d) Instantaneous evaluation functions r_{tripod} (solid line) and r_{wave} (dashed line), and instantaneous average velocity v_{ave} of the asynchronously coupled CA phase oscillators.

T_5 for the period of the clock Clk_5) and the initial values are the same as those chosen in Figs. 8(a) and (b). As shown in these figures, the asynchronously coupled oscillators achieve all the gait transitions between the tripod gait and the wave gait.

Remark 2 (advantage of asynchronous coupling in gait transition). Our extensive simulations reveal that the synchronously coupled CA phase oscillators often fail to realize the proper gait transitions. On the other hand, the asynchronously coupled CA phase oscillators mostly realize the proper gait transitions. Hence, the analyses in this study clarified that the asynchronously coupled CA phase oscillators are suitable to perform the smooth gait transition for the hexapod robot compared to the synchronously coupled CA phase oscillators. It should be noted that this study does not guarantee that the asynchronously coupled CA phase oscillator can always realize the target synchronization patterns. Hence, future work is needed to theoretically analyze the synchronization

phenomena of the coupled CA phase oscillators.

4. FPGA implementation

This section shows the hexapod robot mounted with an FPGA, in which the presented CPG model is implemented, can realize the wave gait, the tripod gait, and their transitions. Figure 9 shows the block diagram of a control system for the i -th leg, where each leg has the two servomotors for the 2-degrees-of-freedom. As shown in this figure, the discrete phase variables Φ_i are converted to Cartesian coordinate representations from polar coordinate representations as follows.

Conversion from polar to Cartesian:

$$\begin{aligned}
 X(\Phi) &\equiv \begin{cases} \lfloor A \cos(2\pi(\pi\Phi/\theta)/N) \rfloor & \text{if } \Phi < N\theta/2\pi, \\ \left\lfloor A \cos\left(2\pi\left(\frac{\Phi - N\theta/2\pi}{2 - \theta/\pi} + \frac{N}{2}\right)/N\right) \right\rfloor & \text{if } \Phi \geq N\theta/2\pi, \end{cases} & X : \mathbf{Z}_N^+ \rightarrow \mathbb{Z}, \\
 Y(\Phi) &\equiv \begin{cases} \lfloor A \sin(2\pi(\pi\Phi/\theta)/N) \rfloor & \text{if } \Phi < N\theta/2\pi, \\ \left\lfloor A \sin\left(2\pi\left(\frac{\Phi - N\theta/2\pi}{2 - \theta/\pi} + \frac{N}{2}\right)/N\right) \right\rfloor & \text{if } \Phi \geq N\theta/2\pi, \end{cases} & Y : \mathbf{Z}_N^+ \rightarrow \mathbb{Z},
 \end{aligned} \tag{24}$$

where $A \in \mathbb{R}$ is a scaling parameter for a pulse-width modulation and these functions are implemented in LUTs. Figure 10(a) shows the example of the signals $X(\Phi_i(t))$ and $Y(\Phi_i(t))$ simulated by Xilinx Vivado Design Suite v2018.2, an integrated design environment for synthesis and analysis of hardware description language (HDL) designs, where the detail of the design is described later. As shown in Fig. 9, the servomotors corresponding to yaw axes are controlled by pulse width-modulated signals of the signals X . Also, the servomotors corresponding to roll axes are controlled by pulse width-modulated signals $\hat{Y}$ of the signals through the following a saturator function.

Saturator:

$$\hat{Y}(Y) \equiv \begin{cases} B & \text{if } Y \geq 0, \\ -B & \text{if } Y < 0, \end{cases} \quad \hat{Y} : \mathbb{Z} \rightarrow \{-B, B \mid B \in \mathbb{Z}\}, \tag{25}$$

where $B \in \mathbb{Z}$ is a scaling parameter for the pulse-width modulation. The dynamics of the presented model are written as a register transfer level (RTL) code using VHDL as follows. The discrete variables Φ_i and P_i are implemented by registers as N-bit and M-bit unsigned integers, where $N = \lceil \log_2 N \rceil$ and $M = \lceil \log_2 M \rceil$, respectively. The function H is implemented in LUTs having a $2(N + 1)$ -bit signed

Fig. 9. Schematic diagram of control strategy for i -th leg's servomotor. The blocks of i -th and j -th oscillators are identical with the circuit in Fig. 2(b).

Fig. 10. (a) Waveforms of signals X and Y simulated by Xilinx Vivado Design Suite v2018.2. (b) Snapshots of tripod gait. (c) Snapshots of wave gait. The parameter values are the same as those chosen in Fig. 7, where T_5 is approximated as 5.88×10^{-4} . Also, $\theta = \pi/3$ for the wave gait and $\theta = \pi$ for the tripod gait.

integer input and a $(M + 1)$ -bit signed integer output in the two's complement format. The functions X and Y are respectively implemented in LUTs having a N -bit unsigned integer input and a L -bit unsigned integer output, where L is depending on the resolution of the pulse-width modulator. The state transitions in Eqs. (5) and (10) are written by sequential statements triggered by the clocks Clk_i . The VHDL code is synthesized by Xilinx Vivado Design Suite v2018.2 and a resulting bitstream file is downloaded into the Xilinx's FPGA, Artix-7 XC7A100T-1CSG324C [25], mounted on Digilent's Nexys 4 DDR evaluation platform [26]. Since the FPGA device does not support asynchronous triggering, the internal clocks Clk_i are generated by frequency-dividing a common clock with a high frequency (100 [MHz]). The FPGA device, in which the presented CPG model is implemented, is mounted on the hexapod robot, the Lynxmotion's MH2 hexapod robot [27], as shown in Fig. 2(a). Figures 10(b) and (c) show snapshots of the hexapod robot controlled by the presented CPG model while performing the tripod gait and the wave gait. The laboratory experiments verified that the hexapod robot can perform the gait transition between the wave gait, the tripod gait, and their transitions.

5. Discussion

5.1 Oscillation periods

In the case of the synchronously coupled CA phase oscillators, e.g., $T_i = T$ for all i , the oscillation period of each oscillator can be easily delivered. Assume

$$\begin{aligned} \Phi(0) \in \Phi^* &\equiv \{(\Phi_1, \dots, \Phi_n) \mid \Phi_i = \Phi_{i+1} + \lfloor N\phi/2\pi \rfloor \text{ for all } i\}, \\ P(0) \in P^* &\equiv \{(P_1, \dots, P_n) \mid P_i = P_j \text{ for all } i \text{ and } j\}, \end{aligned} \quad (26)$$

where $\Phi(t) \equiv (\Phi_1(t), \dots, \Phi_n(t))$ and $P(t) \equiv (P_1(t), \dots, P_n(t))$. This assumption means that the CA phase oscillators are synchronized to a target pattern determined by the parameter ϕ . Under the assumption, from Eq. (8), return values of the function h are always $h(\Delta\Phi_i^+) = 0$ for all i . Then, from Eq. (6), return values of the coupling function H are always $H(\Delta\Phi_i^+) = M - 1$ for all i . Therefore, the oscillation period of each phase variable Φ_i is obtained by

$$\text{Oscillation period} = TMN. \quad (27)$$

In the case of the asynchronously coupled CA phase oscillators, the oscillation period may change from TMN depending on the clock periods T_6 , where $T_i = T$ for $1 \leq i \leq 5$. Figure 11 shows average periods of oscillators for the case of the asynchronously coupled CA phase oscillators. As shown in the figures (a) and (b), the average periods of oscillators are almost the same for the tripod gait and the wave gait.

5.2 Appropriate parameters of N and M for practical use

By choosing a desired oscillation period and a clock period T , the parameters N and M can be determined as $MN = \text{Oscillation period}/T$. Figure 12 shows rates at which the orbits of the oscillators

Fig. 11. Oscillation periods of asynchronously coupled CA phase oscillators in steady states. (a) Tripod gait ($\phi = \pi$). (b) Wave gait ($\phi_i = \pi/3$). The other parameter values are the same as those chosen in Fig. 7, where $T_i = T = 4.347 \times 10^{-4}$ for $1 \leq i \leq 5$.

Fig. 12. Rates at which orbits of oscillators starting from randomly chosen initial values synchronize to tripod gait pattern for N . Total number of trials = 1000 and $M = \text{Oscillation period}/TN$, where the oscillation period and T are chosen as 1.0015488 and 4.347×10^{-4} , respectively. The other parameter values are the same as those chosen in Fig. 6 and Fig. 7.

starting from randomly chosen initial values synchronize to the tripod gait pattern for N , where

$$p = \frac{\text{Number of trials synchronized to tripod gait pattern}}{\text{Total number of trials}}, \quad (28)$$

and N and M are assumed to be integers satisfying Eq. (27). A large value of the rate p correspond to a large attraction basin of the target gait pattern. Therefore, for practical use, the values of the parameters N and M should be chosen such that $p \simeq 1$ (e.g., $N = 36$ and $M = 64$ as shown in Fig. 12).

5.3 Realizable phase locked patterns

Further, under the assumption that $\mathbf{P} \in \mathbf{P}^*$, realizable phase locked patterns in the synchronously coupled CA phase oscillators ($T_i = T_j$ for all i and j) can be analyzed in a similar way in [28]. From Eq. (8), the phases Φ_i are locked if $\mathbf{P} \in \mathbf{P}^*$ and

$$\sin(2\pi\Delta\Phi_i^+/N) = \sin(2\pi\Delta\Phi_{i+1}^+/N) \quad \text{for all } i, \quad (29)$$

where $\phi = 0$. Since $\Delta\Phi_i^+$ is defined on $\{0, \dots, N-1\}$, Eq. (29) can be written in the form

$$(\Delta\Phi_i^+ - \Delta\Phi_{i+1}^+)(\Delta\Phi_i^+ - N/2 + \Delta\Phi_{i+1}^+) = 0 \quad \text{for all } i. \quad (30)$$

Every combination of $\Delta\Phi_i^+$ and $\Delta\Phi_{i+1}^+$ that makes either the first or the second factor equal to zero corresponds to a realizable phase locked pattern. If $\Delta\Phi_i^+$ in Eq. (30) is assumed to be $\lambda \in \mathbf{Z}_N^+$, then $\Delta\Phi_{i+1}^+$ in Eq. (30) should be λ or $N/2 - \lambda$. On the other hand, for all the phase differences $\Delta\Phi_i^+$, the following equation always holds.

$$\sum_i^N \Delta\Phi_i^+ = Nk, \quad (31)$$

where $k \in \{0, \dots, n-1\}$. Therefore, we can take $m\lambda$ and $(n-m)(N/2 - \lambda)$ such that Eq. (31) is satisfied as follows.

$$m\lambda + (n-m)(N/2 - \lambda) = Nk, \quad (32)$$

where $m \leq n$. It follows that every vector $\Delta\Phi^+ \equiv (\Delta\Phi_1^+, \dots, \Delta\Phi_n^+)$ that is a permutation of the following vector corresponds to a realizable phase locked pattern.

$$\Delta\Phi^+ = (\lambda_1, \dots, \lambda_m, N/2 - \lambda_{m+1}, \dots, N/2 - \lambda_n), \quad (33)$$

where $\lambda_i = \lambda_j$ for all i and j . A typical realizable phase locked pattern, described by the following vector, is called an *in-phase synchronized solution*.

$$\Delta\Phi^+ = (0, \dots, 0), \quad (34)$$

Fig. 13. (a1) In-phase synchronized solution. (b1) Elementary solution. (c1) and (d1) Traveling wave solution. $\phi = 0$. (a2) Configuration of (a1) transformed by Eq. (41). (b2) Configuration of (b1) transformed by Eq. (41). (c2), (d2) Configurations of (c2) and (d2) transformed by Eq. (41).

where an example configuration of the CA phase oscillators for the in-phase synchronized solution is shown in Fig. 13(a1). If $N = \text{even number}$, then the following phase locked pattern called an *elementary solution* [28] is realizable.

$$\Delta\Phi^+ = (\underbrace{0, \dots, 0}_m, \underbrace{N/2, \dots, N/2}_{n-m}), \quad (35)$$

where $m \neq n$, and an example configuration of the CA phase oscillators for the elementary solution is shown in Fig. 13(b1). Note that the every vector $\Delta\Phi^+$ that is a permutation of the vector Eq. (35) is the elementary solution. Further, if $N \bmod n = 0$, then the following phase locked pattern called a *traveling wave solution* [28] is realizable.

$$\Delta\Phi^+ = (Nk/n, \dots, Nk/n), \quad (36)$$

where $k \in \{1, \dots, n-1\}$, and example configurations of the phase CA oscillators for the elementary solution are shown in Figs. 13(c1) and (d1).

However, as demonstrated in Section 3.2, the synchronously coupled CA phase oscillators may not oscillate properly even if they are synchronized. For example, when $\Delta\Phi^+$ satisfies

$$\Delta\Phi^+ = (\lambda_-, \dots, \lambda_-) \text{ such that } H(\lambda_-) < 0, \quad (37)$$

the synchronously coupled CA phase oscillators oscillate reversely in the phase locked pattern, where $\mathbf{P} \in \mathbf{P}^*$. In this case, the oscillation period is determined by $T(1 + H(\lambda_-))N$. Figure 14(a) shows an example of the reverse oscillation for the synchronously coupled CA phase oscillators, where $\Delta\Phi^+ = (24, \dots, 24)$, $T = 4.347 \times 10^{-4}$, $H(24) = -8$, and $N = 36$, and thus the oscillation period is 0.1408428. On the other hand, Fig. 14(b) shows the in-phase synchronized solution for the asynchronously coupled CA phase oscillators, where the initial values are the same as those chosen in Fig. 14(a). As shown in this figure, the asynchronously coupled CA phase oscillators do not have the traveling wave solution $\Delta\Phi^+ = (24, \dots, 24)$. Our extensive analyses have revealed that the asynchronously coupled CA phase oscillators rarely have the reverse oscillation. Analysis of detailed occurrence mechanism of the reverse oscillation is an important future problem.

5.4 Stability against perturbations

In order to quantify the in-phase synchronized solution, the following instantaneous evaluation function is introduced.

Fig. 14. Steady-state solutions for synchronously and asynchronously coupled CA phase oscillators. The initial values of both the oscillators are $\Phi(0) = (24, 12, 0, 24, 12, 0)$, $P(0) = (0, \dots, 0)$. (a) Traveling wave solution with reverse oscillation for synchronously coupled CA phase oscillators. $\phi = 0$. The other parameter values are the same as those chosen in Fig. 6. (b) In-phase synchronized solution for asynchronously coupled CA phase oscillators. $\phi_i = 0$. The other parameter values are the same as those chosen in Fig. 7.

$$r_{\text{inphase}}(t) \equiv \frac{1}{n} \left| \sum_{i=1}^n e^{j(2\pi \frac{\Phi_i(t)}{N})} \right|, \quad r_{\text{inphase}} \in [0, 1]. \quad (38)$$

The instantaneous evaluation function $r_{\text{inphase}} = 1$ means the coupled CA oscillators in the in-phase synchronized solution shown in Fig. 13(a1). Figure 15(a) shows the instantaneous evaluation functions r_{inphase} for the synchronously and asynchronously coupled CA phase oscillators for

$$\phi = 0, \quad (39)$$

where the initial values are set on the in-phase synchronized solution, the elementary solution, or the traveling wave solution shown in Figs. 13(a1)–(d1). As shown in this figure, for the asynchronously coupled CA phase oscillators, the in-phase synchronized solution is dominant.

Let us consider the equivalence of the dynamics of the coupled CA phase oscillators for the in-phase synchronized solution and the tripod gait pattern. Note that the coupling function defined by Eq. (6) is basically characterized by Eqs. (8) and (9). Equation (9) for the in-phase synchronized solution (see also Eq. (39)) is as follows.

$$\Delta\Phi_i^+|_{\phi=0} = \Phi_{i+1}(t) - \Phi_i(t) \mod N. \quad (40)$$

Let us introduce the following transformation of the phase variable (see also Figs. 13(a2)–(d2)).

$$\Phi'_i \equiv \begin{cases} \Phi_i & \text{for } i = 1, 3, 5, \\ \Phi_i - \lfloor N/2 \rfloor \mod N & \text{for } i = 2, 4, 6. \end{cases} \quad (41)$$

Equation (9) for the tripod gait pattern (see also Eq. (17)) can be transformed by Eq. (41) into

$$\Delta\Phi_i^+|_{\phi=\pi} = \Phi'_{i+1}(t) - \Phi'_i(t) \mod N. \quad (42)$$

Therefore, the tripod gait pattern can be considered as the special case of the in-phase synchronized solution via the change of variable. In fact, Fig. 15(a) and (b) shows examples of this equivalence.

Fig. 15. (a) Instantaneous evaluation functions r_{inphase} for synchronously and asynchronously coupled CA phase oscillators. The initial values are set on the in-phase synchronized solution, the elementary solution, or the traveling wave solution, where (a1)–(d1) in this figure correspond to (a1)–(d1) in Fig. 13 as follows: $\Phi(0) = (0, 0, 0, 0, 0, 0)$, $(0, 0, 0, 0, 0, 18)$, $(0, 0, 0, 0, 18, 18)$, $(0, 0, 0, 18, 18, 18)$, $(0, 0, 18, 18, 18, 18)$, $(0, 0, 18, 0, 0, 18)$, $(0, 0, 18, 18, 0, 18)$, $(0, 18, 18, 18, 18, 18)$, $(0, 6, 12, 18, 24, 30)$, $(24, 12, 0, 24, 12, 0)$, and $P(0) \in P^*$. $\phi = 0$ and the other parameter values are the same as those chosen in Figs. 6 and 7. (b) Instantaneous evaluation functions r_{tripod} for synchronously and asynchronously coupled CA phase oscillators. The initial values $\Phi(0)$ in (a) transformed by Eq. (41) and the resulting values $(0, 18, 0, 18, 0, 18)$, $(0, 18, 0, 18, 0, 0)$, $(0, 18, 0, 18, 18, 0)$, $(0, 18, 0, 0, 18, 0)$, $(0, 18, 18, 0, 18, 0)$, $(0, 18, 18, 18, 0, 0)$, $(0, 18, 18, 0, 0, 0)$, $(0, 0, 18, 0, 18, 0)$, $(0, 24, 12, 0, 24, 12)$, $(24, 30, 0, 6, 12, 18)$ are used as the initial values. $P(0) \in P^*$. $\phi = \pi$ and the other parameter values are the same as those chosen in Figs. 6 and 7.

Fig. 16. Instantaneous evaluation functions r_{inphase} for synchronously and asynchronously coupled CA phase oscillators with random small perturbations and large perturbations to the phases Φ_i . (a) Synchronous coupling. $\phi = 0$ and the other parameter values are the same as those chosen in Fig. 6. (b) Asynchronous coupling. $\phi = 0$ and the other parameter values are the same as those chosen in Fig. 7.

Figure 15(b) shows the instantaneous evaluation functions r_{tripod} defined by Eq. (19) for the synchronously and asynchronously coupled CA phase oscillators for $\phi = \pi$, where the initial values in Fig. 15(a) are transformed by Eq. (41) for the initial values in Fig. 15(b). As shown in these figures, the coupled CA phase oscillators has the same structure of attraction basins.

Further, Fig. 16 shows the instantaneous evaluation functions r_{inphase} for the synchronously and asynchronously coupled CA phase oscillators with random perturbations. As shown in Fig. 16(a), for the synchronously coupled CA phase oscillators, the in-phase synchronized solution has stability against small perturbations. On the other hand, as shown in Fig. 16(b), for the asynchronously coupled CA phase oscillators, the in-phase synchronized solution has stability against small and large perturbations.

5.5 Future development of Implementation method for the coupling functions

In the implementation of the presented CPG model in Section 4, the coupling function H is directly implemented in LUTs. For further reduction of hardware resources, the COordinate Rotational Digital Computer (CORDIC) algorithm [29] might be applied to implement the coupling function. In addition, if the coupling function H is simplified to three-valued function as proposed in [9], the presented

CPG model is considered to be implemented by fewer hardware resources. However, in this case, careful consideration should be given to synchronous speed for practical use.

6. Conclusions

In this paper, the CPG model based on the asynchronous coupling of CA phase oscillators for the hexapod robot was presented. Analyses using the evaluation functions for the target gait patterns of the hexapod robot clarified that the asynchronously coupled CA phase oscillators are more suitable to perform gait transitions than the synchronously coupled CA phase oscillators. For example, as described in Remarks 1 and 2, the asynchronously coupled CA phase oscillators mostly realize the proper gaits and their transitions while the synchronously coupled CA phase oscillators often fail to realize them. Then, the presented CPG model was implemented in the FPGA. It was verified that the hexapod robot mounted with the FPGA can perform the smooth gait transitions between target gait patterns. Our future work includes: (i) Systematic analyses (e.g., group theoretic approach) of various phase locked patterns [30, 31] and (ii) Investigation of how the pattern transitions will be affected by incorporating external perturbations into the proposed CPG model such as friction and floor reaction forces [32].

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