Paper

A novel cochlea partition model based on asynchronous bifurcation processor

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Abstract: The cochlea is a highly nonlinear biological sound processor the major components of which are lymph (viscous fluid), a basilar membrane (vibrating membrane in the viscous fluid), outer hair cells (active dumpers for the basilar membrane), inner hair cells (neural transducers), and spiral ganglion cells (parallel spikes density modulators). In this paper, a novel cochlea partition model based on a concept of an asynchronous bifurcation processor is presented. It is shown that the presented model can reproduce typical nonlinear responses of partitions of biological cochleae such as nonlinear DC response, nonlinear band-pass filtering, and adaptation. Also, FPGA experiments validate reproductions of these nonlinear responses.

Key Words: asynchronous cellular automaton, bifurcation, cochlea, FPGA

1. Introduction

The mammalian ear is divided into an outer ear, a middle ear, and an inner ear, where sound processing is mainly executed in a cochlea of the inner ear. Figure 1(a) shows a sketch of the mammalian cochlea, which consists of a stapes, lymph, a basilar membrane, outer hair cells, inner hair cells, and spiral ganglion cells [1–3]. Also, Fig. 1(b) shows a cross-sectional view of a partition of the cochlea, which is called a cochlea partition. In the cochlea, a sound stimulation via the stapes induces vibrations of the lymph and the basilar membrane, which work together to realize a mechanical nonlinear Fourier transformation in such a way that higher and lower frequency components in the sound stimulation induce vibrations near and far from the stapes, respectively. As shown in Fig. 1, a huge number of outer hair cells are attached to the basilar membrane. Surprisingly, the outer hair cells change their physical lengths and work as active dumpers to control the mechanical vibrations of the basilar membrane. As shown in Fig. 1, a large number of inner hair cells are also attached to the basilar membrane, where their locations correspond to frequency components in the sound stimulation. Each inner hair cell transforms the mechanical vibration with a specific frequency (called a characteristics frequency) into its internal electrical potential \(V_I\) called a receptor potential. As shown in Fig. 1,
many spiral ganglion cells are attached to one inner hair cell. The spiral ganglion cells encode the receptor potential $V_I$ of the inner hair cell into parallel spike-trains, which are transmitted to the central nervous system. Due to its high nonlinearities, the whole cochlea (i.e., connected compartments of cochlea partition) exhibits a huge variety of nonlinear responses such as nonlinear DC response [4], nonlinear band-pass filtering [5–7], adaptation of spike density [8], multi-tone suppression [9, 10], spike density modulation [11], missing fundamental, first pitch shift, second pitch shift, and so on (see also reviews in [1–3]). Among them, the nonlinear DC response, the nonlinear band-pass filtering, and the adaptation of spike density are important research topics when the cochlea partition is to be investigated like this paper since the other nonlinear responses are caused by connections of the compartments of the cochlea partition. In order to understand such complicated nonlinear responses of the cochlea partition and the whole cochlea, many mathematical models have been presented and analyzed intensively [1–11].

In addition to such fundamental researches, many artificial electronic cochleae [12–24] have been also
Fig. 2. Concepts of the asynchronous bifurcation processor (ab. ABP) and the ABP-based neural system model. (a) The velocity vectors induced by synchronous state transitions are characterized by a finite set. The velocity vectors induced by phase-locked state transitions are characterized by rational numbers. The velocity vectors induced by asynchronous state transitions are characterized by real numbers. (b) The asynchronous transitions of the discrete states realize a smooth velocity vector, a smooth vector field, and thus a smooth bifurcation.

Table I. Hardware-oriented Neural System Modeling Approaches. See also [26, 27] and references therein.

<table>
<thead>
<tr>
<th>Hardware</th>
<th>Time and State</th>
<th>Dynamics</th>
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<tr>
<td>Analog nonlinear circuit</td>
<td>Continuous time</td>
<td>Nonlinear ordinary differential equation [12, 13, 21]</td>
<td>Nonlinearity of circuit element such as MOSFET (not suited for on-chip dynamic parameter update)</td>
</tr>
<tr>
<td>Switched capacitor</td>
<td>Continuous state</td>
<td>Nonlinear difference equation [33] (iterative map)</td>
<td></td>
</tr>
<tr>
<td>Digital processor</td>
<td>Discrete time</td>
<td>Numerical integration [14, 22–24] (hardware resource consuming)</td>
<td>Coefficient of digitally implemented nonlinear function</td>
</tr>
<tr>
<td>Synchronous sequential logic</td>
<td>Continuous state</td>
<td>Synchronous cellular automaton [34–36] (traditional cellular automaton)</td>
<td>Wiring pattern among registers and logic gates (suited for on-chip dynamic parameter update)</td>
</tr>
<tr>
<td>Asynchronous sequential logic</td>
<td>Discrete state</td>
<td>Asynchronous cellular automaton [25–32] (including this paper)</td>
<td></td>
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</table>

Synonyms of “asynchronous sequential logic” from some perspectives:
- Asynchronous sequential logic (hardware perspective)
- Asynchronous cellular automaton (dynamical system perspective)
- Asynchronous numerical integration (computation perspective)
- Asynchronous bifurcation processor (processor perspective)
presented for clinical and engineering applications, e.g., cochlear implant [17–23] and cochlea-inspired sound processor for a mobile phone [24]. Concerning the electronic hardware, major hardware-oriented neural system modeling approaches include the following ones (see also Table I).

- An analog nonlinear circuit approach that uses a nonlinear ordinary differential equation to model the nonlinear dynamics of a neural system.
- A switched capacitor approach that uses a nonlinear difference equation to model the nonlinear dynamics of a neural system.
- A digital processor approach that uses a numerical integration to model the nonlinear dynamics of a neural system.
- A synchronous sequential logic approach that uses a traditional synchronous cellular automaton to model the nonlinear dynamics of a neural system.
- An asynchronous sequential logic approach that uses an asynchronous cellular automaton to model the nonlinear dynamics of a neural system [25–32].

In this paper, the last one, the asynchronous sequential logic approach, is focused on. Figure 2 illustrates concepts of a neural system model based on the asynchronous sequential logic. Figure 2(a) shows a toy example model, which is assumed to have discrete states $V$ and $U$ stored in registers. The registers are connected to each other via logic gates and reconfigurable wires, where the logic gates are used to realize nonlinear functions (say, $F(V,U)$ and $G(V,U)$) and the reconfigurable wires are used as control parameters of the functions. The example toy model is assumed to have clocks $C_V$ and $C_U$, which may depend on the states $(V,U)$ and/or a stimulation $S$. For simplicity of consideration, assume the clocks $C_V$ and $C_U$ trigger the following state transitions.

\[
\begin{align*}
\text{The state } V \text{ transits to } V + F(V,U) \text{ if a clock } C_V = 1 \text{ arrives, where } F \in \{-1, 0, 1\}. \\
The state } U \text{ transits to } U + G(V,U) \text{ if a clock } C_U = 1 \text{ arrives, where } G \in \{-1, 0, 1\}.
\end{align*}
\]  

If the clocks $C_V$ and $C_U$ are in in-phase synchronization, then the velocity vector $(F,G)$ is characterized by a finite set \{(0,0), (0,1), (1,1), (0,1), (-1,1), (-1,0), (-1,-1), (0,-1), (1,-1)\}. If the clocks $C_V$ and $C_U$ are in m : n locking, then the velocity vector $(F,G)$ is characterized by a set of rational numbers. If the clocks $C_V$ and $C_U$ are in asynchronization, then the velocity vector $(F,G)$ is characterized by a set of real numbers. Hence, conceptually speaking, the asynchronous transitions of the discrete states $V$ and $U$ realize a smooth velocity vector $(F,G)$ and a resulting smooth vector field.

As a result, the model with the asynchronous state transitions is expected to have a smoother bifurcation structure compared to the model with the synchronous state transitions. Before explaining advantages and significances of such a concept, let us consider the asynchronous sequential logic from some different perspectives as summarized in Table I. The term \textit{asynchronous sequential logic} represents how the circuit is operated (hardware perspective). Since the asynchronous sequential logic has discrete states with asynchronous transitions, it’s dynamics is described by an \textit{asynchronous cellular automaton} (dynamical system perspective). Since Eq. (1) can be regarded as a one-step explicit numerical integration with asynchronous state updates, it can be regarded as an \textit{asynchronous numerical integration} (computation perspective). Since we use Eq. (1) as a processor to reproduce nonlinear dynamics (especially, bifurcations) of neural systems, it may be acceptable to refer to the equation as an \textit{asynchronous bifurcation processor} (processor perspective). In this paper, the term \textit{asynchronous bifurcation processor} (ab. ABP) is used since this paper presents an asynchronous sequential logic that reproduces nonlinear sound processing of the cochlea partition. Advantages and significances of the ABP-based neural system model include the following points [25–32] (see also Table I and Fig. 2).

- The ABP-based neural system model is designed to have a much smaller resolution of the discrete state space than a digital processor neural system model. However, as illustrated in
Fig. 2, the asynchronicity of the transitions of the discrete states can realize a smooth vector field. Conceptually speaking, in order to realize a smooth vector field and a smooth bifurcation, the ABP-based neural system model *wisely utilizes the continuousness of the time axis*, whereas the digital processor neural system model *straightforwardly utilizes a high resolution discrete state space*. This is the key concept of the ABP-based neural system model.

- The ABP-based neural system model can be implemented by a reconfigurable hardware such as *field programmable gate array* (ab. FPGA). A control parameter of the model is a pattern of reconfigurable wires in the sequential logic circuit, and thus the parameter can be dynamically updated by utilizing a dynamic reconfiguration function of a recent FPGA. Hence the ABP-based neural system model is suited for on-chip dynamic parameter update (on-chip learning). On the other hand, on-chip dynamic parameter update of an analog circuit neural system model is often cumbersome.

- It has been shown that the ABP-based neural system model consumes less hardware resources than the digital processor neural system model for a wide range of reasonable parameter values [31, 32]. Also, unlike the digital processor model, the ABP-based model uses no peripheral circuitry that plays no essential role to reproduce the nonlinear dynamics of a neural system.

- From an academic viewpoint, modeling of the nonlinear dynamics of a neural system by a novel way (e.g., modeling of the cochlea dynamics by the asynchronous cellular automaton like this paper) *per se* is an important fundamental research topic.

So, in this paper, an ABP-based model of a cochlea partition is presented. First, in Section 2, the cochlea partition model is presented and its basic nonlinear dynamics is explained. Second, in section 3, it is shown that the presented model can reproduce typical nonlinear responses of biological cochlea partitions, e.g., nonlinear DC response, nonlinear band-pass filtering, and adaptation. Third, in Section 4, FPGA experiments validate reproductions of these nonlinear responses. Note that this paper presents an ABP-based model of a cochlea partition for the first time, while our group has presented an ABP-based model of pitch shift in a sound processing nervous system [25] and an ABP-based model of a spiral ganglion cell attached to the cochlea [26].

### 2. Novel cochlea partition model based on ABP

In this paper, a novel cochlea partition model, the diagram of which is shown in Fig. 3, is presented. As shown in Fig. 3(a), the cochlea partition model is divided into two components: an outer hair cell model (ab. OHC model) in Fig. 3(b) and a basilar membrane plus inner hair cell model (ab. BM-IHC model) in Fig. 3(c).

#### 2.1 OHC model

As show in Fig. 3(b), the OHC model has two registers storing the following two discrete states.

- **Receptor potential of OHC model:** \( V_O \in \{0, 1, \cdots, M_O - 1\} \),
- **Recovery variable of OHC model:** \( U_O \in \{0, 1, \cdots, N_O - 1\} \),

where the subscript “\( O \)” is used to indicate the OHC model. The positive integers \( M_O \) and \( N_O \) determine the resolution of a discrete state space \( \{(V_O, U_O)| V_O \in \{0, 1, \cdots, M_O - 1\}, U_O \in \{0, 1, \cdots, N_O - 1\}\} \). As shown in Fig. 3(b), the OHC model accepts the following periodic internal clock \( C_O(t) \) with period \( T_O \).

\[
C_O(t) = \begin{cases} 
1 & \text{if } t = 0, T_O, 2T_O, \cdots, \\
0 & \text{otherwise.}
\end{cases}
\]

As preparation to construct a vector field of the OHC model, the following functions are introduced.
The PDM (pulse density modulator) can be easily implemented by the asynchronous bifurcation processor [30] or a standard sigma-delta modulator [37] with a low hardware cost.

Fig. 3. (a) Cochlea partition model. (b) Outer hair cell (OHC) model. (c) Basilar membrane plus inner hair cell (BM-IHC) model.

Fig. 4. A typical sketch of a vector field of the OHC model triggered by the internal clock $C_O(t)$.

Fig. 5. Typical behavior of the OHC model. (a) Time-waveforms. (b) Phase space trajectory.
Functions used to construct a vector field of OHC model:
\[ n_{V_O}(V_O) = \alpha([k_a V_O + k_b], N_O), \quad n_{V_O}(V_O) = \alpha([k_c V_O + k_d], N_O), \]
\[ \alpha(z, N) = \begin{cases} 
-1 & \text{if } z < -1, \\
z & \text{if } -1 \leq z \leq N, \\
N & \text{otherwise,}
\end{cases} \]

where \(\lfloor \cdot \rfloor\) is the floor function and
\[ k_a = \frac{n_a N_O}{M_O}, \quad k_b = \lfloor n_b N_O \rfloor, \quad k_c = \frac{n_c N_O}{M_O}, \quad k_d = \lfloor n_d N_O \rfloor. \]

Then transitions of the discrete states \((V_O, U_O)\) of the OHC model triggered by the internal clock \(C_O(t)\) are described by the following equation.

**State transitions of OHC model triggered by internal clock** \(C_O(t)\):

\[
V_O(t^+) := \begin{cases} 
0 & \text{if } V_O(t) = 0, \\
M_O - 1 & \text{if } V_O(t) = M_O - 1, \\
V_O(t) - 1 & \text{if } V_O(t) \neq 0, \\
V_O(t) + 1 & \text{if } V_O(t) \neq M_O - 1, \\
V_O(t) & \text{otherwise},
\end{cases} \quad (2)
\]

\[
U_O(t^+) := \begin{cases} 
0 & \text{if } U_O(t) = 0, \\
N_O - 1 & \text{if } U_O(t) = N_O - 1, \\
U_O(t) - 1 & \text{if } U_O(t) \neq 0, \\
U_O(t) + 1 & \text{if } U_O(t) \neq N_O - 1, \\
U_O(t) & \text{otherwise},
\end{cases}
\]

where the symbol “\(t^+\)” denotes “\(\lim_{t \to +0} t + \epsilon\)” and the symbol “:=” denotes “instantaneous state transition.” Figure 4 shows a typical sketch of a vector field of the OHC model triggered by the internal clock \(C_O(t)\). As shown in this figure, the internal clock \(C_O(t)\) makes the state vector \((V_O, U_O)\) rotate in the counterclockwise direction. As shown in Fig. 3(b), the OHC model accepts the following stimulation input \(S(t)\) originated from a tone sound stimulation.

**Stimulation input** \(S(t)\) **originated from tone sound stimulation:**
\[ S(t) = A \sin(2\pi ft), \]

where \(f > 0\) is a frequency and \(A \geq 0\) is an amplitude of the stimulation input \(S(t)\). As shown in Fig. 3(b), the OHC model has a pulse density modulator (ab. PDM), which can be easily implemented by the asynchronous bifurcation processor [30] or a standard sigma-delta modulator [37] with a low hardware cost and converts the stimulation input \(S(t)\) to the following stimulation spike-train \(S_O(t)\).

**Stimulation spike-train** \(S_O(t)\) **originated from tone sound stimulation:**
\[ S_O(t) = \begin{cases} 
1 & \text{if } t = \tau_O(1), \tau_O(2), \tau_O(3), \cdots, \\
0 & \text{otherwise},
\end{cases} \]

where the instantaneous density of its spike positions \(\{\tau_O(1), \tau_O(2), \tau_O(3), \cdots\}\) is given as follows.

The instantaneous spike density of \(S_O(t)\) is given by \(S(t) + B\),

where \(B \geq A\) is the DC component of the stimulation spike-train \(S_O(t)\). An example of the stimulation spike-train \(S_O(t)\) is shown in Fig. 5(a). In addition to the internal clock \(C_O(t)\), the stimulation spike-train \(S_O(t)\) also triggers the following transition of the receptor potential \(V_O\).
Transition of receptor potential $V_O$ triggered by stimulation spike-train $S_O(t)$:

\[
V_O(t^+) := \begin{cases} 
0 & \text{if } V_O(t) = 0, \\
V_O(t) - 1 & \text{otherwise.}
\end{cases}
\]  

Figure 5(a) shows typical time-waveforms of the discrete states $(V_O, U_O)$ triggered by the internal clock $C_O(t)$ and the stimulation spike-train $S_O(t)$. Also, Fig. 5(b) shows a typical phase space trajectory of the discrete state vector $(V_O, U_O)$. As shown in Fig. 3(b), the OHC model has another PDM, which can be easily implemented by the asynchronous bifurcation processor [30] or a standard sigma-delta modulator [37] with a low hardware cost and converts the discrete state $V_O(t)$ to the following output spike-train $Y(t)$.

\[
Y(t) = \begin{cases} 
1 & \text{if } t = \tau_Y(1), \tau_Y(2), \tau_Y(3), \cdots , \\
0 & \text{otherwise,}
\end{cases}
\]

where the instantaneous density of its spike positions $\{\tau_Y(1), \tau_Y(2), \tau_Y(3), \cdots \}$ is given as follows.

The instantaneous spike density of $Y(t)$ is given by $K V_O(t)$, where $K \geq 0$ is a parameter characterizing the connection strength from the OHC model to the BM-IHC model. Figure 5(a) shows a typical time-waveform of the output spike-train $Y(t)$.

2.2 BM-IHC model
As shown in Fig. 3(c), the BM-IHC model has three registers storing the following three discrete states.

- **Receptor potential of BM-IHC model:** $V_I \in \{0, 1, \cdots , M_I - 1\}$,
- **Recovery variable of BM-IHC model:** $U_I \in \{0, 1, \cdots , N_I - 1\}$,
- **Adaptation variable of BM-IHC model:** $W_I \in \{0, 1, \cdots , L_I - 1\}$,

where the subscript “$I$” is used to indicate the BM-IHC model. The positive integers $M_I, N_I$, and $L_I$ determine the resolution of a discrete state space $\{(V_I, U_I, W_I)| V_I \in \{0, 1, \cdots , M_I - 1\}, U_I \in \{0, 1, \cdots , N_I - 1\}, W_I \in \{0, 1, \cdots , L_I - 1\}\}$. As shown in Fig. 3(c), the BM-IHC model accepts the following periodic internal clock $C_I(t)$ with period $T_I$.

\[
C_I(t) = \begin{cases} 
1 & \text{if } t = 0, T_I, 2T_I, \cdots , \\
0 & \text{otherwise.}
\end{cases}
\]

As preparation to construct a vector field of the BM-IHC model, the following functions are introduced.

**Functions used to construct a vector field of BM-IHC model:**

\[
n_{V_I}(V_I, W_I) = \alpha([k_e V_I + k_f - k_g W_I], N_I), \quad n_{U_I}(V_I) = \alpha([k_h V_I + k_i], N_I),
\]

where

\[
k_e = \frac{n_e N_I}{M_I}, \quad k_f = \lfloor n_f N_I \rfloor, \quad k_g = \frac{n_g M_I}{L_I}, \quad k_h = \frac{n_h N_I}{M_I}, \quad k_i = \lfloor n_i N_I \rfloor.
\]

Then state transitions of the BM-IHC model triggered by the internal clock $C_I(t)$ are described by the following equation.
As shown in Fig. 3(c), the BM-IHC model accepts the following periodic adaptation clock $C_I(t)$ with period $T_A$.

\[ C_A(t) = \begin{cases} 1 & \text{if } t = 0, T_A, 2T_A, \cdots, \\ 0 & \text{otherwise.} \end{cases} \]

Then the transition of the adaptation variable $W_I$ of the BM-IHC model triggered by the adaptation clock $C_A(t)$ is described by the following equation.

\[ W_I(t^+) = \begin{cases} 0 & \text{if } W_I(t) = 0, \\ W_I(t) - 1 & \text{if } W_I(t) \neq 0, \\ W_I(t) & \text{otherwise,} \end{cases} \]

Figure 6 shows a typical sketch of a vector field (projected onto the $(V_I,U_I)$-plane) of the BM-IHC model triggered by the internal clock $C_I(t)$.

**State transitions of BM-IHC model triggered by internal clock $C_I(t)$:**

If $C_I(t) = 1$, then

\[ V_I(t^+) := \begin{cases} 0 & \text{if } V_I(t) = 0, \\ V_S & \text{if } V_I(t) = V_S, \\ V_I(t) - 1 & \text{if } V_I(t) \neq 0, \\ V_I(t) + 1 & \text{if } V_I(t) \neq V_S, \\ V_I(t) & \text{otherwise,} \end{cases} \]

\[ U_I(t^+) := \begin{cases} 0 & \text{if } U_I(t) = 0, \\ N_I - 1 & \text{if } U_I(t) = N_I - 1, \\ U_I(t) - 1 & \text{if } U_I(t) \neq 0, \\ U_I(t) + 1 & \text{if } U_I(t) \neq N_I - 1, \\ U_I(t) & \text{otherwise,} \end{cases} \]

\[ W_I(t^+) := \begin{cases} 0 & \text{if } W_I(t) = 0, \\ W_I(t) - 1 & \text{if } W_I(t) \neq 0, \\ W_I(t) & \text{otherwise.} \end{cases} \]
Transition of adaptation variable $W_I$ triggered by adaptation clock $C_A(t)$:

$$W_I(t^+) := \begin{cases} L_I - 1 & \text{if } W_I(t) = L_I - 1, \\ W_I(t) + 1 & \text{otherwise.} \end{cases}$$ (5)

As shown in Fig. 3(c), the BM-IHC model has a PDM, which can be easily implemented by the asynchronous bifurcation processor [30] or a standard sigma-delta modulator [37] with a low hardware cost and converts the stimulation input $S(t)$ to the following stimulation spike-train $S_I(t)$.

Stimulation spike-train $S_I(t)$ originated from tone sound stimulation:

$$S_I(t) = \begin{cases} 1 & \text{if } t = \tau_I(1), \tau_I(2), \tau_I(3), \cdots, \\ 0 & \text{otherwise,} \end{cases}$$

where the instantaneous density of its spike positions $\{\tau_I(1), \tau_I(2), \tau_I(3), \cdots\}$ is given as follows.

The instantaneous spike density of $S_I(t)$ is given by $-S(t) + B$, where $B \geq A$ is the DC component of the stimulation spike-train $S_I(t)$. An example of the stimulation spike-train $S_I(t)$ is shown in Fig. 7(a). The stimulation spike-train $S_I(t)$ triggers the following transition of the receptor potential $V_I$.

Transition of receptor potential $V_I$ triggered by stimulation spike-train $S_I(t)$:

$$V_I(t^+) := \begin{cases} V_S & \text{if } V_I(t) = V_S, \\ V_I(t) + 1 & \text{otherwise.} \end{cases}$$ (6)

As shown in Fig. 3(a), the OHC model is connected to the BM-IHC model via the spike-train $Y(t)$. The spike-train $Y(t)$ from the OHC model triggers the following transition of the receptor potential $V_I$ of the BM-IHC model.

Transition of receptor potential $V_I$ triggered by spike-train $Y(t)$ from OHC model:

$$V_I(t^+) := \begin{cases} 0 & \text{if } V_I(t) = 0, \\ V_I(t) - 1 & \text{otherwise.} \end{cases}$$ (7)
Figure 7(a) shows typical time-waveforms of the discrete states \((V_I, U_I, W_I)\) triggered by the internal clock \(C_I(t)\), the adaptation clock \(C_A(t)\), the stimulation spike-train \(S_I(t)\), and the output spike-train \(Y(t)\) from the OHC model. Also, Fig. 7(b) shows a typical phase space trajectory (projected onto the \((V_I, U_I)\)-plane) of the discrete state vector \((V_I, U_I, W_I)\).

2.3 Remark on design philosophy

There exist many physiologically plausible neural system models the dynamics of which are described by higher dimensional differential equations. On the other hand, the presented model in this paper is rather a phenomenological model, which is designed under the following design philosophy.

**Low dimensional phenomenological model:** Recall that the center manifold theorem [38] guarantees that a local bifurcation in a higher dimensional dynamics occurs in a low dimensional center manifold, where the dimension of the manifold is identical with the number of the eigenvalues on the imaginary axis at the bifurcation. Hence, in order to design a phenomenological neural system model, a small number of state variables can be used to reproduce bifurcations in higher dimensional physiologically plausible neural system models. Actually, low dimensional cochlea partition models based on the normal form of the Hopf bifurcation have been presented successfully [17, 18]. Inspired by such models, in this paper, the state variables \(\{V_O, U_O\}\) in the OHC model and the state variables \(\{V_I, U_I\}\) in the BM-IHC model are used to realize the Hopf-bifurcation-type rotating trajectories (not saddle-node-bifurcation-type monotonic trajectories).

**One-way coupling:** It has been shown that the OHC plays essential roles to realize the nonlinear band-pass filtering of the cochlea [1, 2]. In order to model coupling mechanisms from the OHC to the IHC, one-way coupled two oscillators have been utilized successfully [7]. Inspired by such a model, in this paper, the one-way coupled model from the OHC model to the BM-IHC model is used to realize the nonlinear band-pass filtering.

**Nonlinear DC characteristics and adaptation:** The saturation set \(V_S\) is used to realize the nonlinear DC rectifying characteristics of the presented model as is often done to realize a rectifying characteristics. Also, the state variable \(W_I\), which moves slower than the state variables \(\{V_I, U_I\}\), is used to realize the slow adaptation dynamics of the presented model as is often done to realize a slow dynamics.

**Meanings of state variables:** Although the presented model is a phenomenological one, the state variables may have the following hypothetical physiological meanings: the state variables \(\{V_O, V_I\}\) are hypothetically regarded as receptor potentials of the OHC and the IHC, and the state variables \(\{U_O, U_I\}\) are hypothetically regarded to represent other state variables of the OHC and the IHC. Note that such hypothetical regards are inspired by the representative phenomenological neuron model, the Izhikevich model [39], which has a membrane potential and a recovery variable.

2.4 Remark on asynchronous bifurcation processor

The nonlinear dynamics of the presented cochlea partition model is described by Eqs. (2)–(7) and is characterized by the following parameter vector \(p\):

\[
p = (M_O, N_O, M_I, N_I, L_I, n_a, n_b, n_c, n_d, n_e, n_f, n_g, n_h, n_i, V_S, V_{th}, T_O, T_I, T_A, K).
\]

Also, the stimulation spike-trains \(S_O(t)\) and \(S_I(t)\) are characterized by the following parameters:

\[
f, A, B.
\]

It should be emphasized that the transitions of the discrete states \((V_O, U_O, V_I, U_I, W_I)\) are triggered by the clocks \(C_O(t), C_I(t), \) and \(C_A(t)\), which are not necessarily synchronized and generically have different periods. In addition, the transitions of the receptor potentials \((V_O, V_I)\) are triggered by the stimulation spike-trains \((S_O(t), S_I(t))\), which are originated from a sound stimulation (natural input). That is, the presented model has the discrete states the transitions of which are triggered asynchronously. Hence, the presented model can be regarded as “a cochlea partition model based on the concept of the asynchronous bifurcation processor” (see Table I and Fig. 2).
Fig. 8. A conceptual sketch (not a scanned picture) of a nonlinear DC response of a receptor potential of an inner hair cell of a bull frog [4]. The horizontal axis shows a displacement of a stereocilia and the vertical axis shows the receptor potential of the inner hair cell.

Fig. 9. Nonlinear DC response of the presented cochlea partition model. The parameters of the cochlea partition model are \( p = (128, 128, 256, 256, 1024, -0.5, 0.64, 16, 0, -0.9, 0.78, 0.02, 16, -7.45, 135, 85, 0.2 \times 10^{-6}, 0.2 \times 10^{-6}, 0.04 \times 10^{-3}, 625 \times 10^3) \). The frequency of the stimulation input \( S(t) \) is \( f = 1 \times 10^4 \). (a), (b), and (c) are phase space trajectories of the BM-IHC model projected onto the \((V_I, U_I)\)-plane. In (a), (b), and (c), the inverses \( B^{-1} \) of the DC components \( B \) of the stimulation spike-train are \( 0.28 \times 10^{-6}, 0.2 \times 10^{-6}, \) and \( 0.12 \times 10^{-6} \), respectively, and \( A = B \). (d) Nonlinear DC response of the presented cochlea partition model. The points correspond to (a), (b), and (c), respectively.
3. Reproductions of nonlinear responses of biological cochleae

As mentioned in the introduction section, this paper focuses on reproductions of the three typical nonlinear responses of biological cochleae, i.e., the nonlinear DC response, the nonlinear band-pass filtering, and the adaptation.

3.1 Reproduction of nonlinear DC response

Figure 8 shows a conceptual sketch of the characteristics of a receptor potential of an inner hair cell of a bull frog [4]. It can be seen that the receptor potential of the inner hair cell (vertical axis) responds nonlinearly to a displacement of a stereocilia (horizontal axis). On the other hand, Fig. 9(a), (b), and (c) show trajectories \((V_I, U_I)\) of our cochlea partition model for different values of the DC component \(B\) of the density of the stimulation spike-train \(S_I(t)\). In order to characterize such trajectories, the following RMS is introduced.

\[
\text{RMS} = \sqrt{\frac{1}{T} \int_0^T (V_I(t) - V_S)^2 dt}
\]

where \(T\) is sufficiently large. In the cases of Figs. 9(a), (b), and (c), the \(\text{RMS}\) are approximately 21, 37, and 60, respectively. Figure 9(d) shows the characteristics of the \(\text{RMS}\) for the DC component \(B\) of the stimulation. Note the following correspondences between Figs. 8 and Fig. 9(d).

<table>
<thead>
<tr>
<th>Axis</th>
<th>Characteristics of biological inner hair cell in Fig. 8</th>
<th>Characteristics of our model in Fig. 9(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>Displacement of stereocilia (DC stimulation to stereocilia)</td>
<td>DC component (B) of stimulation</td>
</tr>
<tr>
<td>Vertical</td>
<td>Receptor potential of inner hair cell (V_I) of BM-IHC model</td>
<td>(\text{RMS}) of receptor potential (V_I) of BM-IHC model</td>
</tr>
</tbody>
</table>

From the above correspondences, Fig. 9(d) can be regarded as a nonlinear DC response of the receptor potential \(V_I\) of our cochlea partition model. Then, it can be seen in Figs. 8 and Fig. 9(d) that our cochlea partition model can reproduce the nonlinear DC response of the receptor potential of the inner hair cell in a biological cochlea.

3.2 Reproduction of nonlinear band-pass filtering

Figure 10 shows conceptual sketches of the characteristics of two cochlea partitions of a chinchilla cat for input sound frequencies [5]. It can be seen that the cochlea partitions work as nonlinear band-pass filters and thus the characteristics curves in Fig. 10 are called tuning curves [2]. On the other hand, Figs. 11(a), (b), and (c) show trajectories \((V_I, U_I)\) of our cochlea partition model for different values of the frequency \(f\) of the stimulation input \(S(t)\). In order to characterize such trajectories, the following minimum stimulation level \(A_{LV}\) is introduced.

The minimum stimulation level \(A_{LV}\) is the minimum amplitude \(A\) of the stimulation input \(S(t)\) such that the \(\text{RMS}\) of the BM-IHC model is greater than or equal to a threshold value \(\text{RMS}_{th}\).

Figures 11(d) and (e) show characteristics curves of the minimum stimulation level \(A_{LV}\) for the frequency \(f\) of the stimulation input \(S(t)\). Note the following correspondences among Figs. 10, 11(d), and 11(e).

<table>
<thead>
<tr>
<th>Axis</th>
<th>Biological tuning curves in Fig. 10</th>
<th>Tuning curves of our cochlea partition model in Figs. 11(d) and (e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>Frequency of sound stimulation</td>
<td>Frequency (f) of stimulation input (S(t))</td>
</tr>
<tr>
<td>Vertical</td>
<td>The minimum sound level inducing a certain level (threshold level) of firing rate of spiral ganglion cells</td>
<td>The minimum stimulation level (A_{LV}) for a threshold value (\text{RMS}_{th}) of the (\text{RMS}), where the (\text{RMS}) determines a firing rate of a spiral ganglion cell model [26]</td>
</tr>
</tbody>
</table>
Fig. 10. Conceptual sketches (not scanned pictures) of tuning curves of two cochlea partitions of a chinchilla cat [5]. The horizontal axis shows a frequency of a tone sound stimulation and the vertical axis shows the minimum level of the sound stimulation that induces a certain predetermined level of spike density of spiral ganglion cells.

Fig. 11. Nonlinear band-pass filtering of the presented cochlea partition model. (a), (b), and (c) are phase space trajectories of the BM-IHC model projected onto the \((V_I, U_I)\)-plane. In (a), (b), and (c), the parameters are \(p = (128, 128, 256, 256, 1024, -0.5, 0.64, 16, 0, -0.5, 0.63, 0.32, 14, -6.45, 135, 65, 2.3 \times 10^{-6}, 2.6 \times 10^{-6}, 0.32 \times 10^{-3}, 333 \times 10^{3})\) and \(A = B = 417 \times 10^{3}\). Also, in (a), (b), and (c), the frequencies \(f\) of the stimulation input \(S(t)\) are \(0.1 \times 10^{3}\), \(1 \times 10^{3}\), and \(10 \times 10^{3}\), respectively. (d) and (e) are tuning curves of the presented cochlea partition model. In (d), the parameters are \(p = (128, 128, 256, 256, 1024, -0.5, 0.64, 16, 0, -0.9, 0.78, 0.02, 16, -7.45, 135, 85, 2.3 \times 10^{-6}, 2.5 \times 10^{-6}, 0.4 \times 10^{-3}, 333 \times 10^{3})\) and the threshold for the \(RMS\) is \(RMS_{th} = 27.5\). In (e), the parameters are \(p = (128, 128, 256, 256, 1024, -0.5, 0.64, 16, 0, -0.9, 0.78, 0.02, 16, -7.45, 135, 85, 0.26 \times 10^{-6}, 0.28 \times 10^{-6}, 40 \times 10^{-6}, 4.3 \times 10^{6})\) and the threshold for the \(RMS\) is \(RMS_{th} = 27.5\).
Fig. 12. A conceptual sketch (not a scanned picture) of a post stimulus time histogram of a cochlea partition of a cat [1].

Fig. 13. Post stimulus time histogram of our cochlea partition model. The parameters are $p = (128, 128, 256, 256, 1024, -0.5, 0.64, 16, 0, -0.9, 0.78, 0.02, 16, -7.45, 135, 85, 0.2 \times 10^{-6}, 0.2 \times 10^{-6}, 40 \times 10^{-6}, 625 \times 10^{-3})$.

From the above correspondences, Figs. 11(d) and (e) can be regarded as tuning curves of our cochlea partition model. Then, it can be seen in Figs. 10, 11(d), and 11(e) that our cochlea partition model can reproduce the nonlinear band-pass filter characteristics of the biological cochlea.

3.3 Reproduction of adaptation

Figure 12 shows a conceptual sketch of a histogram of the number of spikes from a cochlea partition of a cat for a tone burst input [1], where such a histogram is called a post stimulus time histogram [1]. It can be seen that the cochlea partition responds strongly at the onset of the tone burst input and the strength of the response gradually decreases. Such a response is called an adaptation. In order to characterize an adaptation of our cochlea partition model, the following sampled receptor potential $\hat{V}_I$ is introduced.

$$\hat{V}_I(nD) = \int_{nD}^{(n+1)D} |V_I(t) - V_S| dt, \quad n = 0, 1, 2, \cdots,$$

where $D$ is a bin width. Figure 13 shows a time waveform of the sampled receptor potential $\hat{V}_I$ of our cochlea partition model. Note the following correspondences between Figs. 12 and 13.

<table>
<thead>
<tr>
<th>Axis</th>
<th>Biological post stimulus time histogram in Fig. 12</th>
<th>Sampled receptor potential $\hat{V}_I$ in Fig. 13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>Time</td>
<td>Time</td>
</tr>
<tr>
<td>Vertical</td>
<td>Number of spikes of spiral ganglion cells in each bin</td>
<td>Average of $</td>
</tr>
</tbody>
</table>

From the above correspondences, Fig. 13 can be regarded as a post stimulus time histogram of our cochlea partition model. Then, it can be seen in Figs. 12 and 13 that our cochlea partition model can
Table 14. The upper table shows hardware description language (ab. HDL) synthesis conditions and a resulting design summary. The lower figure shows a register transfer level (RTL) block-level architecture, where bit lengths of the state variables and the signals are labeled for the cases of lengths greater than 1. A standard sigma-delta modulator [37] is used as each pulse density modulator (ab. PDM).

![Fig. 14](image)

Fig. 15. Oscilloscope snapshots of waveforms from an FPGA-implemented cochlea partition model. The parameter values correspond to those in Fig. 11(b). (a) Time-waveform of the receptor potential $V_O$ of the OHC model. (b) Time-waveform of the receptor potential $V_I$ of the BM-IHC model. (c) Phase space trajectory of the BM-IHC model corresponding to that in Fig. 11(b).

![Fig. 15](image)
reproduce the adaptation characteristics of the biological cochlea.

4. FPGA implementation

4.1 FPGA implementation and measurements

Recall that the state transitions of the presented cochlea partition model are described by Eqs. (2)–(7). These equations are written in a VHDL source code, which is compiled by Xilinx’s design software ISE 14.4. Figure 14 shows a resulting design summary and a resulting register transfer level (ab. RTL) block-level architecture, where bit lengths of the state variables are labeled. In this figure, a standard sigma-delta modulator [37] is used as each pulse density modulator (ab. PDM), where the PDMs accept 8-bit sinusoidal input $S(t)$ and its inverted one $-S(t)$, and output density modulated one-bit outputs $S_I$ and $S_O$. As shown in this figure, the presented model accepts the one-bit inputs $\{C_I, C_O, C_A\}$ and the 8-bit input $S$, and outputs the 8-bit output $V_I$. The resulting bitstream file is downloaded to Xilinx’s FPGA XC7Z020-1CLG484. Figure 15 shows oscilloscope snapshots of waveforms from the FPGA-implemented cochlea partition model, where the parameter values correspond to those in Fig. 11(b). It can be seen in Figs. 15(c) and 11(b) that the FPGA-implemented cochlea partition model operates properly. Also, we have confirmed that the FPGA-implemented cochlea partition model can reproduce the three nonlinear responses of biological cochlea partitions, i.e., the nonlinear DC response of the receptor potential, the nonlinear band-pass filtering, and the adaptation.

4.2 Remark on differences from other hardware cochlea models

(a) There exist many hardware cochlea models, where most of them can be categorized into analog circuit models [12, 13, 21] and digital processor models [14, 22–24]. Major control parameters of the analog circuit cochlea models are parameters of linear circuit elements (e.g., resistance) and nonlinearities of nonlinear circuit elements (e.g., shape of nonlinear voltage-current curve), where their dynamic updates are troublesome after implementation. Hence, the presented ABP-based cochlea model has advantages in capabilities of after-implementation or even after-implant dynamic parameter updates.

(b) The digital processor cochlea models need multipliers with long bit-lengths to reproduce smooth vector fields as well as non-hyperbolic behaviors near the local bifurcations. On the other hand, our group has shown that many ABP-based neural system models consumes less hardware resources compared to digital processor neural system models [27, 32]. Hence, the presented ABP-based cochlea model is to have advantages in hardware cost over the digital processor models, where detailed comparisons of the hardware cost are beyond the scope of this paper and will be done in a future paper.

5. Conclusions

The novel cochlea partition model based on the concept of the asynchronous bifurcation processor was presented. The model consists of the two components (i.e., the OHC model and the BM-IHC model) and thus it can be regarded as a coupled system of two asynchronous cellular automaton oscillators. It was shown that the presented model can reproduce the typical nonlinear responses observed in the biological cochlea, i.e., the nonlinear DC response of the inner hair cell, the nonlinear band-pass filtering of the cochlea partition, and the adaptation of the cochlea partition. Also, the FPGA experiments validated the reproductions of these nonlinear responses. Future problems include: (a) detailed bifurcation analysis of the presented model, (b) development of a systematic design method of the presented model, and (c) design of a custom chip of the presented model.

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References


