Paper

The double-assignment method for the exponential chaotic tabu search in quadratic assignment problems

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Abstract: The quadratic assignment problem (QAP) is one of the $\mathcal{NP}$ -hard combinatorial optimization problems. An exponential chaotic tabu search using a 2-opt algorithm driven by chaotic neuro-dynamics has been proposed as one heuristic method for solving QAPs. In this paper we first propose a new local search, the double-assignment method, suitable for the exponential chaotic tabu search, which adopts features of the Lin-Kernighan algorithm. We then introduce chaotic neuro-dynamics into the double-assignment method to propose a novel exponential chaotic tabu search. We further improve the proposed exponential chaotic tabu search with the double-assignment method by enhancing the effect of chaotic neuro-dynamics.

Key Words: double-assignment method, exponential chaotic tabu search, chaotic neural network, quadratic assignment problem, Lin-Kernighan algorithm

1. Introduction

A great deal of effort has been made to solve combinatorial optimization problems, in particular, in the area of operations research and meta-heuristics [1]. Through those researches, various effective search algorithms to obtain approximate solutions were proposed such as the 2-opt algorithm [2], k -opt algorithm [3], Or-opt algorithm [4], Lin-Kernighan (LK) algorithm [5], and stem-and-cycle ejection chain algorithm [6]. In addition, a variety of meta-heuristics, e.g. simulated annealing [7], genetic algorithm, genetic hybrids [8], tabu search [9], reactive tabu search [10], robust tabu search [11] and so on, were proposed to avoid the local minimum problem.

The quadratic assignment problem (QAP) is one well-known $\mathcal{NP}$ -hard combinatorial optimization problem [12–16]. For QAPs, many local searches and meta-heuristics have been proposed and studied [1, 17]. In particular, tabu search and its derivatives with the 2-opt exchanges have been used as

effective meta-heuristic algorithms [18].

On the other hand, since the dynamical memory search was demonstrated in a dynamical associative memory with a chaotic neural network model [19–21], chaotic dynamics have been applied to combinatorial optimization problems. In the early stage of these approaches, chaotic dynamics were used as a noise source [22, 23]. In recent studies, however, chaotic dynamics are more actively used in the following two ways: (a) Chaotic dynamics are introduced into a mutually connected neural network, and (b) a local search is controlled by chaotic neuro-dynamics. In particular, the approach (b), such as an exponential chaotic tabu search, has been shown to be very effective and practical in several combinatorial optimization problems.

The exponential chaotic tabu search, in which the 2-opt exchanges are controlled by chaotic neuro-dynamics, was first applied to the traveling salesman problem (TSP) [24]. The exponential chaotic tabu search, then, has been improved by combining chaotic neuro-dynamics with the Or-opt algorithm, k -opt algorithm, LK algorithm, and stem-and-cycle ejection chain algorithm. As a result, TSPs with the size of 10^5 to 10^7 can be efficiently solved through the chaotic search [25–30]. In addition to TSPs, the exponential chaotic tabu search with chaotic neuro-dynamics was also applied to QAPs [31, 32], motif extraction problems [33], vehicle routing problems with time window [34], and packet routing problems [35]; furthermore, effectiveness of the exponential chaotic tabu searches was demonstrated through these problems.

Salient features of the exponential chaotic tabu search are an exponential decaying continuous tabu (exponential tabu) effect, and an efficient dynamical search in the state-space through chaotic itinerancy [36]. The exponential tabu is naturally introduced in the exponential chaotic tabu search dynamics by the refractoriness of the chaotic neuron [19, 21, 31, 32]. The tabu intensity in the exponential chaotic tabu search takes continuous value, and decreases exponentially by the refractoriness of the chaotic neuron [31, 32]. Moreover, the intensity and decay time-constant of the tabu effect can be easily controlled by the refractory parameters of the chaotic neurons [19, 21].

On the other hand, chaotic itinerancy refers to universal dynamics in high-dimensional dynamical systems, showing itinerant motion among varieties of low-dimensional ordered states (quasi-attractors or attractor ruins) through high-dimensional chaos [36]. The chaotic attractors usually have fractal structures. The chaotic search dynamics search the solutions along such chaotic attractors with zero Lebesgue measure in the state space, so that the chaotic search is expected to be efficient. Moreover, the itinerant motion near the attractor ruins would make the local search deep, and at the same time, help the local search to escape from local minima.

Complexity, richness, and computational power inherent in chaotic dynamics are due to the complexity of real-numbers [37–39]. Therefore, it is mandatory to implement a chaotic dynamical system with an analog circuitry, which can handle real-numbers through continuous variables [40, 41]. In addition, an analog circuit system can achieve truly and massively parallel processing as its nature. In this respect, we have constructed a general purpose chaotic neuro-computer with a switched-capacitor analog sampled-data integrated-circuit technique [40, 41]. Moreover, we have implemented an exponential chaotic tabu search based on the 2-opt exchanges into a hardware system by using switched-current analog integrated circuits [41–43]. By means of experiments, we have demonstrated high-abilities of these hardware systems in solving QAPs. Furthermore, we analyzed high-dimensional spatio-temporal chaotic neuro-dynamics, which are necessary and effective to the exponential chaotic tabu search, through spatio and temporal mutual information [41, 44]. In addition, a completely parallel updating scheme of the neurons has been proposed for a quick execution of the exponential chaotic tabu search with the 2-opt algorithm. The execution time of the parallel hardware systems would be independent from the size of QAPs thanks to the truly parallel nature inherent in the analog circuitry [44, 45].

As mentioned above, the exponential chaotic tabu search, in which the 2-opt algorithm is controlled by chaotic neuro-dynamics, was successfully applied to QAPs showing high solving performance. In this paper, we propose a further effective exponential chaotic tabu search for QAPs with a novel local search. To this end, we first propose in Section 3 a double-assignment (DA) method as a local search for QAPs [46]. In the DA method, features of the LK algorithm for TSPs are employed. We

then show through numerical simulations that the DA method has better performance than the 2-opt method as a local search. Secondly, in Section 4, we propose an exponential chaotic tabu search in which chaotic neuro-dynamics control the DA method. Thirdly, we modify the DA method (modified double-assignment (MDA) method) to enhance the chaotic neuro-dynamics effect. We also evaluate the MDA method as a local search. We then adopt the MDA method in the exponential chaotic tabu search. We finally compare the performance of the exponential chaotic tabu searches with the DA method, with the MDA method, and with the 2-opt exchanges.

2. Quadratic assignment problem

The quadratic assignment problem (QAP) [13, 14, 16] is one of the $\mathcal{NP}$ -hard combinatorial optimization problems. In this section, we briefly review the QAP using the problem of plant locations of size N as an example. The goal of the plant location problem is to find an assignment of plants to unique cities that minimizes the cost function in Eq. (1).

$$F(\mathbf{p}) = \sum_{i=1}^N \sum_{j=1}^N a_{ij} b_{p(i)p(j)} \quad (1)$$

where a_{ij} gives the distance between city i and city j , and $b_{p(i)p(j)}$ gives the flow (weight) from plant $p(i)$ to plant $p(j)$. The distances between each pair of cities, and the flows between each pair of plants are given by the distance matrix $\mathbf{A}$, and the flow matrix $\mathbf{B}$, respectively, as in Eq. (2).

$$\mathbf{A} = \begin{bmatrix} a_{11} & \cdots & a_{1j} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{i1} & \cdots & a_{ij} & \cdots & a_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{Nj} & \cdots & a_{NN} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & \cdots & b_{ij} & \cdots & b_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{N1} & \cdots & b_{Nj} & \cdots & b_{NN} \end{bmatrix} \quad (2)$$

Furthermore, feasible solutions can be expressed by the permutation $\mathbf{p}$ in Eq. (3) [15].

$$\begin{aligned} \text{index : } & 1 \quad 2 \quad \dots \quad i \quad \dots \quad j \quad \dots \quad N \\ \mathbf{p} : & \{ p(1), p(2), \dots, p(i), \dots, p(j), \dots, p(N) \} \end{aligned} \quad (3)$$

In Eq. (3), the index i of $\mathbf{p}$ corresponds to city i , and the element $p(i)$ corresponds to the plant that is assigned to city i .

The number of feasible solutions to a size- N QAP is $N!$, and thus it is difficult to obtain the optimum solution in a reasonable time by evaluating all feasible solutions. Therefore, some heuristic methods have been proposed to quickly obtain sub-optimum solutions [1, 8–13, 16–18]. The 2-opt exchange [47] is one of the most basic local search heuristics. It exchanges two plants if the exchange decreases $F(\mathbf{p})$. An example of the 2-opt exchange method for QAPs is illustrated in Fig. 1 for a size-5 QAP, where plants 2 and 5 are to be exchanged.

Fig. 1. An example of the 2-opt method for a size-5 QAP; (a) is the initial permutation, and (b) is the resulting permutation after the exchange of plants 2 and 5.

3. The double-assignment method

In this section, we propose a local search, double-assignment (DA) method, which can be available for the exponential chaotic tabu search. The DA method employs features of the LK algorithm; therefore, we start from a short review of the LK algorithm.

3.1 The Lin-Kernighan algorithm for TSPs

In the TSP [1, 48], which is also an $\mathcal{NP}$ -hard problem, the task is to find a shortest possible tour that visits all cities in a list exactly once. Many heuristics have been proposed for the TSP [1, 48], including the 2-opt method, the k -opt method, the Or-opt algorithm, the LK algorithm, and stem-and-cycle ejection chain algorithm [2–6].

The LK algorithm [5] is known to be one of the efficient methods for solving TSPs. It swaps links like the k -opt method, but k changes dynamically. In addition, the LK algorithm goes through infeasible solutions during the search for better tours, as shown in Fig. 2. We first choose a city t_1 at random (Fig. 2(b)). Then, we choose a city t_2 next to t_1 , and trim the path x_1 between t_1 and t_2 . Next, we add the route y_1 between t_2 and a city t_3 , such that y_1 is shorter than x_1 (Fig. 2(c)). After that, we choose a city t_4 next to t_3 (Fig. 2(d)), and trim the path x_2 between t_3 and t_4 . We then evaluate the cost improvement G^* when we temporarily add a path T' between t_1 and t_4 . We next choose a city t_5 and a route y_2 between t_4 and t_5 such that $|x_2| + |x_1| > |y_2| + |y_1|$ (Fig. 2(e)), where $|x_1|$, $|x_2|$, $|y_1|$ and $|y_2|$ are the lengths of x_1 , x_2 , y_1 and y_2 , respectively. If $(|x_2| + |x_1|) - (|y_2| + |y_1|) > G^*$, we continue to delete and add routes (Fig. 2(f)). Moreover, if $(|x_2| + |x_1|) - (|y_2| + |y_1|) \leq G^*$, or we cannot add y_2 , we halt and update the current tour to T' (Fig. 2(g)).

As shown in Fig. 2, the number of link exchanges depends on the cost improvements and the lengths of the added and removed paths. Furthermore, the tours in Figs. 2(b) to 2(e) are not closed, and so are infeasible solutions. This greatly broadens the solution search space, giving the LK algorithm good performance.

Fig. 2. An example of the Lin-Kernighan algorithm: (a) the initial tour; (b) delete the path x_1 between t_1 and t_2 ; (c) add the path y_1 between t_2 and t_3 ; (d) delete the path x_2 between t_3 and t_4 , and evaluate the temporary path T' between t_1 and t_4 ; (e) add the path y_2 between t_4 and t_5 ; (f) continue the procedure, or (g) halt.

From the above, we can conclude that the LK algorithm has the following two salient features.

(F-1) The number of exchanged links changes dynamically.

(F-2) Feasible solutions are searched for through infeasible solutions.

In this respect, in this paper we propose the DA method which can be used in the exponential chaotic tabu search for QAPs, introducing feature **(F-2)** of the LK algorithm [46]. Namely, the DA method searches for feasible solutions through infeasible solutions.

3.2 The double-assignment method for QAPs

The DA method first constructs an infeasible solution in which some cities have no plants assigned, and some cities have two plants. A new feasible solution is then constructed by reassigning over-assigned plants to empty cities. In the following, we illustrate the DA procedure using an example where plant c is initially doubly assigned to city d . In **Step 1** and **Step 2** below, we first deliberately construct an infeasible solution, and from that construct a feasible solution in **Step 3**.

Step 0: Initialize the counters m and n as $m = 0$ and $n = 0$, where the parameter m is a count of the number of runs of **Step 1** and **Step 2**, and n is a count of the runs of **Step 3**.

Step 1: Increment m by one (now, $m = 1$). Arbitrarily choose a plant c and a city d from the list of plants and cities, respectively, and doubly assign c to d . Denote city d as x_1 , and the city to which plant c was assigned as y_1 .

Step 2: Increment m by one ($m = m + 1$). Randomly choose a city y_m from among the cities that have only one plant. Doubly assign the plant in city y_m to city x_m for which the assignment results in the minimum cost increase. Note that the list of cities from which x_m is chosen excludes the cities x_1 to x_{m-1} and y_1 to y_m . If $m + 1 \leq N/2$, repeat **Step 2**, otherwise proceed to **Step 3**.

Step 3: Increment n by one ($n = n + 1$). Choose from among those plants assigned to cities x_1 to x_m the plant that gives the minimum cost increase when it is assigned to city y_n , and assign it to y_n . Relabel the city where the chosen plant resided as z_n . If $n = m$, the algorithm ends. Otherwise, repeat **Step 3**.

A flow diagram of the DA method is shown in Fig. 3 for the case of a size-5 QAP. Figure 3(a) shows **Step 1** in the case when “city d ” = “city II,” and “plant c ” = “plant 1.” There, we first doubly assign plant 1 to city II. We then denote city II as x_1 , and city III as y_1 . Figure 3(b) shows **Step 2** when city V is chosen as y_2 . In this step, we first calculate the cost increase when plant 4, which was assigned to city V, is temporarily assigned to cities I and IV, respectively, each of which has only one plant. If the assignment of plant 4 to city I gives the smallest cost increase, we doubly assign the plant 4 to city I. The first procedure of **Step 3** is shown in Fig. 3(c). In this step, we evaluate the increases in the cost when plants 1, 2, 3, and 4 are each assigned to city III. If the assignment of plant 2 to city III results in the smallest cost increase, we reassign plant 2 to city III as a consequence. As a result, z_1 = “city II.” Figure 3(d) shows the successive procedure of **Step 3**. We again evaluate the increases in the cost when the remaining plants 3 and 4 are assigned to city V, respectively. If when plant 4 is assigned to city V the cost increase is smaller than the case where plant 3 was assigned, we reassign plant 4 to city V. As demonstrated in Fig. 3, the DA method constructs a new feasible solution through infeasible solutions.

The total cost improvement by one execution of double assignment can be evaluated as the sum of the cost changes after each step. Moreover, in **Step 2** and **Step 3**, we need to calculate the increase in cost when we choose city x_m , where the plant of city y_m is assigned, and the plant that is reassigned to city y_n . Equations (4) and (5) are used to evaluate the cost increases in **Step 1** and **Step 2**, respectively.

Fig. 3. An example of the double-assignment method: (a) **Step 1** when plant 1 is chosen as c , and city II is chosen as d ; (b) **Step 2** when city V is chosen as y_2 ; (c) **Step 3** when plant 2 is chosen for reassignment to city y_1 ; and (d) **Step 3** when plant 4 is chosen for reassignment to city y_2 .

$$\begin{aligned} \Delta F_{ij}^1 &= -(a_{ij}b_{p(i)p(j)} + a_{ji}b_{p(j)p(i)}) + \sum_{k=1, k \neq i, j}^N (b_{p_1(i)p_1(k)})(a_{kj} - a_{ki}) \\ &+ \sum_{k=1, k \neq i, j}^N (b_{p_1(k)p_1(i)})(a_{jk} - a_{ik}) \end{aligned} \quad (4)$$

$$\begin{aligned} \Delta F_{ij}^2 &= -(a_{ij}b_{p(i)p(j)} + a_{ji}b_{p(j)p(i)}) + \sum_{k=1, k \neq i, j}^N (b_{p_1(i)p_1(k)} + b_{p_1(i)p_2(k)})(a_{kj} - a_{ki}) \\ &+ \sum_{k=1, k \neq i, j}^N (b_{p_1(k)p_1(i)} + b_{p_2(k)p_1(i)})(a_{jk} - a_{ik}) \end{aligned} \quad (5)$$

where ΔF_{ij}^1 is the cost increase when the plant in city i is doubly assigned to city j in **Step 1**, and ΔF_{ij}^2 is that in **Step 2**. Moreover, $p(i)$ and $p(j)$ are the plants assigned to city i and city j , respectively. In addition, $p_1(i)$ is the plant originally assigned to city i , and $p_2(i)$ is the plant newly assigned to city i . On the other hand, the cost increase in **Step 3** is given by Eq. (6).

$$\begin{aligned} \Delta F_{ij}^3 &= a_{lj}b_{ip(l)} + a_{jl}b_{p(l)i} + \sum_{k=1, k \neq i, j}^N (b_{ip_1(k)} + b_{ip_2(k)})(a_{kj} - a_{kl}) \\ &+ \sum_{k=1, k \neq i, j}^N (b_{p_1(k)i} + b_{p_2(k)i})(a_{jk} - a_{lk}) \end{aligned} \quad (6)$$

where ΔF_{ij}^3 represents the change in cost when plant i is reassigned to city j in **Step 3**. Moreover, l is the city to which plant i was originally assigned, and $p(j)$ and $p(l)$ are the plants assigned to city j and city l , respectively.

3.3 Comparison of the 2-opt and double-assignment methods

In this section, we compare the solving performance of the DA method and the 2-opt method as local searches through numerical simulations to confirm the effectiveness of feature **(F-2)** of the LK algorithm for solving QAPs. Because these methods are local searches, they would finally fall into a local minimum. Therefore, comparisons are made for the same iteration numbers defined below for each method.

In simulations of the 2-opt method, the following procedure (**2opt-1** and **-2**) is treated as a single iteration:

2opt-1: Examine the results of all possible 2-opt exchanges to the current solution, π .

2opt-2: If π_{best} is better than π , replace π with π_{best} .

On the other hand, in the simulations for the DA method the following procedure (**DA-1**, **2**, **3** and **4**) is treated as a single iteration.

DA-1: Initialize each of the counters i and j to 1.

DA-2: Temporarily execute the DA method, which initially assigns plant i to city j in **Step 1**, to the current solution π .

DA-3: If $j < N$, increment j by one, and return to **DA-2**. If $j = N$ and $i < N$, increment i by one. At the same time, reset j to $j = 1$, and go back to **DA-2**. If $j = N$ and $i = N$, go to **DA-4**.

DA-4: Take the best solution π_{best} from all the solutions obtained from **DA-2** and **DA-3**. If π_{best} is better than π , we replace π by π_{best} .

In the numerical simulations, we used 5,000 iterations for each trial. We executed 30 trials with different initial conditions for each method. Table I summarizes the average gap from the optimum or known-best solution (AG), and the success rate in obtaining the optimum or known-best solution (SR), for local searches employed in the exponential chaotic tabu searches, using benchmark problems from the QAP library [16]. From the left two columns (2-opt and DA) of Table I, it is clear that the DA method results in better solutions than the 2-opt method. This result confirms that solution searches through infeasible solutions, feature **(F-2)** of the LK algorithm, are effective in solving QAPs.

4. The exponential chaotic tabu search with the double-assignment method

Although the DA method has better solving performance than the 2-opt method, like the 2-opt method it can be trapped in local minima. To solve the local minima problem, some meta-heuristic methods such as the simulated annealing [7], genetic algorithm, genetic hybrids [8], tabu search [9], reactive tabu search [10], and robust tabu search [11] have been proposed for QAPs. In addition, chaotic dynamics have been effectively utilized. In particular, the exponential chaotic tabu search for QAPs, in which the 2-opt exchanges are controlled by chaotic neuro-dynamics, is one of the effective searches which exploit high-dimensional chaotic dynamics [31, 32]. In the exponential chaotic tabu search, the tabu effects exponentially decay due to the inherent refractoriness of the chaotic neurons.

The DA method is employed in the exponential chaotic tabu search in place of the 2-opt method as a local search in this section. We first briefly review the original exponential chaotic tabu search with the 2-opt method; then the chaotic neuro-dynamics is introduced into the DA method.

4.1 The original exponential chaotic tabu search with the 2-opt method

Candidate exchanges in the 2-opt method are determined by chaotic neuro-dynamics from a chaotic neural network in the exponential chaotic tabu search [31, 32]. In the exponential chaotic tabu search, chaotic neurons in the neural network are asynchronously updated. If the (i, j) th neuron fires, we assign plant i to city j , and at the same time we assign plant $p(j)$ in city j to city $q(i)$, where plant i

Table I. Comparisons among local searches used in the exponential chaotic tabu searches for QAPs. The average gap from the optimum or known-best solution (AG), and the success rate in finding the optimum or known-best solution (SR) through 30 numerical simulation trials with 5,000 iterations for each trial, using the 2-opt method (2-opt), the double-assignment method (DA), and the modified double-assignment method (MDA). The best result and second best for each problem are shown with bold and italic numbers, respectively.

Instance	2-opt		DA		MDA	
	AG (%)	SR (%)	AG (%)	SR (%)	AG (%)	SR (%)
Chr20a	45.852	0	8.750	0	<i>10.53</i>	0
Chr20b	43.394	0	6.545	3.33	<i>7.008</i>	0
Chr20c	78.226	0	9.263	6.66	<i>18.13</i>	<i>3.33</i>
Had20	1.174	3.33	<i>0.499</i>	<i>3.33</i>	0.175	53.3
Nug20	3.709	0	1.089	<i>3.33</i>	<i>1.344</i>	6.66
Nug30	2.310	0	<i>1.595</i>	0	0.894	0
Rou20	5.058	0	1.426	0	<i>1.467</i>	0
Scr20	8.684	0	1.609	<i>6.66</i>	<i>1.654</i>	10.0
Sko42	3.560	0	<i>3.125</i>	0	1.205	0
Tai20a	6.079	0	<i>1.772</i>	0	1.757	0
Tai30a	3.499	0	<i>2.624</i>	0	2.203	0
Tai40a	5.087	0	<i>4.921</i>	0	2.539	0
Tai20b	16.560	0	<i>0.325</i>	<i>40.0</i>	0.317	50.0
Tai30b	16.518	0	<i>1.243</i>	0	1.802	13.3
Tai40b	5.087	0	3.826	0	<i>4.384</i>	10.0

was assigned. When the (i, j) th neuron fires, the (i, j) th and $(p(j), q(i))$ th neurons will not fire for a while, as determined by the refractoriness of the (i, j) th and $(p(j), q(i))$ th neurons, which naturally implement exponentially decaying tabu effects.

Figure 4 illustrates the exponential chaotic tabu search, in which the 2-opt exchange is controlled by the firing of chaotic neurons [31, 32]. The figure also demonstrates the exponential tabu effect through the refractoriness of the chaotic neuron model [19, 21]. In Fig. 4, the $(2, 5)$ th neuron fires, so plant 2 is assigned to city V by a 2-opt exchange. In addition, the $(2, 5)$ th neuron and the $(4, 2)$ th neuron will not fire for a while because of the exponentially decaying refractory effects.

The performance of the exponential chaotic tabu search with the 2-opt exchanges, such as the average gap from the known-best solution and the CPU-time, has been carefully and extensively compared with that of other meta-heuristic approaches, such as the stochastic simulated annealing, the robust tabu search and chaotic simulated annealing, using the instances from the QAP library in [32]. As a result, they showed that the exponential chaotic tabu search with the 2-opt exchanges is comparable to or better than the robust tabu search in terms of quality of solutions and speed. Moreover, we have confirmed their results by repeating their simulations. Therefore, we employ this original exponential chaotic tabu search with the 2-opt method as an evaluation standard for the proposed exponential chaotic tabu searches in the following simulations.

4.2 Introducing the exponential chaotic tabu search into the double-assignment method

As in the original exponential chaotic tabu search [31, 32], we use a chaotic neural network with $N \times N$ chaotic neurons for a size- N QAP in order to control the DA method. The dynamics of the (i, j) th chaotic neuron in the network are given by

$$\zeta_{ij}(t+1) = k_r \zeta_{ij}(t) - \alpha x_{ij}(t) + R \quad (7)$$

$$\xi_{ij}(t+1) = \beta(F_1^{\mathbf{P}}(t) - F_{ij}^{\mathbf{P}}(t)) \quad (8)$$

$$x_{ij}(t+1) = f(\zeta_{ij}(t+1) + \xi_{ij}(t+1)) \quad (9)$$

Fig. 4. The 2-opt method driven by chaotic neuro-dynamics for a size-5 QAP: (a) the (2, 5)th neuron is updated, and fires; (b) the 2-opt exchange assigns plant 2 to city V; (c) refractory effects on the (2, 5)th neuron and (4, 2)th neuron; (d) the 2-opt exchanges that assign plant 2 to city V and plant 4 to city II are prohibited for a while because of the refractoriness.

where $\zeta_{ij}(t+1)$ corresponds to the tabu effect for the assignment of plant i to city j , and $\xi_{ij}(t+1)$ gives the gain resulting from the assignment. k_r is a decay parameter of the refractory effect ($0 < k_r < 1$), α is a scaling parameter of the refractory effect ($\alpha > 0$), R is an external bias, and $f(\cdot)$ is a sigmoidal output function of the neuron. In addition, $F_1^P(t)$ represents the current cost at time t , and $F_{ij}^P(t)$ is the cost after the DA method which initially doubly assigns plant i to city j in **Step 1**. When we asynchronously update the (i, j) th neuron, we first calculate $\zeta_{ij}(t+1)$ and $\xi_{ij}(t+1)$ by Eqs. (7) and (8), respectively. In order to obtain $\xi_{ij}(t+1)$, we temporarily execute the DA method, which initially assigns plant i to city j in **Step 1**, to evaluate $F_{ij}^P(t)$. We denote the resulting temporal solution as $\mathbf{p}_{temp}$. Then, if the (i, j) th neuron fires, that is, $x_{ij}(t+1) > \frac{1}{2}$, we accept $\mathbf{p}_{temp}$ as a new solution, and proceed to update the next neuron. Otherwise, we discard $\mathbf{p}_{temp}$, and proceed to the next neuron.

We used numerical simulations to compare the performance of the exponential chaotic tabu search with the DA method with the original exponential chaotic tabu search, which is the evaluation standard in this paper as mentioned at the end of Section 4.1. In the simulations, we executed 30 trials with each method, and the number of exchanges in the permutation is fixed at $100n$ in each trial for fair comparisons [32], where n is the size of the problem. The left two columns, CS-2opt and CS-DA, of Table II summarize the simulation results for the exponential chaotic tabu searches with the 2-opt method and with the DA method, respectively. The results were obtained with Intel Core2 Quad CPU Q9650 @ 3.00 GHz with 3.6 GB of memory. The OS was Linux 2.6.27-67vl5, and the compiler was gcc++ with -O3 option. As shown in Table II, the exponential chaotic tabu search with the DA method resulted in better average gaps from the optimum or known-best solution than that with the 2-opt method, in 9 instances out of 15. In contrast, however, the exponential chaotic tabu search with the DA method consumed much longer CPU-time than that with the 2-opt method for all instances.

Table II. Comparisons among exponential chaotic tabu searches. The average gap from the optimum or known-best solution (AG (%)), the success rate in finding the optimum or known-best solution (SR (%)), and the CPU-time (CPU (sec.)) through 30 numerical simulation trials using the original exponential chaotic tabu search with the 2-opt method (CS-2opt), the exponential chaotic tabu search with the DA method (CS-DA), and the exponential chaotic tabu search with the MDA method (CS-MDA). Each run is cut at $100n$ exchanges of the permutation, where n is the size of the problem. The best result and second best for each problem are shown with bold and italic numbers, respectively.

Instance	CS-2opt			CS-DA			CS-MDA		
	AG	SR	CPU	AG	SR	CPU	AG	SR	CPU
Chr20a	14.91	0	<i>0.571</i>	<i>13.37</i>	0	3.000	11.22	3.33	0.138
Chr20b	14.32	0	<i>0.584</i>	<i>9.460</i>	0	2.501	8.631	0	0.124
Chr20c	20.73	0	<i>0.576</i>	7.757	0	2.277	<i>19.30</i>	0	0.118
Nug20	<i>0.566</i>	<i>13.3</i>	<i>0.655</i>	1.639	0	5.128	0.451	26.7	0.303
Nug30	<i>0.932</i>	0	1.093	3.091	0	33.45	0.499	3.33	<i>12.84</i>
Had20	<i>0.251</i>	0	<i>0.503</i>	0.679	0	1.624	0.234	13.3	0.151
Rou20	<i>2.358</i>	0	<i>1.575</i>	3.356	0	1.683	1.203	3.33	0.200
Scr20	6.941	0	<i>0.421</i>	<i>4.819</i>	0	2.715	0.987	26.7	0.148
Sko42	<i>1.971</i>	0	6.577	4.421	0	62.45	0.514	0	<i>19.33</i>
Tai20a	3.345	0	<i>1.629</i>	<i>2.303</i>	0	7.339	2.091	0	0.278
Tai30a	4.584	0	0.102	<i>4.399</i>	0	55.21	1.552	0	<i>27.70</i>
Tai40a	<i>2.364</i>	0	12.38	5.478	0	233.3	1.951	0	<i>44.37</i>
Tai20b	5.644	0	<i>0.391</i>	2.042	0	1.087	<i>3.133</i>	0	0.121
Tai30b	4.360	0	0.526	2.188	0	6.657	<i>2.622</i>	0	<i>1.155</i>
Tai40b	5.135	0	1.178	<i>2.702</i>	0	18.39	2.561	0	<i>3.334</i>

The DA method calculates three cost changes, ΔF_{ij}^1 , ΔF_{ij}^2 , and ΔF_{ij}^3 , given by Eqs. (4), (5), and (6), respectively. Moreover, the calculations for these cost changes take a large amount of time. This would be one of the reasons for the slow execution of the exponential chaotic tabu search with the DA method. In summary, although the DA method is superior to the 2-opt method as a local search as demonstrated in Table I, the introduction of chaotic neuro-dynamics into the DA method did not satisfactorily improve the performance of the DA method. Therefore, we modify the DA method in such a way that the chaotic search dynamics more effectively control the DA mechanism, in the next subsection.

4.3 The exponential chaotic search with the modified double-assignment method

In the original exponential chaotic tabu search with the 2-opt method mentioned in Section 4.1, firing of the corresponding neuron directly specifies the candidate for the 2-opt exchange; thus, chaotic neuro-dynamics completely control the 2-opt exchanges in the exponential chaotic tabu search. In contrast, in the exponential chaotic tabu search with the DA method, only the assignment in **Step 1** is determined by the firing of a chaotic neuron. Therefore, chaotic neuro-dynamics do not completely control the double assignments. This may be the reason for the poor improvements indicated by Table II. We therefore propose the exponential chaotic tabu search with a modified DA (MDA) method, which enhances the effect of **Step 1**.

To that end, we impose a maximum value for m to restrict the number of executions of **Step 2** and **Step 3** in the MDA. In practice, we set the maximum m to 5 for a size-20 QAP, and 10 for size-30 and size-42 QAPs. The performance of the DA method itself, however, may be affected by the limitation on m . We therefore first evaluate the performance of the MDA method with limited m as a local search. The results from the simulations of the MDA method are shown in the right column (MDA) of Table I. As shown in the table, the MDA method results in much better solutions than the 2-opt method for all problems, and has some advantages in the overall performance over the DA method. However, the MDA and DA methods are mostly comparable to each other as local searches.

We finally evaluate the performance of the exponential chaotic tabu search with the MDA method through numerical simulations as shown in the right column (CS-MDA) of Table II. The table shows that the exponential chaotic tabu search with the MDA method resulted in the best, or at least the second best, average gaps from the optimum or known-best solutions for all instances. Moreover, the success rates in obtaining the optimum or known-best solutions of the exponential chaotic tabu search with the MDA method gave the best results. In terms of the CPU-time, the exponential chaotic tabu search with the MDA method is comparable to that with the 2-opt method. In fact, the exponential chaotic tabu search with the MDA method was quickest in 9 instances out of 15. These results confirm that the limitation in m in the MDA not only enhances the effect of chaotic neuro-dynamics improving the quality of solutions, but also contributes to the short CPU-time. As a consequence, we can conclude that the exponential chaotic tabu search with the MDA method has the best overall performance.

5. Conclusions

We first proposed the double-assignment (DA) method for QAPs based on the LK algorithm, which finds suboptimal solutions through temporary infeasible solutions, as a local search suitable for the exponential chaotic tabu search. In addition, we have shown that feature **(F-2)** of the LK algorithm for TSPs is also effective in solving QAPs. Second, we introduced chaotic neuro-dynamics to the DA method to implement the exponential decaying tabu effect and dynamical search to realize the exponential chaotic tabu search with the DA method. Furthermore, we imposed a maximum on m in the DA method (MDA) in order to enhance the effect of chaotic neuro-dynamics on the DA mechanism. Finally, we proposed the exponential chaotic tabu search with the MDA method, and further, we confirmed that the proposed MDA method has excellent performance in solving QAPs when it is used in the exponential chaotic tabu search.

In future studies, we will improve the proposed method by also considering feature **(F-1)** of the LK algorithm. Furthermore, we will propose an automatic tuning method to find an appropriate maximum value for m . In addition, we will incorporate synchronous updates of the neuron states in the proposed method for massively parallel hardware implementations with an analog circuitry to drastically accelerate the execution time especially for large-size QAPs [44, 45].

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