Paper

Analog CMOS circuit implementation of a pulse-coupled phase oscillator system and observation of synchronization phenomena

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Abstract: Analog CMOS circuit implementation of a system of pulse-coupled phase oscillators is proposed. A CMOS circuit that achieves the dynamics of pulse-coupled oscillators has been designed and fabricated using a 0.25- μm CMOS technology. The proposed oscillator circuits with continuous-time operation interact with each other via a pulse at each firing time. Update of the oscillator state is achieved by integrating the phase sensitivity function with the pulse width time span. The phase sensitivity function is generated by the combination of binary functions, while the function consists of three-values $\{-1, 0, 1\}$. Introducing a zero-value span in the function leads to fast synchronization and robustness to parameter fluctuation due to LSI device mismatches, which facilitates VLSI implementation. Using the fabricated CMOS circuit, we have observed not only in- and anti-phase but also out-of-phase synchronization.

Key Words: analog CMOS circuit, pulse-coupled phase oscillator, synchronization

1. Introduction

Collective synchronization phenomena occur ubiquitously in nature. Such phenomena can be modeled by ensembles of nonlinear limit-cycle oscillators. In general, an ensemble of coupled limit-cycle oscillators can be reduced into a system of coupled phase oscillators by means of the phase reduction technique [1–5].

Coupled phase oscillators are classified into two types: *phase*-coupled and *pulse*-coupled oscillators, according to the interaction among the oscillators. In the conventional theories about coupled phase oscillators, mutual interactions among phase oscillators are represented as a phase coupling function, which is a function of the phase difference between two oscillators [1, 2].

In order to realize large-scale coupled oscillator systems for various practical applications, CMOS LSI implementation of coupled phase oscillator systems is essential. In our previous work, a CMOS

circuit for the *phase*-coupled oscillators has been proposed [6] by using the pulse-width and pulse-phase modulation (PWM/PPM) approaches [7]. We applied the circuit technique to phase-locked loop neural networks [8] and visual image processing [9]. The circuit can realize functions of phase-coupled oscillators with an arbitrary phase coupling function. However, it has limitations in the operation speed and efficiency due to discrete-time operation based on the external clocks [6]. To overcome these limitations, we have proposed an analog CMOS circuit for *pulse*-coupled oscillators based on continuous-time operation [10].

There has been earlier work about analog CMOS circuit implementation of pulse-coupled oscillators [11–13]. They targeted at applications of communication devices, and achieved fast synchronization with low power consumption. However, the aim of this work is to achieve various synchronization phenomena, such as in-phase, anti-phase, and out-of-order synchronization, by using a simple phase sensitivity function, which leads to simple circuit implementation. A promising application of our system is image processing [14].

In this paper, we describe the analog CMOS circuit for pulse-coupled oscillators in detail. We designed and fabricated the circuit using a 0.25- μm CMOS technology. We demonstrate that the circuit exhibits in-/anti-phase and different (out-of-phase) synchronization through experiments using the fabricated circuit.

2. Pulse-coupled phase oscillator model

The concept of coupled phase oscillators has been firstly proposed by Winfree [3] and its dynamics is expressed as follows:

$$\frac{d\phi_i}{dt} = \omega_i + Z(\phi_i)S(t), \quad (1)$$

where ϕ_i is the i -th phase variable with 2π periodicity, ω_i the i -th natural angular frequency, $Z(\phi_i)$ the phase sensitivity function, which gives the response of the i -th oscillator, and $S(t)$ the summation of inputs from other oscillators. We assume here the pulse inputs as follows:

$$S(t) = \frac{K_0}{N} \sum_{j=1}^N \sum_{n=1}^{\infty} \delta(t - t_{jn}) \quad (2)$$

where K_0 the coupling strength, N the number of oscillators, and t_{jn} a firing time. Function δ is mathematically Dirac's delta function, and only represents the input spike timing without a pulse width. However, in the real circuit, a spike pulse has a definite width Δt , during which ϕ_i is updated according to the value of $Z(\phi_i)$. If it is assumed that $Z(\phi_i) = -\sin(\phi_i)$ as a specific case, the dynamics of the *pulse*-coupled oscillators are schematically illustrated in Fig. 1.

Pulse-coupled oscillators may be transformed into a canonical form of phase-coupled oscillators [15]. By replacing $t - t_{jn} = [\phi_j(t) - 2\pi n]/\omega_0$ and averaging in time, we can obtain the following equivalent dynamics [1]:

$$\frac{d\phi_i}{dt} = \omega_i - \frac{K}{N} \sum_{j=1}^N \sin(\phi_i - \phi_j) \quad (3)$$

where $K = 2\pi K_0/\omega_0$. This dynamical equation is widely known as the Kuramoto model [1, 2].

Despite of this approximate equivalence in theoretical analysis between *pulse*-coupled and *phase*-coupled oscillator models, from a hardware implementation point of view, *pulse*-coupled systems are more suitable than *phase*-coupled systems. In the latter systems, information transported between oscillators is represented by analog values, which must be buffered by analog circuits, and this leads to less efficiency and more power consumption of the systems. On the other hand, in the former systems, information is represented by pulses, which can be buffered by digital circuits.

In our system, we use three-value functions as $Z(\phi_i)$; a typical function shape is shown in Fig. 2. The three-value functions are expanded in the time domain and have values: $-1, 0, 1$. In the update operation, “ -1 ” and “ 1 ” correspond to decrease and increase of ϕ_i , respectively, and “ 0 ” means no updating. Increasing ϕ_i results in leading of next spike firing timing and vice versa.

Fig. 1. Dynamics of pulse-coupled phase oscillator model.

Fig. 2. An example of the shape of function $Z(\phi_i)$.

We newly introduce parameter α as the span during which $Z(\phi_i) = 0$, as shown in Fig. 2. By setting $\alpha > \Delta t$, we can prevent the connected oscillators from over-updating when they approach to a synchronization state. Even in such a case, if $Z(\phi_i)$ is like that shown in Fig. 2, either oscillator is updated, and finally the system reach a synchronization state. Compared with sinusoidal functions, the three-value functions lead to faster convergence and simpler circuit implementation.

3. CMOS circuit for pulse-coupled oscillator systems

The oscillator unit circuit consists of oscillator *OSC*, phase sensitivity function generator *ZGEN*, and phase state updater *UPD*, as shown in Fig. 3.

3.1 Oscillator circuit

Figure 4 shows the oscillator circuit, which stores the phase variable of the oscillator. The phase variable is converted into spike pulses by thresholding operation using comparators, and output spike pulses to other oscillators. The circuit consists of a capacitor, two switched current sources, two comparators each of which is followed by a pulse generator *PG*, and a reset switch, as shown in Fig. 4(a). The capacitor stores a charge as the phase variable, and switched current source I_ω charges a constant charge to the capacitor for realizing ω_i in Eq. (1). Switched current source I_{set} sets the initial phase state by using a PWM signal given from terminal *input*. Two sets of comparators *comp* shown in Fig. 4(b) and pulse generators *PG* shown in Fig. 4(c) are used for generating spike outputs, *slope* and *reset*.

The timing diagram for circuit operation is shown in Fig. 4(d). The terminal voltage of the capacitor, V_ϕ , increases continuously by the current I_ω from the current source. When V_ϕ reaches constant

Fig. 3. Oscillator unit circuit architecture.

Fig. 4. Oscillator circuit *OSC*: (a) oscillator (*OSC*), (b) comparator (*comp*), (c) spike pulse generator (*PG*), and (d) timing diagram for circuit operation.

threshold voltage V_{middle} , an output spike pulse is generated and transmitted to other oscillators. Then, when V_ϕ reaches another constant threshold voltage V_{high} , signal *reset* is generated, and V_ϕ is reset at the lowest voltage V_{low} .

3.2 Phase sensitivity function generator

We use binary functions shown in Fig. 5(a) and (b) as function $Z(\phi)$, which approximates $\pm \sin(\phi)$. The phase sensitivity function generator *ZGEN* is shown in Fig. 5(c), and the timing diagram is shown in Fig. 5(d). The circuit generates signals Z_p and Z_n , which represents function $Z(\phi)$ in the time domain, using received spike pulses, a comparator output, and an external signal *sign* that determines the function type of Z . Signals Z_p and Z_n represent positive and negative parts of function Z , respectively. By combining these two signals, we can generate “ $-\sin$ ”-type function Z^- and “ $+\sin$ ”-type function Z^+ . It is noted that a zero-value time span (gap), α , exists between non-zero time spans.

3.3 Phase state updater

Phase state updater *UPD* is shown in Fig. 6. The circuit updates V_ϕ with currents from the switched current sources. The current flows only during the time span corresponding to the pulse width of the spike pulse, Δt , received from the other oscillators. When a spike pulse is fed into *UPD*, only if Z_p is “High”, the charging side of the current source I_{c+} is active, and V_ϕ increases; on the other hand, only if Z_n is “High”, the discharging side of the current source I_{c-} is active, and V_ϕ decreases.

It is noted that because Δt is finite, the slope of a change in V_ϕ is also finite. This means that $Z(\phi)$ is not binary but actually piecewise-linear, as shown in Fig. 6(b). In this circuit configuration, we can increase the connections by the number of *UPD* connected to *OSC*.

Fig. 5. Phase sensitivity function generator $ZGEN$: (a) “ $-\sin$ ”-type function Z^- , (b) “ $+\sin$ ”-type function Z^+ , (c) circuit configuration, and (d) timing diagram for generating functions $Z^\pm$.

Fig. 6. Phase state updater UPD : (a) circuit configuration and (b) timing diagram for update operation.

Fig. 7. Circuit design result: (a) layout result and (b) microphotograph of two pulse-coupled oscillator unit circuits.

3.4 LSI circuit design

We designed and fabricated a CMOS circuit in which two unit circuits are connected each other using TSMC $0.25 \mu\text{m}$ (1-Poly, 5-Metal) CMOS technology, in order to construct a system of two pulse-coupled oscillators. The layout result and the microphotograph of the core part of the fabricated circuit are shown in Fig. 7, and the circuit specification is shown in Table I.

4. Measurement results and discussion

4.1 Observation of synchronization states

Measurement results for this pulse-coupled phase oscillator system are shown in Fig. 8. We set mean firing period $T = 1 \mu\text{s}$, $\Delta t = 20 \text{ ns}$, $\alpha = 50 \text{ ns}$, $V_{\text{high}} = 2 \text{ V}$, $V_{\text{middle}} = 1.5 \text{ V}$, $V_{\text{low}} = 1 \text{ V}$, and updating current I_c is set about $1 \mu\text{A}$. As shown in Fig. 8(A), if we used function Z^- and set $\Delta t \leq \alpha < \pi/2$, two oscillators synchronized with in-phase, where the initial phase difference was 0.95π . In contrast, if we used function Z^+ and set α as the same condition, they synchronized with anti-phase, where the initial

Table I. Circuit specification.

Technology	TSMC 0.25 μm 1-Poly 5-Metal CMOS
Layout area	$400 \times 150 \mu\text{m}^2$
Supply voltage	3.3 V
Update period	1 μs
Power consumption	840 μW (estimated by SPICE)

phase difference was 0.05π , as shown in Fig. 8(B). In addition to those, if we used functions Z^- and Z^+ with $\alpha > \pi/2$, we found other synchronization states, as shown in Fig. 8(C) and (D), respectively. This different-phase (out-of-order) synchronization was indicated in another literature [16], where the synchronization occurred due to time delay of spike pulses. On the other hand, in our circuit, the synchronization is induced by a certain value of parameter α , as described below.

Synchronization mechanisms for the four cases shown in Fig. 8 are explained using Fig. 9 as follows.

Regarding in-phase synchronization case (A), if the initial state is in about an anti-phase condition, as shown in Fig. 9(A)(a), ϕ_j leads because Z_j is positive when a spike pulse from oscillator i reaches oscillator j , and ϕ_i lags because Z_i is negative when a spike pulse from oscillator j reaches oscillator i . Then, two oscillators reach the state shown in Fig. 9(A)(b), where one oscillator (oscillator i in this case) is not updated because of gap α , but the other is still updated. Finally, the system reaches the steady state shown in Fig. 9(A)(c). If some fluctuation occurs and synchronization is broken, the system goes back to the state Fig. 9(A)(b), and then reaches the steady state again. Thus, this system has stability to stay at the in-phase synchronization state.

Regarding anti-phase synchronization case (B), if the initial state is in about an in-phase condition, as shown in Fig. 9(B)(a), one oscillator (j in this case) is updated, and ϕ_j lags because Z_j is negative when a spike pulse from oscillator i reaches oscillator j . On the other hand, oscillator i is not updated because of gap α . Then, the phase difference $\Delta\phi$ becomes large, and the system reaches the state shown in Fig. 9(B)(b), where both oscillators are updated and $\Delta\phi$ becomes larger. Finally, the system reaches the steady state shown in Fig. 9(B)(c). In the similar way to case (A), this system also has stability to stay at the anti-phase synchronization state.

Figure 9(C) shows a different-phase synchronization state when using function Z^- . As in case (A), the initial state is in about an anti-phase condition, as shown in Fig. 9(C)(a). One oscillator (i in this case) is updated, and ϕ_i lags because Z_i is negative when a spike pulse from oscillator j reaches oscillator i . Then, the system reaches the state shown in Fig. 9(C)(b), where both oscillators are not updated because $\alpha > \pi/2$ and both spike pulses are in gap α . If some fluctuation occurs and $\Delta\phi$ becomes larger, the system still stay at the same state, but if $\Delta\phi$ becomes smaller, and state shown in Fig. 9(C)(c) is realized, the system reaches the in-phase synchronization state.

Figure 9(D) shows another different-phase synchronization state when using function Z^+ . As in case (B), the initial state is in about an in-phase condition, as shown in Fig. 9(D)(a). One oscillator (j in this case) is updated, and ϕ_j lags because Z_j is negative when a spike pulse from oscillator i reaches oscillator j . Then, the system reaches the state shown in Fig. 9(D)(b), where both oscillators are not updated as in the state shown in Fig. 9(C)(b). If some fluctuation occurs and $\Delta\phi$ becomes smaller, the system still stay at the same state, but if $\Delta\phi$ becomes larger, and state shown in Fig. 9(D)(c) is realized, the system reaches the anti-phase synchronization state.

4.2 Measurement results about settling time

We investigated the relationship between updating current $I_{c\pm}$ and the settling time until synchronization. Circuit simulation (HSPICE) results about the relationship between the number of spikes and phase-difference $\Delta\phi$ in transition states toward the in-phase synchronization state are shown in Fig. 10(a) as a parameter of $I_c (= I_{c\pm})$. The simulation condition is the same as that described in 4.1. Measurement results using the fabricated circuit about spike trains in the same transition states are shown in Fig. 10(b). The convergence timing in each condition is marked by a red circle in Fig. 10(b), and they are nearly identical with the simulation results shown in Fig. 10(a). Each update amount is proportional to charges updated in capacitor C_ϕ , as shown in Fig. 6(a), and it is given by $I_c \cdot \Delta t$.

Fig. 8. Measurement results of synchronization states in the CMOS pulse-coupled phase oscillator system: (A) in-phase state ($\Delta\phi = 0$, function Z^-), (B) anti-phase state ($\Delta\phi = \pi$, function Z^+), (C) different-phase state ($\Delta\phi \neq 0, \pi$, function Z^-), and (D) different-phase state ($\Delta\phi \neq 0, \pi$, function Z^+): (a) observed waveforms and (b) $V_{\phi_i} - V_{\phi_j}$ plot.

Therefore, $\Delta\phi$ is expected to change in proportion with I_c , and equivalently the number of spikes required to reach a steady state, N_{conv} , is inversely proportional to I_c , as shown in Fig. 6(c).

It can also be found from these measurement results that the convergence time to reach synchronization states in our *pulse*-coupled oscillator system is much shorter than that in the *phase*-coupled phase oscillator system that we previously proposed [6], in which the convergence time was more than $200 \mu s$ (100 time step).

Fig. 9. Synchronization mechanisms: (A) to (D) correspond to those in Fig. 8, and (a) initial state, (b),(c) transient state or steady state.

Other circuit simulation results are shown in Fig. 11, where $\alpha = \Delta t = 20$ ns and other conditions are same as in Fig. 10(a). Period-2 oscillations are observed in synchronization states at large update amounts. This is because update operation is performed in the time span caused by delays in spike pulse transmission between oscillators and by operation delays in circuits *OSC* and *ZGEN*. On the other hand, such period-2 oscillations are not observed in Fig. 10(a). From these results, we can conclude that the effect of transmission/operation delays can be eliminated by setting $\alpha > \Delta t$. This feature is also effective to compensation for calculation precision degradation due to CMOS device mismatches and other nonidealities in analog circuits. In our measurement, timing fluctuation of about 1.5 ns was observed at a mean firing period of 1 μ s. This nonideality was successfully compensated by the setting regarding α , and steady synchronization behavior was observed.

5. Conclusion

We proposed an analog CMOS circuit for a system of pulse-coupled oscillators. The circuit was

Fig. 10. Relationship between updating amount and settling time until synchronization where $\Delta t = 20$ ns, $\alpha = 50$ ns: (a) circuit simulation (HSPICE) results, (b) circuit measurement results, and (c) relationship between I_c and $1/N_{conv}$.

designed and fabricated using a $0.25 \mu m$ CMOS technology. We observed in/anti-phase and other types of synchronization states in the measurement using the fabricated chip. We also found that such synchronizations occur very quickly because of instantaneous updating with spike timing, which will

Fig. 11. Circuit simulation (HSPICE) results of relationship between updating amount and settling time until synchronization, where $\alpha = \Delta t = 20$ ns).

lead to high-speed, highly-effective spike-based computing systems. A promising application of such systems is image processing, and integrated circuit design for large-scale networks of pulse-coupled oscillators will be expected as future work.

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