Effects of configuration of inhibited in-coming synaptic connections in sensitive response of chaotic wandering states

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Abstract: In the present paper, we investigate the dependence on the configuration of inhibited in-coming synaptic connections in Nara & Davis chaotic neural network model related with the sensitive response of the chaotic wandering state to memory pattern fragments. It has been shown that when a memory pattern fragment was given, the chaotic wandering state in Nara & Davis chaotic neural network model suddenly converges into the target memory pattern with a high robustness. However, the potentiality depends on dynamical properties of the chaotic wandering states, which are affected with the configuration of the inhibited in-coming synaptic connections. Therefore, we investigate the sensitivity to memory pattern fragments for two types of the configuration, (i) which denotes randomly inhibited in-coming synaptic connections except for ones from memory pattern fragments referred as a partly random configuration hereafter and (ii) which defines randomly inhibited in-coming synaptic connections referred as a fully random configuration hereafter. From the computer experiments, the success ratio for the partly random configuration becomes much higher than the fully random ones, and the accessing time becomes shorter. In addition, from Lyapunov dimension, the system with the partly random configuration reveals higher developed chaos.

Key Words: Nara & Davis CNN, memory pattern fragments, sensitive response

1. Introduction

Based on the discovery of chaos in biological systems, chaos would play important roles in realizing sophisticated and excellent information processing occurring in brain [1–8]. For instance, Skarda
and Freeman have shown that chaos could play important roles in a learning process and recalling process [3]. In a learning process, chaos could provide driving activity essential for memorizing novel inputs. In a recalling process, chaos ensure rapid and unbiased access to previously trained patterns. From the theoretical approach, on the other hand, Nara & Davis have shown the interesting results that the chaotic wandering states instantaneously converge into a basin of the target memory pattern attractor when a memory pattern fragment is given [7, 8]. Kuroiwa and his colleagues also have shown that the property of the sensitive response to memory pattern fragments in chaotic wandering states is general even though the mechanism of chaos is different [9]. Thus, the sensitive response to memory pattern fragments with chaotic wandering states could play important roles in realizing the rapid and unbiased access in various chaotic neural networks.

Recently, we start investigations of the mechanism of the sensitive response in chaotic neural network models from the viewpoint of an instability of orbit [10–12]. For instance, we have investigated effects of the configuration of inhibited in-coming synaptic connections [12]. It has been shown tentatively that the configuration, which denotes randomly inhibited in-coming synaptic connections except for ones from memory pattern fragments referred as a partly random configuration hereafter, could realize a more sensitive response than the configuration which represents randomly inhibited in-coming synaptic connections referred as a fully random configuration hereafter. Unfortunately, we did not consider the normalization factor for the strength of the memory pattern fragment, then the comparison of the sensitivity between the partly random configuration and the fully random one is almost meaningless and we can expect that the sensitivity of the partly random one is higher qualitatively. In addition, since the strength of the memory pattern fragments was too large, we could not investigate the relationship between the sensitivity and the instability of orbit. Therefore, the purposes of the present paper are (i) to introduce the the normalization factor into the strength of memory pattern fragment, (ii) to compare the sensitivity between the partly random configuration and the fully random one, and (iii) to investigate the sensitivity from the instability of the chaotic wandering orbit.

In section 2, we explain a recurrent neural network model referred as RNN hereafter and Nara & Davis chaotic neural network model referred as Nara & Davis CNN hereafter. In section 3, we denote a evaluation method of the sensitivity to the memory pattern fragment and a calculation method of the instability of the chaotic wandering orbit. In section 4, results of the sensitivity and the instability are given. Section 5 devotes discussions and in section 6, conclusions are given.

2. Chaotic neural network model

2.1 Recurrent neural network model with multi-cycle memory patterns

Let us explain RNN, briefly. The updating rule of RNN is given as follows:

\[ u_i(t+1) = \sum_{j=1}^{N} w_{ij} z_j(t), \]  

(1)

where \( u_i(t) \) represents an internal state of the \( i \)th element at discrete time \( t \), \( z_i(t) \) describes its output, \( w_{ij} \) denotes a synaptic connection from the \( j \)th neuron to the \( i \)th one, and \( N \) is the total number of neurons in RNN.

In the present paper, the output is given by the following function,

\[ z_i(t) = \tanh(\beta u_i(t)), \]  

(2)

where \( \beta \) corresponds to the steepness of the function.

In the present paper, the synaptic connection is defined as,

\[ w_{ij} = \sum_{a=1}^{L} \sum_{\nu=1}^{P} \nu^a \nu^{\nu+1} (v_j^a \nu)^{\dagger}, \]  

(3)

where \( v^a \mu \) denotes \( \mu \)th memory pattern in \( \alpha \)th cycle, and \( L \) and \( P \) are the number of cycles and the number of patterns per cycle, respectively. Note that we employ cycle memory patterns as shown in
Fig. 1, thus $\mathbf{v}^a_{P+1} = \mathbf{v}^a_1$, $L = 5$, $P = 6$ and $N = 400$. The dagger vector of $(\mathbf{v}^a \mu)^\dagger$ is given by,

$$(\mathbf{v}^a \mu)^\dagger = \sum_{b=1}^{L} \sum_{\nu=1}^{P} (\mathbf{o}^{-1})_{a \mu b \nu} (\mathbf{v}^b \nu)^T,$$

where the symbol “T” denotes the transpose operator, and $\mathbf{o}^{-1}$ is the inverse of $\mathbf{o}$ defined by,

$$(\mathbf{o})_{a \mu b \nu} = \sum_{k=1}^{N} \mathbf{v}^a_k \mathbf{v}^b_k.$$

### 2.2 Nara & Davis chaotic neural network model

Let us explain Nara and Davis CNN, briefly. The updating rule of Eq. (1) is rewritten by,

$$\eta_i(t + 1) = \sum_{j=1}^{N} w_{ij} \epsilon_{ij}(d) z_j(t).$$

The output $z_i(t)$ is given by Eq. (2), and $\epsilon_{ij}(d)$ denotes a matrix of binary activity values, that is,

$$\epsilon_{ij}(d) = \begin{cases} 0 & (j \in F_i(d)) \\ 1 & (\text{otherwise}) \end{cases},$$

where $F_i(d)$ represents a configuration which defines randomly inhibited in-coming synaptic connections of the $i$th neuron. The parameter $d$ represents the number of remaining synaptic connection, that is, $\sum_j \epsilon_{ij}(d) = d$, referred as the connectivity, hereafter. The connectivity of $d$ and the configuration of $F_i(d)$ are system parameters in Nara & Davis CNN. In the computer experiments, we employ the following two types of configurations, (i) the partly random configuration in which inhibited in-coming synaptic connections are randomly selected except for ones from memory pattern fragments and (ii) the fully random configuration in which inhibited in-coming synaptic connections are randomly selected.

### 3. Effects of the configuration in sensitive response

#### 3.1 Evaluation method of sensitivity

At first, let us explain a calculation method of the sensitive response to the memory pattern fragments. We update Nara & Davis CNN by Eq. (6) until $T_0$. After $T_0$, we apply one of memory pattern fragments into Nara & Davis CNN. Then, the update rule is rewritten by,

$$\eta_i(t + 1) = \sum_{j=1}^{N} w_{ij} \epsilon_{ij}(t) z_j(t) + \rho \sigma_i I_i,$$

where $\mu : \text{pattern index}$ and $a : \text{cycle index}$.
where $I_i$ represents a memory pattern fragment which consists of black and white pixels in Fig. 2 and is composed of 40 pixels among 400 ones, $\rho$ denotes the strength of the fragments, and $\sigma_i$ describes a standard deviation of the internal state of the $i$th neuron in Nara & Davis CNN which corresponds to a normalization factor. By performing a preliminary experiment, we have evaluated $\sigma_i$ before we apply the fragments. In the preliminary experiment, $\sigma_i$ is calculated within 10,000 steps in chaotic wandering state excluding initial $T_0 = 19,997$ steps according Eq. (6).

In the present paper, we apply RNN to determine whether the system accesses to the target basin or not. Thus, at each step of updating according to Nara & Davis CNN of Eq. (6), we employ outputs of the system as an initial configuration of RNN of Eq. (1). We check which attractors RNN converges into. If the attractor corresponds to the target memory pattern, we regard that the system can access to the basin of the target memory pattern. Otherwise, we update the system with Eq. (8) again, and we perform the same procedures until the system accesses to the basin of the target memory pattern.

In order to investigate the sensitivity, we focus on the following two problems.

1. Success ratio: How many times does it reach the target basin within 30 steps starting from different points of chaotic wandering state? In other words, is the searching procedure by means of chaotic wandering assured?

2. Accessing time: How many steps does it take to reach the target basin related with a memory fragments? In other words, how short is the “access” time for the memory basin corresponding to the external input of a memory fragments?

It should noted that high success ratio and short accessing time is practical in realizing an instant memory search. In the computer experiments, we evaluate the sensitive response to memory pattern fragments as follows. If the system converge into the pattern corresponding to the memory pattern fragments within 30 steps while memory pattern fragments is applying, we identify the search procedure of memory pattern as success. On the other hand, if the system doesn’t converge into the pattern, we regard that the search procedure misses and accessing time takes 30 steps.

3.2 Instability of orbit

In the present paper, we investigate the sensitivity from the instability of the chaotic wandering orbit. In an evaluation of the instability of the orbit, we employ Lyapunov dimension given by,

$$D_L = j + \sum_{k=1}^{j} \frac{\lambda_k}{|\lambda_{j+1}|},$$  \hspace{1cm} (9)

where $j$ is the maximal index satisfying the condition of $\sum_{k=1}^{j} \lambda_k \geq 0$ for descending ordered Lyapunov spectrum. In the present paper, if all the Lyapunov exponents takes positive values, the Lyapunov dimension is defined by $D_L = N(= 400)$.

In the present paper, Lyapunov spectrum $\lambda_i$ are evaluated by,

![Fig. 2. Memory pattern fragments. The below figures show the memory pattern fragments corresponding to each face pattern in the upper figures. The black pixel correspond to 1, the white pixel corresponds to −1, and the blue pixel correspond to 0. Thus, the memory pattern fragments correspond to black and white pixels and consists of 40 pixels.](image)
where $\delta_i(t)$ is a perturbation term defined by,

$$\delta_i(t) = \varepsilon \sigma_i,$$

and the $\delta_i(t + 1)$ at the next time step is evaluated by,

$$\delta_i(t + 1) = \sum_j w_{ij} \epsilon_j(d) f(u_j(t) + \delta_j(t)) - \eta_i(t + 1),$$

where $\varepsilon$ is a perturbation parameter and $\sigma_i$ denotes the normalization parameter which corresponds to the standard deviation of $\{\eta_i(t)\}$. In the present paper, we employ $\varepsilon = 0.1$, 0.01 and 0.001. The normalization factor $\sigma_i$ is necessary that $\{\eta_i(t)\}$ takes a different value depending on the connectivity of $d$. In the present paper, we evaluate the perturbation $\delta_i(t)$ by a numerical method, not an analytical method based on a Jacobian matrix. In Nara & Davis CNN, for the larger connectivity $d$, almost elements of the Jacobian matrix becomes zero. Thus the analytical method is inappropriate in evaluating $\delta_i(t)$.

4. Results

4.1 The sensitivity to memory pattern fragments

In computer experiments of the sensitive response, we employ the parameters $\beta = 100$ and $\rho = 0.1$. We evaluate the success ratio and accessing time within 1,000 steps in chaotic wandering state excluding

![Fig. 3. Success ratio for 5 different partly random configurations.](image)

![Fig. 4. Accessing time for 5 different partly random configurations.](image)
initial $T_0 = 19,997$ steps.

For a comparison, we apply 5 different configurations for the partly random and the fully random, respectively. For each configurations, we evaluate the success ration and the accessing time for 5 different memory pattern fragments individually, as shown in Fig. 2, and calculate an average for them. The success ratio and the accessing time for the partly random configuration are given in Fig. 3 and Fig. 4, respectively. In addition, the success ratio and the accessing time for the fully random configuration are given in Fig. 5 and Fig. 6, respectively. It should be noted that the range of $x$ axis is different between Figs. 3 and 4, and Figs. 5 and 6. In the case of the partly random configurations, for each neurons, 40 in-coming synaptic connections are not inhibited, which come from neurons corresponding to memory pattern fragments. Therefore, the connectivity $d$, which denotes the number of remaining in-coming synaptic connections, can take from 0 to 360. On the other hands, in the case of the fully random configurations, for each neuron, it is possible to inhibit all the 400 in-coming synaptic connections. Thus, the connectivity $d$ can take from 0 to 400. Therefore, the range of $x$ axis is different.

At first, we explain results for the partly random configuration in details. In Fig. 3(a) and Fig. 4(a), until the connectivity $d = 278$, the success ratio takes 20% and the accessing time takes 24.2 steps, reflecting a feature of the chaotic orbit that the system chaotically wanders in basins of memory patterns within a certain intra-cycle. As the connectivity $d$ increases from $d = 278$, the success ratio increases and the accessing time becomes shorter, except for the connectivity of $d = 313, d = 317$

![Fig. 5. Success ratio for 5 different fully random configurations.](image)

![Fig. 6. Accessing time for 5 different fully random configurations.](image)
and $d = 321$. For $d = 313$, the success ratio takes 0.5% and the accessing time takes 29.9 steps. For $d = 317$, the success ratio takes 0.8% and the accessing time takes 29.2 steps. For $d = 321$, the success ratio takes 24.9% and the accessing time takes 26.5 steps. From $d = 324$ to $d = 360$, the success ratio takes almost 92% and the accessing time takes almost 9.6 steps, showing the higher sensitivity to memory pattern fragments. Especially, for $d = 347$, the success ratio takes the maximum value of 100% and the accessing time takes the shortest value of 5.5 steps. We can observe similar behaviors in Fig. 3 and Fig. 4.

At last, we give results for fully random configuration in detail. In Fig. 5(a) and Fig. 6(a), until $d = 308$, the success ratio takes 20% and the accessing time takes 24.2 steps, showing the similar results for the partly random configuration. As the connectivity $d$ increases from $d = 308$, the success ratio increases and the accessing time becomes shorter, except for the connectivity of $d = 382$. From $d = 387$ to $d = 395$, the success ratio takes almost 89% and the accessing time takes almost 11.8 steps. For $d = 393$, the success ratio takes the maximum value of 92% and the accessing time takes the shortest value of 10.7 steps. We can observe similar behaviors in Fig. 5 and Fig. 6.

Thus, the averaged accessing time of 9.6 steps in the partly random configurations is shorter than 11.8 steps in the fully random configurations. In addition, the best accessing time of 5.5 steps in the partly random configurations is quite shorter than 19.7 steps in the fully random configurations. The results suggest us that the sensitivity for the partly random configuration is higher than the fully random configuration. The difference between the partly random configuration and the fully random one becomes more clear as the strength of the memory pattern fragment becomes larger as shown in Fig. 7. In the partly random configuration, the best accessing time of 1.8 steps with the success ratio of 100% for $d = 319$. On the other hand, in the fully random configuration, the best accessing time of 5.7 steps with the success ratio of 94.7% for $d = 379$. Therefore, the sensitive response of chaotic wandering states for the partly random configuration reveals higher potentiality in the accessing time and the success ratio.

4.2 Instability of orbit

In evaluations of the Lyapunov dimension, we apply the parameters $\beta = 100$ and $\varepsilon = 0.1, 0.01$ and 0.001. We calculate the Lyapunov dimension for 5 different configurations for the partly random and the fully random as same as the evaluation of the sensitivity. The results are given in Fig. 8 and Fig. 9, respectively. The Lyapunov dimension for $\varepsilon = 0.1$ is quite different from the others. However,
the behavior is qualitatively similar, suggesting the validity of the evaluations.

At first, we investigate the difference of the Lyapunov dimension between the partly random configuration and the fully random one. From Fig. 8 and Fig. 9, the Lyapunov dimension for the partly random one is meaningful larger. Thus, the higher instability of the orbit introduces the higher sensitivity to the memory pattern fragment.

At last, we focus on the results for the partly random configuration in detail. In Fig. 8, from the connectivity \( d = 278 \), the Lyapunov dimension suddenly increases, corresponding to the sensitive response. As the connectivity increases more than \( d = 345 \), the Lyapunov dimension suddenly decreases. Among the range of \( d = [345, 360] \), its behavior fluctuates and the value becomes smaller than the range of \( d = [278, 345] \). However, the response to the memory pattern fragment is most sensitive among the range of \( d = [345, 360] \). We can observe similar behaviors for the other partly random configurations. The results arise the problem that the higher instability of the orbit does not always give the higher sensitivity. It is a further problem why the sensitivity becomes higher among the range of \( d = [345, 360] \).

**Fig. 8.** Lyapunov dimension for 5 different partly random configurations.

**Fig. 9.** Lyapunov dimension for 5 different fully random configurations.
5. Discussion

In the present paper, we introduce a normalization factor into the strength of the memory pattern fragment, which corresponds to $\sigma_i$ in Eq. (8). The results of the standard deviation $\sigma_i$ for the first neuron and the 351th neuron are given in Fig. 10 for 5 different partly random configurations and in Fig. 11 for 5 different fully random ones. Note that the first element in each memory pattern takes $-1$, on the other hand, the 351th depends on the 30 memory patterns.

In all the configurations, the standard deviation changes for the connectivity $d$. Especially, for the first element, the standard deviation monotonically decreases as $d$ increases. In addition, the value is almost similar between the partly random configurations and the fully random configurations, reflecting the fact that for all the memory patterns, the first element takes the same value of $-1$. On the other hand, for the 351th, the variation of standard deviation is more complicated. In addition, among the different configurations, the standard deviation is quite different. The standard deviation for almost neurons reveals similar behavior. The results indicate that it is important to introduce the normalization factor of $\sigma_i$ in comparing the sensitivity among the connectivity $d$ or the different configurations.

Although 5 different configurations are employed in the simulation, for smaller $d$ which is the almost same value, the success ratio takes a low value of 0.2 as shown in Fig. 3. For smaller $d$, the chaotic...
orbit wanders among basins of the cyclic memory patterns corresponding to the initial configuration of $a = 1$ and $\mu = 1$ pattern. Therefore, in the case of smaller strength of $\rho = 0.1$, the chaotic wandering orbit can access to the 1st cycle only and misses to visit to the other 4 cycles. Then the success ratio takes 0.2 ($= 1/5$). According to Fig. 8, Lyapunov dimension suddenly becomes larger around $d = 270$, suggesting that the randomness of the chaos becomes larger. Therefore, for larger $d = 270$, chaotic wandering orbit can access to the other cycles, resulting the success ratio takes larger value.

6. Conclusions
In the present paper, we investigate the effects of configurations of inhibited in-coming synaptic connections by introducing the normalization factor into the memory pattern fragment related with the sensitivity. Results are as follows:

- The sensitivity for the partly random configuration becomes much higher than the fully random one.
- The instability of orbits for he partly random configuration becomes also much higher than fully random one.
- The normalization factor is important to compare the sensitivity among the connectivity $d$ or the different configurations of inhibited in-coming synaptic connections.

Therefore, the sensitive response of chaotic wandering states for the partly random configuration reveals higher potentiality in the accessing time and the success ratio. It is a further problem why the higher instability of the orbits does not always introduce the higher sensitivity related with edge of chaos.

References